Dark energy as zero energy of gravity field

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Homogeneous isotropic empty space is described here based on its metric. The only parameter of this metric is taken to be equal to the Hubble constant. The tensor of energy-momentum of empty space is calculated. Upon this, the density of the empty homogeneous space turned out to be equal to the expected density of the Universe. In other words, the density of empty space is "dark energy". Of not less importance is the fact that the same metric imposes the exponentially increasing law of redshift asymptotically close to the Hubble law at low redshift values. The solution has no singularities.

Key words: dark energy, metric of Universe, redshift, accelerated expansion, density of Universe.

1. Introduction

There is a plain and simple kind of energy and this is the energy of the gravity field. In other words, this is the energy of an empty space, in which both substance and electromagnetic fields are absent.

Is this energy equal to zero in absence of a gravity field? There are no reasons for the energy of the empty space being equal to zero.

For calculating of the (allegedly equal to zero) energy of the empty space, the metric of the homogeneous isotropic space assists. This means that the dark energy is nothing but the energy of the empty space. Notably, it is possible that the dark matter is nothing but thickening (concentration) of the same dark energy, which – as any other energy – possesses a weight.

2. Homogeneous Universe

A.A. Friedman and A. Einstein considered the Universe filled up with substance. The paper [1] laid the foundations to the concept of the non-stationary Universe. The discovery of the cosmological red shift and A. Einstein's support [2] determined the development of cosmology for many years to come.

The effects of the zero fluctuations (of physical vacuum) have been confirmed very precisely by the well-known experiments. This is one of the greatest successes of the scientific physics. However, the attempts of a vacuum energy density determination failed both in calculations and experiments. Any estimations result in incredibly large values of the vacuum density. To be successful in connecting of the vacuum energy with the Einstein's "empty space" is possible only with aid of the "imagination play", though often some authors consider this imagination to be rather ill.

Here we shall make an attempt to describe the Universe background space having no substance based only on the relativity principle and the Riemannian geometry.

The constancy of the speed of light is the necessary condition of the relativity principle. For the metric

$$ds^2 = dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2,$$

the speed constancy follows from the fact that the equation $ds^2 = 0$ describes the wave front having the fixed speed c.

Einstein and Fokker [3] noted that the reference system with the constant (local) speed of light is described with the metric of the following kind:

$$ds^2 = v^2 \left(dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 \right), \quad (1)$$

notably, v is function of position coordinates*.

The space with the metric (1) can be qualified as conformally-Galilean [4].

Let us give a consideration to the space with the homogeneous acceleration of the gravity field, i.e. with the homogeneous derivative coordinates on time of second order. In the coordinates system, in which a point has the zero speed, the components of the point acceleration look like as follows [5]:

$$\frac{d^2 x^i}{dt^2} - c^2 \Gamma_{00}^i = c^2 k_i .$$

Let us require that the metric (1) is homogeneous on acceleration, i.e. that the acceleration $c^2 k_i$ does not depend on coordinates. The following metric meets this condition: †

$$ds^2 = \exp\left(2k_i x_i\right)\left(dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 \right).$$

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* The authors [3] came from this result to the generally covariant Nordstrom equation. However, the Nordstrom scalar theory failed in the competition with the general relativity theory.

† This metric can be considered as a generalization of the exact result obtained by Einstein as early as in 1907 [6] for the homogeneous field of accelerations $d\tau = \exp\left(ax / c^2\right) dt$. Besides, it is compatible with the homogeneous one-dimensional metric by Harry Lass [7].

In the case that $k_i = 0$ is the metric of the Minkowski space, if at least one of $k_i \neq 0$, anyway, $k_0 = 0$ is the metric of the homogeneous accelerated system [8]. The assumption that the Universe as a whole has non-zero acceleration in one direction, is most unbelievable. The only possible metric left is one with the non-zero component of the vector k_0, which takes the following form:

$$ds^2 = \exp\left(2k_0 x_0\right)\left(dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2\right). \qquad (2)$$

The constant k_0 has the same dimensionality as the Hubble constant H_0. Now let us make the most reasonable step and replace the constant in the equation (2) with the Hubble constant.

$$ds^2 = \exp\left(2H_0 x_0\right)\left(dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2\right). \qquad (3)$$

The metric (3) describes the space-time continuum with the monotonously increasing scale factor (see Fig.). It is essential that the Einstein’s tensor of the metric (3) is non-zero.

3. Energy of empty space

It is well-known that the covariant divergence of the Einstein’s tensor is equal to zero. This allows assuming that, following Einstein, it is proportional to the tensor of energy-momentum. The proportionality rate between these values is known from the asymptotical behavior of these tensors in the Newtonian approximation[‡]. These considerations allow calculating the tensor of energy-momentum as follows:

‡ The reversed Einstein equation.

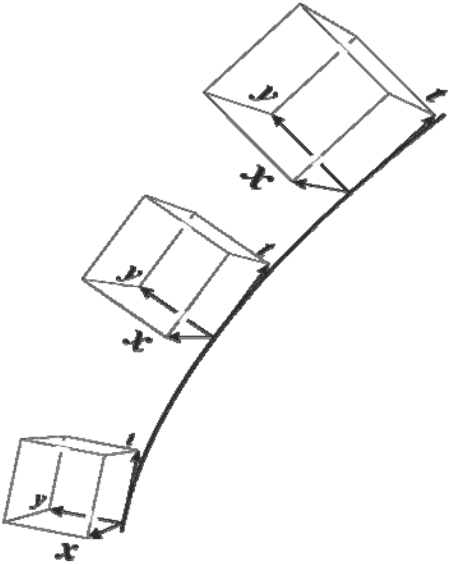

Fig. Attempt to depict space evolution with growing scale factor

$$t_{ik} = \frac{c^4}{8\pi G}\left(R_{ik} - \frac{1}{2}g_{ik}R\right) = \begin{pmatrix} 3p & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}, \qquad (4)$$

notably $p = \frac{c^2 H_0^2}{8\pi G}$. The expression (4) does not depend on coordinates and thus indeed the metric (2) describes the homogeneous space-time continuum having a constant positive density and a negative pressure in every point of the space. The metric (2) has a negative scalar curvature $R = -6k_0^2 \exp\left(-2k_0 x\right)$.

Surprisingly, the space density of the homogeneous metric $\rho = 3p / c^2$ coincides with the Friedman's critical density [1]

$$\rho = \frac{3H_0^2}{8\pi G}.$$

This confirms the ideas by A. A. Friedman. The value ρ coincides with the expected value of sum of densities of the dark energy and the dark matter in our Universe. Hence, ρ is the natural contender for the place of the **dark energy** of a homogeneous Universe.

In the real Universe, the tensor of energy-momentum (4) must acquire a non-zero energy flow, for example, at action of a gravity field. This way, a non-homogeneous density ρ distribution in the space is induced, mainly in massive galaxies or clusters of galaxies. This is already quite plausible model of the **dark matter**.

4. Cosmological red shift

From the same metric of empty space (3), it follows that for a fixed in space observer, the source own time is connected with the observer's time on the distance x_0 via the expression:

$$d\tau = \exp\left(k_0 x_0\right) dt.$$

If $T = x_0 / c$ is the signal delay time, then if to assume that $ck_0 = H_0$, we shall obtain the red shift being close to the Hubble law:

$$z = \exp\left(H_0 T\right) - 1 \approx H_0 T. \tag{5}$$

However, with the distance increase to a source, the red shift grows faster than it is predicted by the Hubble law, which – in fact – corresponds to the observations.

5. Conclusion

It is readily expected that the tensor of energy-momentum (3) of a homogeneous field is constant through the entire space. From this result, it follows that an observer – *in any space point and at any instant of time* – would find himself in the same conditions and would observe the identical red shift (5). This shift is equivalent to the expanding at a growing rate Universe. We see that this is not necessary that the Hubble law extrapolation to the past makes the Universe shrinking to a point.

Let us pay attention that this result was obtained with relying on neither the Friedman equation nor any hypotheses with use of the lambda member.

Perhaps this is the beginning of a new point of view on the theory of relativity [8], [9].

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