The Relativity Principle and the Kinetic Scaling of the Units of Energy, Time and Length

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The special theory of relativity (STR) has successfully predicted that proper clocks run more slowly as their speed relative to an observer on the earth's surface increases. A straightforward means of describing this phenomenon is to assume that the unit of time varies from one inertial system to another. A key theoretical assumption that has been verified by means of experiments with airplanes and satellites is that all clocks in a given rest system slow down in exactly the same proportion (Q) when they are accelerated, thereby making it impossible to observe any change in their rates on the basis of exclusively *in situ* measurements, consistent with the relativity principle (RP). The same line of argumentation leads to a similar conclusion for *in situ* length measurements, since any change in the distance between two objects co-moving with an observer must be matched by a strictly proportional change in the standard device employed to measure it. It is argued on this basis that the unit of length must vary with the state of motion in the same proportion Q as that of time in order for the speed of light to be constant for all observers (Einstein's second postulate of STR). The use of such a (rational) set of units in each rest frame requires that a different space-time transformation (Global Positioning System-Lorentz transformation GPS-LT) be introduced into relativity theory from that given by Einstein in his original work. Moreover, the unit of energy must also vary in exactly the same manner as that of time (Q), since the energies of accelerated objects are known to increase in direct proportion to their lifetimes, even though *in situ* measurements are again incapable of detecting such changes because of the RP. The latter conclusion indicates that the ratio of the energy of photons to their frequency (Planck's constant h) varies as Q^2 with the state of motion of the light source relative to the observer, and an experiment involving the photoelectric effect is suggested to test this prediction. More generally, the ratio of the units of any other mechanical quantity must vary as Q^n between the same two rest frames, where n is an integer determined from the composition of this quantity in terms of the basic units of energy, time and length.

I. Introduction

The first postulate of Einstein's special theory of relativity (STR [1]) states that the laws of physics are the same in all inertial systems. In essence, Einstein was simply taking over Galileo's relativity principle (RP) of the early 17th century and adapting it to the interpretation of experiments that had only become possible nearly 300 years later, particularly those in the field of electricity and magnetism. One of the most interesting predictions of the new mechanical theory was that the rates of natural clocks depend on their state of motion. The simplest way to understand this *time dilation* effect in the context of the RP is to assume that the *unit of time varies* with the speed of the clock relative to the observer.

Accordingly, the laws of physics are indeed the same in all inertial systems, but the system of physical units in which they are expressed varies in a systematic manner between different rest frames.

In this connection, it is important to recall that Einstein's second postulate of STR [1] states that the speed of light c is independent of the state of motion of the observer. On this basis, the unit of length must vary in the same manner as the unit of time, since only then can the speed of light be the same for two observers with different clock rates. This conclusion raises an interesting question about the Fitzgerald-Lorentz contraction effect (FLC) of STR [1], however. The latter holds that two observers will disagree on the magnitudes of distances measured along the line of their relative motion, but not those in a perpendicular orientation. *How then can each observer's respective unit of length be the same in all directions?* The object of the following discussion is to develop an internally consistent relativistic theory that takes account of the way in which the units of all physical quantities vary with the state of motion of the observer, while still remaining consistent with the above two postulates of STR. It will be seen that the only way to accomplish this goal is to adopt a space-time transformation that is consistent with the principle of remote simultaneity of events.

II. Simultaneity and the Unit of Time

In order to make the definition of physical units quantitative, it is essential to understand how a given quantity varies with an object's speed v relative to an observer O. According to STR [1], for example, the decay lifetime τ of a meta-stable system varies as $\gamma(\mathrm{v}) = \left(1 - \mathrm{v}^2 c^{-2}\right)^{-0.5}$, where c is the speed of light in free space (299792458 ms^{-1}). Because of the RP, however, an observer M traveling with the object will measure its lifetime to be τ, the same value that O measures when $v = 0$. Consistency is restored by simply agreeing that M's unit of time is $\gamma(v)$ times larger than O's. In order for these definitions to be of any practical value, however, it is necessary for the resulting system of units to be *rational*. This means that *the ratio of elapsed times* for different events measured by two observers in relative motion must always be the same. This clock-rate ratio principle (CRP) is the underlying theoretical assumption of the Global Positioning System (GPS) for measuring distances on the earth's surface to a high degree of accuracy, as has been discussed in a recent publication [2].

A simple way to make this procedure quantitative is to assign a proportionality constant α_O to each inertial system O. This constant shall be referred to in the following as *its clock-rate parameter*. It may be defined, for example, to have a value of unity in a standard laboratory E located on the earth's surface ($\alpha_E = 1$). The ratio of the elapsed time on a proper clock that is stationary in E to that obtained with an identical clock which is stationary in O's rest frame is then defined to have a value of α_O. For example, in the familiar case in which O has been accelerated to speed v relative to E, $\alpha_O = \gamma(v)$. It is implicitly assumed thereby that it does not matter what kind of (proper) clock is used for this purpose.

If we refer to the corresponding elapsed times measured by observers O and M as T(O) and T(M), respectively, the following relationship then holds:

$$T(O) = \frac{\alpha_M}{\alpha_O} T(M). \tag{1}$$

One can express the same relationship by stating that each observer's unit of time is proportional to the clock-rate parameter associated with his rest frame. The unit of time in the standard laboratory itself is defined to be 1.0 s, so the corresponding unit in rest frame O is α_O s. According to eq. (1), the measured value of an elapsed time is *inversely proportional* to the unit of time in that rest frame. The situation is exactly the same as when measured values of distances and masses in a given rest frame are reported; they are also inversely proportional to the respective unit in which they are expressed.

The conclusion based on eq. (1) is that *absolute* timings are actually the same for all observers in different inertial systems. In other words, O and M have different *numerical* values for the elapsed time of a given event, but this is only because the units in which they are expressed are not the same. This position is verified by the workings of the GPS navigation technology. The "pre-corrected" clock on a satellite simply runs at a different rate from its uncorrected counterpart, namely at the same rate as an identical proper clock on the earth's surface. Thus, events that are simultaneous for an observer at rest on the satellite must also occur simultaneously for an observer on the earth's surface [2].

The fact that Einstein's STR [1] foresees the opposite result (non-simultaneity) based on the Lorentz transformation (LT) therefore proves that this formulation of relativity theory cannot be applied successfully in this important case. More generally, it should be noted that the LT also predicts that clock rates in different rest frames will differ by a constant factor (time dilation). Therefore, elapsed times Δt and $\Delta t'$ measured for the same event are predicted to satisfy a simple proportionality relation, i.e., $\Delta t = X\,\Delta t'$. Given this proportional time-dilation prediction of the theory, it is clearly unacceptable to claim in the same application that one of these time differences can be zero (simultaneous observation) without the other being so as well.

It is impossible for STR to operate consistently on the basis of a rational system of units. It has always been claimed on the basis of STR, for example, that two clocks in relative motion can both be running slower than the other at the same time (symmetry principle) [3]. This conclusion eliminates any possibility of employing eq. (1) to make timing comparisons, thus making it necessary to use complicated arguments to explain the results of experiments carried out with atomic clocks carried onboard airplanes and satellites [4,5] and still hold to the belief in the non-simultaneity of events for different observers in relative motion.

Fortunately, there is a simple way to incorporate simultaneity into relativity theory and still satisfy the two postulates that Einstein used to derive the LT [1]. One can take advantage of a degree of freedom in the definition of any space-time transformation, as pointed out by Lorentz [6] several years before Einstein's original paper on STR. The exact form of an alternative Lorentz transformation (ALT), or Global Positioning-Lorentz transformation (GPS-LT), that incorporates the principle of absolute simultaneity into relativity theory has been given in earlier work [2] and is illustrated by means of the diagram shown in Fig. 1.

The latter shows a light pulse traveling across a laboratory located on a satellite in which observer M is at rest. The satellite is moving with speed v along the x axis relative to another observer O. Both observers agree that the speed of the light pulse is c, in agreement with Einstein's second postulate. *They also agree to both use O's unit of time* in order to express their measured value for the elapsed time $dt = \Delta T(O)$ required for the light pulse to arrive at its detector on the satellite. The corresponding distance traveled by the light pulse from M's perspective is $dr' = A = cdt$. It

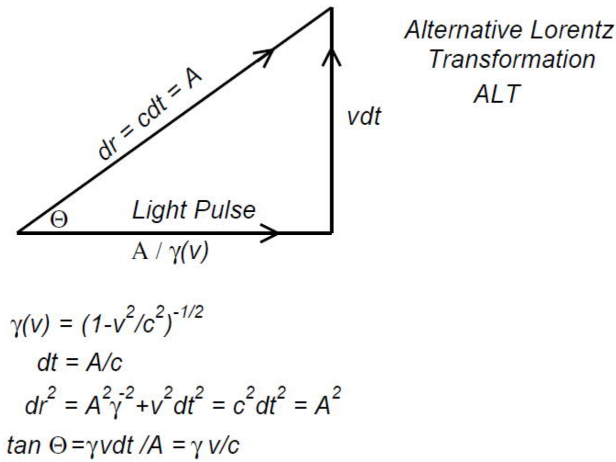

Fig. 1. Diagram showing a light pulse traveling in a transverse direction on a satellite that is moving with speed v relative to observer O. The distance dr = A traveled by the light pulse is computed from O's vantage point by employing the alternative Lorentz transformation (ALT) discussed in the text [2]. He finds that the light pulse travels at an angle $\Theta = \tan^{-1}(\gamma\, v/c)$ and with speed equal to c, so that his measured elapsed time is dt = A/c. Another observer (M) at rest on the satellite, who agrees to use the same unit of time as O, i.e. the same clock rate, also finds the light pulse to travel the same distance from his perspective (dr' = A) at the same speed c, but in a different direction (Θ = 0). The elapsed time for this event measured by M is therefore dt' = A/c, the same value as for O. The light pulse therefore arrives at the same time at its detector on the satellite for the two observers, despite the fact that they are in relative motion to each other.

is directed along the y axis for M, that is, perpendicular to the velocity of the satellite relative to O. In accordance with the simultaneity principle, O finds that the elapsed time for the light pulse to arrive at the detector is also $\mathrm{dt} = \frac{\mathrm{A}}{\mathrm{c}}$. Because of the motion of the satellite, however, the two observers differ on the direction traveled by the light pulse. The GPS-LT [2] finds that the y-component of the vector distance traveled from O's perspective is $dy = \left(1 - \frac{v^2}{c^2}\right)^{0.5} A = \frac{A}{\gamma}$ (see Fig. 1). Combining this with the corresponding x-component, $dx = vdt = v\frac{A}{c}$, he finds that the total distance traveled by the light pulse is $dr = \left(\gamma^{-2}A^2 + v^2dt^2\right)^{0.5} = A\left[1 - v^2c^{-2} + v^2c^{-2}\right]^{0.5} = A$, the same value (dr') as found by M. The corresponding angle of approach is $\Theta = \tan^{-1}\left(\frac{dx}{dy}\right) = \tan^{-1}\left(\gamma\ \frac{v}{c}\right)$ from O's perspective, the same value as found for the aberration of star light from the zenith [7].

Note that the latter equality for the distance traveled from the two observers' perspectives (dr and dr') does not result from use of the LT [1]. In that case O and M agree on the value of the y-component as $dy = dy' = A$, but O finds the total distance traveled by the light pulse from his perspective to be $dr = \left[A^2 + v^2dt^2\right]^{0.5}$, that is, a larger value than for M ($dr' = A$). Consequently, the two observers also disagree on the elapsed time since the speed of light is the same for both ($dt = \frac{dr}{c}$ for O but $dt' = \frac{A}{c}$ for M). The conclusion from the LT is therefore that the light pulse does not arrive at the detector on the satellite at the same time for O and M, which result stands in direct contradiction with the experience of the GPS technology [2]. If M uses the "pre-corrected" atomic clock on the satellite (and therefore the same unit of time), he obtains the same value for the elapsed time (A/c) as does the observer on the earth's surface.

The two observers can also use eq. (1) to convert the above timing results to M's system of units. This clearly does not change the fact that the light pulse arrives at the detector at the same time for both. It simply means that the numerical value for this elapsed time ($A/\gamma c$) differs *for each of the observers* by the same factor $Q^{-1} = \alpha_O / \alpha_M = 1/\gamma$ relative to the value of A/c obtained above using O's system of units. The resulting two values are still equal to one another in M's system of units.

One also has to make corrections for gravitational effects on clocks [8], as was done in the experiment with circumnavigating airplanes carried out by Hafele and Keating [4], as well as in the GPS technology. When this has been done, the ratio of two different clock-rate parameters can be obtained by observations of the transverse Doppler effect [9]:

$$\frac{\alpha_M}{\alpha_O} = \frac{\lambda^M(O)}{\lambda^O(O)} = \frac{\nu^O(O)}{\nu^M(O)}. \tag{2}$$

In this equation, $\lambda^M(O)$ is the wavelength and $\nu^M(O)$, the frequency, that O measures when the light source is co-moving with M, and $\lambda^O(O)$ and $\nu^O(O)$ are the corresponding *in situ* values.

It is important to recognize that the clock-rate parameters are also convenient for defining the way in which the units of other physical quantities vary with changes in velocity, however, as will be discussed below.

III. Scaling of the Unit of Length

One of the most basic features of Einstein's STR [1] is the FitzGerald-Lorentz length contraction effect (FLC). It states that two observers will measure different values for distances parallel to their direction of relative motion, but that they will agree on the values measured perpendicular to this direction. Based on the foregoing discussion about the unit of time in different inertial systems, one would be led to conclude from the FLC that the unit of length is *directionally dependent.*

In considering this point, it is helpful to go back to Einstein's second postulate. If the speed of light is the same for two observers in relative motion, and one knows from experiment that their respective clocks are running at different rates, it follows that there must be an *exact compensation* in their respective measurements of the distance the light has traveled. Hawking [10] has argued that the observer on a rocket ship must find distances to be *γ times shorter* than does his counterpart in the rest frame of the earth. On this basis, since this is exactly the factor by which his clock has slowed, the observer on the rocket ship should also measure the speed of light to be c, consistent with the second postulate. There is a flaw in this argument, however. According to STR, the amount of the length contraction varies with the orientation of the object. There is no contraction at all when the distance measured is transverse to the direction of relative motion, for example. Therefore, the speed of light would not be the same in all directions for the observer on the rocket ship according to the FLC, in contradiction to the second postulate of STR. Moreover, Hawking's above position about radial length contraction does not agree with STR either. The FLC asserts that the observer on a rocket ship must find distances in his rest frame to be γ times *longer* than does his counterpart in the rest frame of the earth. Rather than the two effects cancelling, one must conclude that they actually reinforce each other in this case. Accordingly, the observer in the rest frame of the earth must find that the speed of light in the rest frame of the rocket ship is $\gamma^{-2}c$ (longer time, *shorter* distance traveled) when the distance is in the radial direction, not c as the light-speed postulate demands.

The GPS technology provides a definitive test for the FLC, however, as discussed in previous work [2]. It shows that the predictions of STR in this regard are also inconsistent with the CRP introduced in Sect. II. The pre-corrected clock on the satellite runs Q times faster than the proper clock (Q>1) (after a correction is made for gravitational effects). Since the length of an object is defined as the product of the speed of light c and the corresponding elapsed time required for a light pulse to traverse it, it follows that the measured value based on the pre-corrected clock, which runs at exactly the same rate as its identical counterpart on the earth's surface, will be Q times *larger* than that based on the local (uncorrected) clock on the satellite. Moreover, this result will be the same independent of the orientation of the object relative to the line of motion between the satellite and the earth. In short, the time-dilation effect for clocks on the satellite is only consistent *with isotropic expansion* of the lengths of all objects in its rest frame. This means in effect that the standard unit of length (meter stick) *increases* in direct proportion to Q as its speed relative to the observer on earth increases, exactly the same ratio as for the periods of co-moving clocks [see eq. (1)]. Only in this way can one explain how the speed of light can be the same for both the observer on the satellite and his counterpart on the earth's surface (again after correcting for gravitational effects), even though the latter's clock runs Q times faster than that on the satellite.

Under the circumstances, there is nothing that stands in the way of simply defining the unit of length (m) to vary in direct proportion to the unit of time. The analogous relationship to eq. (1) for lengths L (O) and L (M) measured by observers O and M for the distance between a given pair of objects therefore holds, namely

$$L(O) = \frac{\alpha_M}{\alpha_O} L(M). \tag{3}$$

The transverse Doppler effect offers a means of checking the above result. It is known from experiment [9] that the wavelength of light increases with the speed v of the source relative to the observer (O) by a factor of γ (after the correction for the first-order effect has been taken into account so that the measured result is directionally independent). Consistent with Einstein's second postulate, other experiments [11] have shown that the corresponding frequency decreases by the same factor, so that the product remains constant with a value of c (again assuming that O is stationary in the rest frame of the earth in this example). Because the unit of time in the inertial system of the source's rest frame (M) is $Q = \gamma$ times larger, as discussed in the previous section, it follows that the *in situ* value of the frequency is independent of the speed v, however.

Is there a corresponding independence of the *in situ* value of the wavelength for M? This question has recently been answered in the affirmative by means of experiments carried out with a cavity resonator over a period of 190 days [12]. The resonance condition within the cavity has been found to be highly stable over this period, despite the fact that the orbital speed of the earth around the sun changes significantly over time. This result proves to quite high accuracy that the values of wavelengths measured *in situ* do not vary with the state of motion of the observer. In view of the results of Doppler experiments [9,11], it therefore follows that *the length of the cavity resonator itself must also have increased by the same factor*. The fact that M does not detect this change shows that there is a uniform length expansion in his accelerated rest frame, including the meter stick he employs to carry out his measurements. His unit of length must therefore be γ m if his unit of time is γ s .

IV. The Units of Energy and Inertial Mass

The question that will be considered in the present section is how the units of other physical quantities vary with the state of motion of the observer. Perhaps the best place to start is the argument presented by Lewis and Tolman [13] based on the law of momentum conservation. These authors considered the example of two particles undergoing an elastic collision. They concluded that the Einsteinean time-dilation effect implies that the inertial mass of the particles must vary in direct proportion to the time of the collision as measured by a given observer O. In practice, this means that as a particle's speed v increases relative to O, its inertial mass increases by a factor of $\gamma(v)$ for him. Experiments carried out by Bucherer [14] with charged particles moving in a transverse magnetic field subsequently verified this relationship.

Because of the mass/energy equivalence relation [1], the variation of inertial mass also implies that the relativistic energy E (O) of the particle must vary by the same factor. As with decay lifetimes, however, it is clear that an observer M co-moving with the particle will notice no change in the energy he measures *in situ*. On this basis, one must conclude that the respective energy values measured by O and M differ in exactly the same manner as elapsed times in eq. (1) and lengths in eq. (3), that is,

$$E(O) = \frac{\alpha_M}{\alpha_O} E(M) . \tag{4}$$

Similarly as with lengths and time, this relationship can also be expressed by stating that observer O's unit of energy is proportional to α_O, whereby the corresponding unit in the standard laboratory E on earth might be defined to be 1.0 Joule (J). Clearly, the same argument must be made for the unit of inertial mass:

$$m_I(O) = \frac{\alpha_M}{\alpha_O} m_I(M). \quad (5)$$

The ratio $Q = \alpha_M/\alpha_O$ plays a key role in the above determinations. In the usual situation in STR, it has a value of γ (v) because O finds that lifetimes and energies of objects co-moving with M both have this dependence on the relative speed v. The discussion to this point has emphasized that the units of energy, length and time all vary as Q. The central principle to be followed in determining the way other physical quantities vary is that they must be chosen in such a way as to ensure that certain mechanical equations hold in each inertial system. There also must be complete consistency in these choices.

The first example demonstrating the latter requirement is the unit of speed/ velocity. Since it is defined as the ratio of a distance to an elapsed time, its unit must be equal to the corresponding ratio of these two fundamental units. One is thus led unequivocally to the conclusion that the unit of velocity must vary as Q^0 and the unit of acceleration (a) as Q^{-1}. The unit of force F must also be independent of Q because it is the quotient of the unit of energy with the unit of length (1.0 N = 1.0 J/m). The unit of inertial mass is then determined by the requirement that both Newton's Second Law ($F = dp/dt = m_I a$) and Einstein's mass/energy equivalence relation must hold in each inertial system. For this purpose it is easier not to use kg [15] as the standard unit for inertial mass, but rather 1.0 N s^2 / m = 1.0 J s^2 / m^2. Because of the manner in which

Table 1. Variation of the units of physical quantities with state of motion. The object of the measurement is in the rest frame of inertial system M with clock-rate parameter α_M (as defined in Sect. II), whereas the observer carrying out the measurement is at rest in inertial system O with clock-rate parameter α_O. The ratio of the respective units of a given quantity in the two inertial systems is conveniently given as a power n of the ratio $Q = \alpha_M / \alpha_O$ in each case.

Physical Quantity	Standard Unit (mks system)	Power of ratio Q n
Time	s	1
Length	m	1
Energy	J = Nm	1
Force	N	0
Velocity	m/s	0
Gravitational Mass	kg	0
Inertial Mass	Ns^2/m	1
Momentum	Ns	1
Acceleration	m/s^2	-1
Angular Momentum (Planck's constant)	Nms	2
Torque	Nm	1
Radiative Frequency	s^{-1}	-1

each of the latter quantities varies, it then follows that the unit of inertial mass must vary as Q, as already specified in eq. (5). Accordingly, the unit of momentum **p** also varies as Q because its standard unit is N s (Second Law). Angular momentum l varies as Q^2 because of its definition as the product of momentum and distance.

The unit of gravitational mass m_G is independent of Q, however. This follows directly from experience with Newton's inverse square law that shows that the gravitational mass of the sun and its planets does not vary with their state of motion. Since the unit of inertial mass varies as Q, this means that these two quantities are simply proportional to one another, a fact which is quite basic to gravitational theory [16]. The fact that all objects are subject to the same gravitational acceleration at a given point in space (Galilean unicity principle) is a manifestation of this proportionality.

The basic premise in the above discussion is that it must be possible for the units of physical quantities to vary without affecting the validity of the fundamental laws of physics. The conservation laws of momentum and energy, for example, are mathematical equations. As such it is possible to multiply them on both sides by the same constant without affecting the condition of equality. Knowing what these fundamental equations are puts definite restrictions on the manner in which a given physical quantity may vary.

The procedure outlined above is appropriately called "kinetic scaling." One can summarize the results of this discussion quite succinctly in terms of the scaling factor $Q = \alpha_M/\alpha_O$. To each physical quantity discussed above corresponds an integral power of Q, as given in Table 1. The quantities considered in this table are restricted to those of a strictly kinematic nature. It is possible to carry out the same program for the myriad of physical quantities that appear in the field of electromagnetism, but this aspect has been considered elsewhere [17]. The very fact that a consistent formulation of relativity theory can be achieved in this manner suggests that a similar approach might be successful in describing gravitational effects. In this case the units of physical quantities are assumed to vary with the location of the object and observer relative to a distant mass. The corresponding "gravitational scaling" of units is discussed in an earlier publication [8].

Returning to the subject of kinetic scaling, it is clear that one has to have a means of actually determining the value of Q in Table 1 in order to make quantitative predictions on this basis. If the inertial system O is *an objective rest system* (ORS [18]) for M, that is, M has at some point in time moved away from the rest frame of O to attain the relative speed v, the value of Q is simply $\gamma(v)$. Otherwise, it is necessary to know the values of the clock-rate parameters, α_O and α_M, on the basis of experimental observations in order to fix the value of Q, as already noted at the end of Sect. II.

V. Dependence of Measured Values on Clock-rate Parameters

Knowledge of the way in which the units of physical quantities vary between inertial systems, as summarized in Table 1, allows one to make definite predictions of how a given experimental value depends on both the state of motion of the observer and the object of the measurement. The following procedure will be adapted to obtain the necessary relationships. First, an experiment is carried out in which both the object (M) and the observer (O) are at rest on the surface of the Earth where the clock-rate parameter is defined to be $\alpha_E = 1$. Next, the object undergoes acceleration so that a clock co-moving with it runs α_M times slower than before. At the same time, the observer is accelerated so that his clock now runs α_O times slower (note that the results obtained with this procedure are equally valid if the clock-rate parameters α_O and α_M are less than unity). As discussed above, a key assumption in compiling Table 1 is that the speed of light is independent of the state of motion of both O and M, in accord with Einstein's second postulate of STR.

If an elapsed time τ is measured in the original experiment on the earth's surface, the corresponding value obtained by O after the acceleration phase must be $Q\tau = (\alpha_M/\alpha_O)\ \tau$. Since M's clock has slowed by a factor of α_M, the elapsed time for an equivalent event is by definition equal to $\alpha_M\tau$ for an observer at rest on the earth's surface (i.e., O = E). For the usual case in the laboratory when the object is accelerated to speed v, $\alpha_M = \gamma(v)$, i.e., Einsteinean time dilation has occurred. Because O has also changed his state of motion in the above procedure, however, his clock now runs α_O times slower than when it was at rest in the laboratory on earth. Consequently, his measured value for this elapsed time must be inversely proportional to α_O, i.e. it will be $Q\tau = (\alpha_M/\alpha_O)\ \tau$, as indicated above. An observer traveling with the object will detect no change in elapsed time, consistent with the RP. For such an *in situ* measurement, $\alpha_O = \alpha_M$ by definition, and this leads to the latter conclusion based on eq. (1).

Exactly the same analysis can be given for energy values. The energy of an accelerated object increases in the same proportion as do elapsed times from the standpoint of a stationary observer on earth. Such variations also cannot be detected via *in situ* measurements, however. A rational system of units [19] requires that the measured value depend on the state of motion of the observer, and this is reflected quantitatively through the $Q = \alpha_M/\alpha_O$ factor in the general formula given in Table 2.

Table 2. General formulas for the variation of measured values for various physical quantities as a function of the clock-rate parameters α_O and α_M for the observer O and the object M. The measured *in situ* value ($\alpha_M = \alpha_O$) is given in the second column in each case. The corresponding value when the object is at rest in inertial system M and the observer is at rest in inertial system O is given in the third column. For the entries with equations, a quantity in quotation marks in the third column indicates that the general value given in an earlier row should be employed.

Physical Quantity/Equation	*in situ* Value	General Value
Speed of Light	c	c
Length	L	$(\alpha_M/\alpha_O)\, L$
Time	T	$(\alpha_M/\alpha_O)\, T$
Energy	E	$(\alpha_M/\alpha_O)\, E$
Frequency	ν	$(\alpha_O/\alpha_M)\, \nu$
Inertial Mass	m_I	$(\alpha_M/\alpha_O)\, m_I$
Mass/Energy	$E=m_I c^2$	"E"= "m_I" c^2
Gravitational Mass	m_G	m_G
Momentum	p	$(\alpha_M/\alpha_O)\, p$
Planck's Constant	h	$(\alpha_M/\alpha_O)^2\, h$
Energy/Frequency	$E=h\nu$	"E"= "$h\nu$"
Wavelength	λ	$(\alpha_M/\alpha_O)\, \lambda$
Phase Velocity	$\lambda v = c$	"λv" = c
Velocity	v	v
Force	F	F
Angular Momentum	$l = m_I v r$	$(\alpha_M/\alpha_O)^2\, l$
$e^2/4\pi\varepsilon_0$	$F_C r^2$	$(\alpha_M/\alpha_O)^2 F_C r^2$
Fine-structure constant	$\alpha = e^2/2\varepsilon_0 h c$	"α" = "$e^2/2\varepsilon_0 h$" c = α
Hartree	$E_H = m_e e^4/4\varepsilon_0^2 h^2$	"E_H"= "$m_e e^4/4\varepsilon_0^2 h^2$"
Hartree	$E_H = \alpha^2 m_e c^2$	"E_H" = α^2 "m_e" c^2
Bohr radius	$a_0 = \varepsilon_0 h^2/\pi m_e e^2$	"a_0" = "$\varepsilon_0 h^2/\pi m_e e^2$"
Atomic unit of time	$t_0 = 2\varepsilon_0^2 h^3/\pi m_e e^4$	"t_0" = "$2\varepsilon_0^2 h^3/\pi m_e e^4$"

It also is not possible to detect any change in the length of the accelerated object by means of *in situ* measurements. As has been pointed out in Sect. III, the unit of length must always be the same in all directions if light is to propagate isotropically in every inertial system, as experiment has found. In Table 1 it has been assumed that this unit also varies in the same manner as time, consistent with the fact that the speed of light is independent of both the state of motion of the observer and of the light source. This choice also requires that observers in relative motion agree on the values of all *relative velocities*, however [20]. The latter conclusions assume that appropriate gravitational corrections have been applied to the measured results in each case [4, 8].

The values of measured radiative frequencies must vary in a consistent manner as the corresponding periods, i.e. in inverse proportion to them. Hence, if an atom emits a frequency ν in M's rest frame, it follows that the observer O in the above procedure must find that it has changed to a value of $(\alpha_O/\alpha_M)\, \nu = Q^{-1}\, \nu$. This result is consistent with the transverse Doppler effect [9, 11]. Again, in the usual case, $\alpha_M = \gamma$ and $\alpha_O = 1$, and thus the observer in the laboratory on earth finds that the radiation is red-shifted to a value of ν/γ. The *in situ* value is always ν, consistent with the RP and the above general formula.

Einstein's STR [1] defines the inertial mass m_I of an object as the ratio of its energy E to the square of the speed of light. It has already been shown on this basis that the unit of inertial mass must vary as $Q = \alpha_M/\alpha_O$ (Table 1). The governing ratio of clock-rate parameters for momentum **p** can be determined from the relationship p = E/c for photons. The appropriate factor is obtained by dividing the ratio Q for energy by that for the speed of light (Q^0) in Table 2, which again gives the result of $Q = \alpha_M/\alpha_O$ for this factor. In this connection it is important to recall (Sect. IV) that all *relative* velocities are independent of the state of motion because of the kinetic scaling of units. The original definition of momentum is simply as the product of inertial mass and velocity. Multiplying the corresponding factors, Q and Q^0, for the latter two quantities also gives the above result for the variation of momentum, as required. The corresponding ratio for angular momentum measurements is Q^2, whereas torque varies as Q. The values of force measurements are completely independent of the state of motion, however, since they involve ratios of energy and length (Table 2).

The variation of Planck's constant with acceleration is obtained by dividing the factors for energy and frequency (or multiplying the energy and time factors). The result is that h varies as the square of $Q = (\alpha_M/\alpha_O)^2$ (Table 2). This finding can be verified by experiment, as will be discussed in the following section. Another quantum mechanical relationship can be used to obtain the variation of wavelength with acceleration, $\lambda = h/p$. Appropriate division finds that on this basis wavelengths also vary as $Q = \alpha_M/\alpha_O$, the same as for lengths in general. The same result is obtained from the definition of the phase velocity of light ($c = \lambda\nu$), since ν is inversely proportional to Q and c is constant.

The fine-structure constant represents an interesting case for the kinetic scaling procedure. As a dimensionless quantity, it should not change in value for any consistently defined set of units. Its standard definition is given in terms of quantities that have already been considered in the present context, however, so one must also ensure that no inconsistency arises on this basis. In standard texts the fine structure constant is usually written simply as $\alpha = e^2/\hbar c$, where $\hbar = h/2\pi$. The quantity e^2 itself is defined through the Coulombic force equation, and thus in the Giorgi or mks set of units an extra factor of $4\pi\varepsilon_0$ is required in the corresponding definition [17]. Since lengths have been assumed to vary as Q while forces are independent of the state of motion (Q^0), this means that $e^2/4\pi\varepsilon_0$ (or simply e^2 in the former system of units) must transform as Q^2, as shown in Table 2. Since the scaling factors for h and c are Q^2 and Q^0, respectively, it follows that the fine-structure constant is indeed independent of the state of motion of both the observer and the object of the measurement.

The fine-structure constant α frequently appears in quantum mechanical expressions in which the various quantities are given in atomic units. The unit of energy in this system is the hartree (E_H), which is defined as $m_e e^4/\hbar^2$. The ionization potential of the hydrogen atom is 0.5 E_H (1.0 Rydberg) and must therefore transform as energy upon acceleration of the observer and/or the atom. Substituting the appropriate clock-rate parameters for the above quantities (Q for m_e and Q^2 for e^2 and h) shows that observer O will find a value for "E_H" based on his measurements of an object co-moving with M to be Q E_H, i.e., that it varies in the same manner as energies in general. The formula for the hartree in terms of the fine structure constant is $E_H = \alpha^2 m_I c^2$, which is also consistent with the above conclusion since m_I varies as Q and both α and c are constants.

Finally, the Bohr radius, $a_O = \hbar^2/m_e e^2$, must vary as Q since it is a unit of length. Substitution of the various kinetic scaling factors for h, m_e and e^2 again verifies that this is the case. When one computes the hartree as e^2/a_0, the result ("E_H") again transforms as energy because of the variation of e^2 and a_0. Another example for the atomic unit of time, $t_0 = \hbar^3/m_e e^4$, is also given in Table 2. More details about uniform kinetic scaling may be found elsewhere [19].

VI. Planck's Radiation Law: A Proposed Experimental Test

The units of energy and time vary in the same manner in eqs. (1, 4), and this raises an interesting point that does not seem to have been recognized earlier. Since the unit of frequency must be the reciprocal of the unit of time, it follows that the ratio of the unit of energy to the unit of frequency is not constant: it must vary as $Q^2 = (\alpha_M/\alpha_O)^2$, as already pointed out in the previous section. According to Planck's radiation law [21], the energy E of a light quantum is equal to h times the frequency ν of the associated light waves. The unit for Planck's constant is therefore J s in the mks system. The RP demands that Planck's law hold in every inertial system.

The fact that the unit J s differs for observers in relative motion (Table 1) means that *they generally do not agree on the value* of the energy/frequency ratio, however. According to the above discussion of the manner in which physical units vary with the state of motion of the observer, there is only one way to reconcile these two statements: the energy/frequency ratio only has a value of h in the system of units for the observer who carries out the necessary measurements *in situ*. More specifically, if the light source is in M's rest frame, the value of the energy/frequency ratio found by an observer O on the basis of his measurements in another inertial system is Q^2 h. The relativistic version of Planck's law thus becomes

$$\mathrm{E}(\mathrm{O}) = \left(\frac{\alpha_{\mathrm{M}}}{\alpha_{\mathrm{O}}}\right)^2 \mathrm{h}\,\nu(\mathrm{O}). \qquad (6)$$

This result can be tested experimentally by means of the transverse Doppler effect discussed at the end of the previous section. As the speed v of the light source increases, the frequency of the emitted radiation is known to decrease by a factor of γ [11]. Yet for the same observer (O = E), all energies of objects co-moving with the source (M) *increase* by the same factor, as indicated in eq. (4). The RP requires that the energy of light quanta vary in the same manner, so that the stationary observer O must therefore find the value of the energy/frequency ratio to be γ^2 h, consistent with eq. (6).

Historically, the most accurate determination of Planck's constant has been achieved through the study of the photoelectric effect [22]. In order to test the validity of eq. (6) one would have to carry out such experiments with light emitted from a source moving at a high speed relative to the metallic surface from which electrons are to be ionized. Since the frequency is decreased because of the Doppler effect [11], one might predict that the kinetic energy of the ejected electrons would also decrease as the speed v of the source is increased. Since the energy of the photons is increasing with v, however, the opposite behavior should occur, consistent with eq. (6). The electronic kinetic energy should increase as γ even though the measured frequency obtained via the transverse Doppler effect [9] is decreased by this factor.

A key point in this discussion is that the fractional increase in the total energy of an object does not depend on the direction of motion. This is in stark contrast to the variation of the measured frequency of the light, which must satisfy the relativistic Doppler formula. If v is the speed of the light source and χ is the angle between its direction and the line of observation, eq. (6) can be generalized to

$$\mathrm{E}(\mathrm{O}) = \left(\frac{\alpha_{\mathrm{M}}}{\alpha_{\mathrm{O}}}\right)^2 \left(1 + \frac{\mathrm{v}}{\mathrm{c}}\cos\chi\right) \mathrm{h}\,\nu(\mathrm{O}) \tag{7}$$

This relationship shows that the largest effect is obtained when the source is moving away from the observer ($\chi = 0$). The *effective* change in Planck's constant is then of first-order in v/c, rather than second-order (with $\alpha_M/\alpha_O = \gamma$) as in the case of transverse motion of the source.

Finally, it is interesting to compare the way the two fundamental constants, the speed of light and Planck's constant, vary with the speed of the observer and the corresponding light source. The *in situ* value (i.e. $\alpha_M = \alpha_O$ and $v = 0$) of the latter is always equal to h according to eq. (7), but it changes with the speed of the light source (M) relative to the observer (O) by a factor of $Q^2 = (\alpha_M/\alpha_O)^2$. By contrast, the speed of light is completely independent of the speed of the source (transverse Doppler effect [9, 11]) as well as the state of motion of the observer, in accord with Einstein's second postulate of relativity.

VII. Conclusion

Experiment has confirmed the prediction of Einstein's STR that clocks slow down as their speed v relative to a stationary observer increases. It has been similarly confirmed that the energy, momentum and inertial mass of an object increase in the same proportion, $\gamma(v)$, as the clock rates decrease. A convenient way of expressing these relationships is to assume that the units of physical quantities vary in a well-defined manner with the state of motion. For example, the fact that clocks slow down by a certain factor in a given rest frame simply means that the unit of time there has increased in that proportion. In addition, in order to have a *universally consistent* set of units, it is necessary that certain physical laws be valid in every inertial system. These include

Einstein's mass/energy equivalence relation and Planck's radiation law.

Einstein also predicted that distances are contracted along the line of relative motion, although not in a transverse direction (FLC). This prediction is incompatible with his second postulate, however. The only way two observers can agree on the value of the speed of light when their clocks run at different rates is if there is a compensating difference in the lengths of their meter sticks. Moreover, this change must occur isotropically because the speed of light is observed to be the same in all directions. Since clocks slow down upon acceleration, as demonstrated in the Hafele-Keating experiment with circumnavigating airplanes [4], this means that co-moving meter sticks must *increase* in length in exactly the same proportion as the clock rates decrease. In short, *isotropic length expansion* must occur with time dilation, not the anisotropic length contraction predicted by the FLC.

This conclusion has received stunning verification in experiments with cavity resonators [12]. The ratio of the length of the apparatus to the wavelength of light has been found to remain perfectly constant over a long period of time. Since one knows from the transverse Doppler effect [9] that the wavelength itself is constantly varying for an observer located at the sun, for example, this proves that the length of the apparatus is also changing at the same rate from his vantage point. The effect is real, just as the slowing down of clocks upon acceleration, but it cannot be detected by purely *in situ* observations because of the RP.

The fact that the FLC is contradicted by experiment proves that the LT from which it is derived in STR is not a physically valid space-time transformation. The same conclusion results from consideration of its prediction of the non-simultaneity of events for two observers in relative motion [2]. The experimental fact that two clocks are running at different rates (use a different unit of time) in no way permits events to be simultaneous based on one of them but not so on the other. Failure to recognize this point has led to a general reluctance to define a rational system of units [19] in each inertial system.

One can incorporate simultaneity in relativity theory by eliminating the LT as its space-time transformation and replacing it with an *alternative Lorentz transformation* ALT [2, 23] or GPS-LT [24]. This space-time transformation also satisfies both of Einstein's postulates of relativity. Specifically, Einstein's first postulate (the RP) is satisfied by demanding that the laws of physics be expressed in the same form in each inertial system but generally in a different system of units.

The GPS-LT accomplishes all the above objectives by simply using the simultaneity condition ($dt' = dt/Q$) to describe the relationship between measured elapsed times obtained by observers in relative motion. What is meant thereby is that different units of time be used in the two rest frames, the conversion factor thereof is $Q = \alpha_M/\alpha_O$ defined in eq. (1). The GPS technology provides a clear example of how this can be done in actual practice. The observer on the satellite uses a "pre-corrected" clock that runs at exactly the same rate as the clock used by his counterpart on the earth's surface. They also must agree on the units of all other physical quantities, in particular, that with which they base their measurements of length. It is a simple matter to go from one system of units to another by using a consistent set of conversion factors. The procedure is no different in principle than to go from the cgs to the mks system of units, for example, or from the metric to the British set of units.

The actual scaling of units with the state of motion is conveniently effected by means of a single parameter α_O in each rest frame that is proportional to the rate of slowing down of clocks. It can be defined to have a value of unity ($\alpha_E = 1$) in the rest frame of the earth's surface as standard, for example. When observers in different inertial systems O and M wish to compare their measurements, it is necessary for them to know the ratio $Q = \alpha_M/\alpha_O$. It has been demonstrated that the ratio of their respective measured values for any purely mechanical quantity will then be proportional to an integral power of Q. This power of Q is +1 for time, length, energy, momentum and inertial mass, zero for velocity, force and gravitational mass, -1 for radiative frequencies and the rates of chemical and nuclear reactions, and 2 for angular momentum and Planck's constant h (Table 1). These values are internally consistent and can be determined for derived quantities by simply knowing their definitions in terms of the fundamental physical quantities: time (s), distance (m), force (N) and gravitational mass (kg), which scale with the powers of Q of 1, 1, 0, and 0, respectively, according to the above definitions. A key example is inertial mass. According to Newton's Second Law, its units are Ns^2/m. Adding the corresponding powers for force (0), time (1) and distance (1) gives a value for the integral power of 1, as noted above.

The system of physical units described above is *rational*. This means that if M's clock is ticking *half* as fast as O's, then necessarily O's must be ticking *twic*e as fast as M's. Existing texts dealing with STR often claim something quite different. The claim is made that O will find that M's clock has slowed down because it is in motion relative to him and he is at rest, but that there is a *symmetrical* relationship for M. Accordingly, since he thinks it is he who is at rest and that O is moving relative to him, it somehow follows that M will find that O's clock is running slower than his by the same margin.

That such a symmetric interpretation is fallacious can be experimentally proven. One only has to bring the two clocks back together in the same rest frame to see that one of them has indeed been running more slowly than the other. Recognition of this logical difficulty has led many authors to conclude that the symmetric relationship assumed above does not exist in the present case because of differences in the accelerations of the two observers during the course of the experiment. Experiments such as that carried out by Hafele and Keating [4] indicate that the "moving" clocks run *continuously* at a lower rate, however. The amount of the discrepancy relative to the "stationary" clock on the earth's surface (or one traveling in the opposite direction) can be computed quite accurately under this assumption. The success of the GPS technology is ultimately due to its rejection of such a symmetry principle in favor of the definition of a completely rational set of units on both the satellite and on the earth's surface. Similar remarks hold for *all* physical quantities measured by two observers in relative motion. The ratios of respective measured values can always be computed on the basis of the kinetic scaling procedure summarized in Tables 1 and 2.

One further test of the present conclusions can be made by carrying out measurements of the photoelectric effect for light emitted from sources moving at high speed relative to the observer. Planck's constant h has units of J s, so according to the arguments discussed above, its value should change by a factor of $Q^2 = (\alpha_M/\alpha_O)^2$ as the speed of the light source M relative to the observer O is varied. This means that even though the frequency of the light decreases because the source is moving away from O (Doppler effect), the energy of the photons must be increasing at the same time.

Finally, the above tables are restricted to strictly mechanical quantities, but the corresponding results for quantities that appear in the theory of electromagnetism can also be derived [17]. It also is possible to give a similar table for the variation of all these quantities with position in a gravitational field [8]. The result is a coherent relativistic theory that is consistent with all known experimental observations, but one whose predictions for a number of experiments proposed in the present study are qualitatively different from those that are inferred from Einstein's original version of STR [1].

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