How the Generalized Maxwell Equations Can Be Derived from the Einstein Postulate?*

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We show that the Gersten derivation of Maxwell equations can be generalized. It actually leads to additional solutions of `S=1 equations'.

In a recent paper [1] the author (Dr. A. Gersten) studied the matrix representation of the Maxwell equations, both the Faraday and Ampere laws and the Gauss law. His consideration is based on equation (9)

$$\left[(E/c)^2 - \mathbf{p}^2\right]\mathbf{\Psi} = \left[(E/c)I^{(3)} - \mathbf{p}\cdot\mathbf{S}\right]\left[(E/c)I^{(3)} + \mathbf{p}\cdot\mathbf{S}\right]\Psi - \mathbf{p}\left(\mathbf{p}\cdot\mathbf{\Psi}\right) = 0 \qquad \text{(Eq. (9) of ref. [1])}$$

which is equivalent to the Einstein postulate on the relation between energy, momentum and mass.

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Furthermore, he claimed that the solutions to this equation should be found from the set

$$\left(\frac{E}{c} I^{(3)} + \mathbf{p}\cdot\mathbf{S}\right)\mathbf{\Psi} = 0 \qquad \text{(Eq. (10) of ref. [1])}$$

$$(\mathbf{p}\cdot\mathbf{\Psi}) = 0 \qquad \text{(Eq. (11) of ref. [1])}$$

Thus, Gersten concluded that his equation (9) is equivalent to the Maxwell equations (10,11). As he also correctly indicated, such a formalism for describing S = 1 fields has been considered by several authors before. See, for instance, [2-10]; with those authors mainly considering the dynamical Maxwell equations in the matrix form.

However, we straightforwardly note that the equation (9) of [1] is satisfied also under the choice[1]

$$\left(\frac{E}{c} I^{(3)} + \mathbf{p}\cdot\mathbf{S}\right)\mathbf{\Psi} = \mathbf{p}\chi\,, \tag{1}$$

$$(\mathbf{p}\cdot\mathbf{\Psi}) = \frac{E}{c}\chi\,, \tag{2}$$

with some arbitrary scalar function χ at this stage. This is due to the fact that[2]

$$(\mathbf{p}\cdot\mathbf{S})^{jk}\mathbf{p}^{k} = i\varepsilon^{jik}\mathbf{p}^{i}\mathbf{p}^{k} = 0 \tag{3}$$

(or after quantum operator substitutions **rot grad** χ =0). Thus, the generalized coordinate-space Maxwell equations follow after a similar procedure as in [1]:

$$\nabla \times \mathbf{E} = -(1/c)\,\partial\,\mathbf{B}/\partial t + \nabla\, Im\,\chi \tag{4}$$

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[1] We leave the analysis of possible functional non-linear (in general) dependence of χ and $\partial\chi/\partial x^{\mu}$ on the higher-rank tensor fields for future publications.

[2] See the explicit form of the angular momentum matrices in Eq. (6) of the Gersten paper.

$$\nabla \times \mathbf{B} = +(1/c)\, \partial \mathbf{E}/\partial t + \nabla\, Re\, \chi \qquad (5)$$

$$\nabla \cdot \mathbf{E} = -(1/c)\, \partial Re\chi/\partial t \qquad (6)$$

$$\nabla \cdot \mathbf{B} = +(1/c)\, \partial Im\chi/\partial t \qquad (7)$$

If one assumes that there are no monopoles, one may suggest that $\chi(x)$ is a *real* field and its derivatives play the role of charge and current densities. Thus, surprisingly, on using the Dirac-like procedure[3] of derivation of "free-space" relativistic quantum field equations, Gersten might in fact have come to the *inhomogeneous* Maxwell equations![4] Furthermore, we are not aware of any proofs that the scalar field $\chi(x)$ should be firmly connected with the charge and current densities, so there is sufficient room for interpretation. For instance, its time derivative and gradient may also be interpreted as leading to the 4-vector potential. In this case, we need some *mass*/*length* parameter as in [11a,d]. Both these interpretations were present in the literature [9,11] (cf. also [12]).

The conclusion is: it is the *generalized* Maxwell equations (many versions of which have been proposed in the last 100 years, see, for instance, [13]) that should be used as a guideline for proper interpretations of quantum theories.

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3 That is to say, on the basis of the relativistic dispersion relations $(E^2 - \mathbf{p}^2 c^2 - m^2 c^4)\Psi = 0$, Eq. (1) of ref. [1].

4 One can also substitute -(2 i h /c) **j** and - (2 i h) ρ in the right hand side of (1,2) of the present paper and obtain equations for the current and the charge density $(1/c)\, \nabla \times \mathbf{j} = 0$ and $\frac{1}{c^2}\frac{\partial \mathbf{j}}{\partial t} + \nabla\rho = 0$, which coincide with equations (13,17) of [9b]. The interesting question is: whether such defined **j** and ρ may be related to $\partial\chi / \partial x^\mu$.

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