On Mathematical Misconceptions Masquerading as Physics

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In considering the relationship of mathematics to physics, one is readily reminded of Little Jack Horner, who "stuck in his thumb and pulled out a plum and said 'what a good boy am I.'" At every stage of the history of physics the presumption is that congratulations are in order, the theory is almost perfected, prosperity is just around the corner, and a smooth, monotonic, upward curve of *progress* in mathematical insight is bearing the profession of physics to ever loftier heights. Second thoughts are heresy. In short, the trumpet is perpetually sounding that has never called retreat. On this basis the profession of theoretical physics has become loaded (dare I say over-loaded?) with mathematicians. The result is a tripartite sociological division of physics: First-class citizens: mathematicians; second-class: other (misc.) theorists; third-class: experimentalists.

During the last century Percy Bridgman recommended to physicists the idea of *operationalism*; namely, that practitioners of physics should pay attention to how their concepts are realized through measurement. That was the last heard about measurement. Mathematization of the subject has since progressed to the point that it is becoming difficult to get even crucial experiments done. But the resulting pain is mitigated by the widespread conviction that no more experiments are really needed, because we already know it all … or at any rate practically all that is worth knowing.

It is high time someone blew the whistle – although the fate of whistle-blowers is well-known. There must be limits to academic hubris, as well as to Jack Horner-style narcissism. One way to put a spoke in this great, ever-churning wheel is to show by specific examples that the accepted theory defining classical physics is rotten in so many places that to express wonder at the "fit" between physics and mathematics is to marvel at what does not exist. That unpopular task will guide the present essay. My theme is simple: physics is passed off nowadays as something practically finished; whereas in fact it is barely begun. The time for viewing it as some mystical collusion of mathematical magic and stern reality is not yet; and, to judge by past performance, will never be. Still, the felt need for magic is time-invariant; so the marvelling will go on regardless of error. The mistakes I shall enumerate are so elementary that their establishment will hardly require mathematics. But where math is needed it is supplied in endnotes.

The poster child for mathematical misconceptions is *universal covariance*. This is instructive in that it shows that the best way to protect an error from discovery is to universalize it. Once it fills the limited mental space of mathematical conceptualization, there is no room left for competitive ideas; so semi-eternal life is assured. Before covariance, there was another, less problematic, variety of form preservation known as *invariance*. That topic will be central to what I have to convey. Suffice it to say that some kind of mathematical form preservation under inertial transformations is mandatory, as a means of enabling our mathematics to mimic the stability attribute of the real world (independence of the observer and his doings).

But why covariance? The truth is that covariance provides not even a successful method of form preservation, since what is preserved is not the transformed quantity, $X'_\mu = X_\mu$, but a redefined form, $X'_\mu = \sum_\nu \alpha_\mu^\nu X_\nu$, of that quantity. It does not require a mathematical education to recognize that a redefined quantity is not a preserved quantity. (Indeed, only a sufficiently mathematical mind could harbor confusion on this elementary issue.) If one wants true form preservation, one must demand true form invariance. Yet even the curiosity requisite to the search for an honestly invariant alternative has been sabotaged by universalizing the myth of covariance.

What physics lies behind covariance? One searches in vain for something more substantial than *spacetime symmetry*. The slightest touch of the Bridgman philosophy withers this to inanity. There is simply no similarity, much less symmetry, in the methods used to *measure* space and time. Spacetime symmetry is simply another myth, complementary to that of covariance. They are mutually supportive and equally non-physical.

Next on our role-call of hitherto unchallenged physics myths comes the Lorentz transformation. Students of relativity have been taken up on the mountain, where it has been revealed to them that the Lorentz transformation (LT) is a better mathematical representation of physical inertial (unaccelerated) motion than the old-fashioned Galilean transformation (GT). Here, I must regrettably enter upon speculation, since I am not aware that any decisive experiment has been done, either to support or deny conclusively this vital hypothesis. Almost any "relativity" experiment is automatically *interpreted* as supporting the claim of LT superiority. But said superiority eludes me. The LT adduces two new allegedly physical effects, the Lorentz contraction and time dilation. The first has only dubiously been observed, despite scores of earnest attempts, and the second introduces a logically impossible symmetry. Time dilation is better handled by a simple scheme of time measurement invented by the scientists of the Global Positioning System (GPS). To them it occurred that coordinating a global system of earth satellites would be facilitated if all clocks could be induced to run in step (that is, to tell the same "time"). This they achieved by the simple expedient of correcting out all physical effects of clock motion and gravity change. (Cesium clocks were used and the corrections were made by counter settings that varied the *number* of atomic oscillations used to define the "second" of elapsed time.) Thus the method of time *measurement* was contrived to produce variable clock running rates, as needed to get "time" to flow uniformly.

GPS time is thus essentially Newtonian time, which (ironically) is anathematized by relativity experts. I believe I am on firm ground in asserting that, if GPS time is used as the time parameter in the Galilean transformation, an entirely workable representation of physical inertial motions is obtained. No Lorentz contraction is predicted, but time dilation is automatically accommodated by the GPS clock compensations. Hence, there is no spacetime symmetry; but we have already recognized that as smoke and mirrors. Further, the time dilation produced by the GPS timekeeping scheme is asymmetrical between the inertial rest system of a Master clock and all other inertial systems (wherein corrected "slave" clocks may reside *ad lib.*). This Master-slave clock-setting *asymmetry* is essential to avoid the logical bind of the twin paradox, a well-known false consequence of the LT's unearthly symmetry between inertial systems, which gives rise to a whole chapter of relativity apologetics. A clock moved between two truly symmetrical inertial systems could not change its running rate. No?

Proper time τ (the only invariant quantity in Einstein's world), which is measured by uncorrected clocks, has a number of deficiencies that make it inferior to GPS time, which I shall designate t_0. In particular, $\mathrm{d}\tau$ is an inexact differential, whereas $\mathrm{d}t_0$ is exact. Clocks in different states of motion run at different proper time rates. This, together with the inexactness (path dependence) of the proper time differential, makes proper time inferior to GPS or Newtonian time for treating the many-body problem, which is central to physics. The fact that relativity theory relies on proper time and denies the "existence" of Newtonian time has the effect of permanently crippling Einstein's theory in regard to simple handling of the many-body problem.

In summary, then, it appears that multiple myths mutually support special relativity theory. These include covariance, spacetime symmetry, the Lorentz transformation, probably the Lorentz contraction, and the claimed obsolescence of Newtonian time (the sort that flows uniformly and unidirectionally, as *measured* by suitably corrected clocks everywhere throughout space). From this position it is no great stretch to recognize the mythological nature of relativity theory itself, a major pillar of established physics, favored with the blind allegiance of almost all physical and mathematical scholars of our day.

Next, let us consider another pillar of sacrosanctity, Maxwell's equations. Those equations have their part to play in the relativity mythology, in that they suffer from the malady of covariance (remember universal covariance?) – which means they are invariant under no known transformation. They are invariant not even at first order. Indeed, if we were to interrogate history in search of the Ur-justification for spacetime symmetry, hence for covariance, we would be inexorably led to the symmetry of *space and time partial derivatives* in Maxwell's equations. (Here it is worth remembering that Maxwell himself never laid eyes on "Maxwell's equations.") Actually, a very slight mathematical change is needed to correct the problem. This change was first discovered by Heinrich Hertz[1]. It turns out that what spoils invariance under the Galilean transformation is the presence of the *partial time derivative* operator, which is non-invariant [$(\partial/\partial t)' = (\partial/\partial t') = (\partial/\partial t + \vec{v}\cdot\vec{\nabla}) \neq (\partial/\partial t)$, where $\vec{v}$ is the relative velocity of two inertial reference systems].

Hertz found that the electromagnetic field equations (Maxwell's equations, as we know them) could be made strictly invariant by substituting everywhere for the non-invariant $\partial/\partial t$ the invariant total time derivative, $d/dt_0 = \partial/\partial t_0 + \vec{v}_d \cdot \vec{\nabla}$, where $\vec{v}_d$ denotes field detector (absorber) velocity relative to the observer's inertial frame, as measured by GPS clocks. Here and elsewhere I have chosen to use GPS time exclusively, although invariance does not require that specialization. (That the total time derivative is GT-invariant is proven in endnote [1].) One other change in Maxwell's equations is required. The fact that detector velocity $\vec{v}_d$ now enters the field equations explicitly means that the field detector can move with respect to the observer; hence, the measured source current is modified by a convective term, $j_{measured} = j_{Maxwell} - \rho\vec{v}_d$. Otherwise, Maxwell's equations are unaltered. This is all it takes to make them invariant[2] rather than covariant.

Apart from the switch to invariance, are there other gains realized from this modification? Yes, the fact that the new field equations contain ra detector velocity parameter $\vec{v}_d$ previously appearing only in an add-on relationship known as the Lorentz force law is highly significant. It suggests that we may be able to dispense with an extra force law entirely; that the new field equations may be able to do the whole job of predicting electromagnetic force without extra help. In endnote [2] it is proven that this is indeed the case. There it is shown that the newly predicted force law is that of Lorentz, plus an extra term dependent on vector potential. Thus, our new-found law of force on charge q is $q\vec{E} = \vec{F}_{Lorentz} - q\vec{\nabla}\left[\left(\vec{v}_d/c\right)\cdot \vec{A}\right]$

This is worth studying. Observe that the extra force term is the gradient of a scalar quantity. This means that it integrates to zero around any closed curve. Currents were said by Maxwell to flow only in closed circuits. So, it is evident that ordinary experiments with closed circuits are not going to reveal the presence of this term. But certain cases are exceptional. In particular, currents flow irregularly in *plasmas*, without regard to circuit closure. Thus in experiments with Tokamak-type devices this extra term would be expected to come into play.

What is the nature of the extra force term? It contains the vector potential, which is a vector directed in the same direction as current flow. This vector is dotted into another, $q\vec{v}_d$, which is a current, if we recognize that the "field detector" is what Lorentz would call the *test charge*. The nature of this extra force term is therefore similar to the once- famous, long-forgotten "Ampère longitudinal force[2]," which the father of electrical physics deduced as necessary to satisfy Newton's third law of action-reaction equality. That equality, as it happens, got quietly dropped when the Lorentz law was credited with telling the whole story, as part of the unqualified triumph of relativity, post-1905.

Suppose Ampère longitudinal forces are real. Then the billions spent on hot fusion experiments, and recognized as thrown away because of mysterious plasma instabilities of "unknown" origin, may be seen as of origin *known* since the beginning of the nineteenth century, but disregarded because a subsequently dominant false ideology prevented giving the history of science so much as a glance. One of the more shameful chapters of that history, by the way, is the one devoted to relativistic apologetics for the treatment (disregard, actually) of Newton's third law in electrodynamics.

It is time for a summation. I began by making fun of the endeavor to perceive magic in the miraculous fit of mathematics to physics. My criticism was based on showing by repeated, mutually-reinforcing examples that the classical physics and relativity theory on which we place unquestioning reliance is, often as not, plain wrong. It is thus ludicrous to marvel at a fit that does not exist. But in the position I have adopted there is a strong element of paradox. For I found it no trouble at all, once the mind has been cleared of its current universal fog of political correctness, to find perfectly workable theoretical alternatives, based on universal invariance, invariant electromagnetism, corrected clocks, etc. One merely has to replace one ideology with another. Having done so, the original problem resurges – why the close fit of (corrected) mathematics to (modified) physics?

It seems to me that a key idea here is that of *progress*. As long as progress in the science of physics is recognizable as real, needed, and possible, physics and its associated mathematics will run on ever more nearly parallel tracks (although, I insist, never on identically the same track). One must recognize that the two tracks distort each other by proximity, the "physics" being always subject to "interpretation" that bends it all out of shape. (Case in point: the bending of the GT into the LT to represent physical inertial motion.) The prevalence of progress is the condition for physics to remain a healthy science. But let progress become an accomplished fact, so that physics becomes merely a taught and learned static doctrine – expressing eternal truths beyond which there is no progress – then the subject will sink into the doldrums and fall into the hands of mathematicians, much as it has done of late …and much as it did in the time of Ptolemy. So, the big question for modern physics is *how over-mathematized will it become before the idea of progress becomes laughable?*

Notice that I have said not a word here about genuinely modern physics – that of the last half-century, built on what I have just debunked. I have confined my criticisms to classical physics that is supposed to be "settled science." When people tell me I must know all about semi-groups in order to understand modern physics, I am ready to claim that the physics track is already bent out of shape in order to match the more inflexible math track. This sort of thing is bound to happen when the empirical underpinnings of physics (the reality checks) are warped or absent. In the physical domains of both the very large and the very small, this is unavoidable: experiments are few and costly; so mathematical imagination is free to run wild. The rewards and prizes (which already tend to run wild) multiply in inverse proportion to the worth of the things prized.

References:

[1] H. R. Hertz, *Electric Waves* (Dover, New York, 1962), Chap. 14 (translated by D. E. Jones).

[2] T. E. Phipps, Jr., *Old Physics for New* (Apeiron, Montreal, 2012).

Endnotes:

[1] *Proof of GT-invariance of the total time derivative*: From $\vec{\nabla}' = \vec{\nabla}$, d/dt_0 the definition, and the Galilean velocity addition law, $\vec{v}'_d = \vec{v}_d - \vec{v}$, we get

$$\begin{aligned}(d/dt_0)' &= (\partial/\partial t_0 + \vec{v}_d \cdot \vec{\nabla})' = \partial/\partial t_0 + \vec{v}'_d \cdot \vec{\nabla}' \\ &= (\partial/\partial t_0 + \vec{v} \cdot \vec{\nabla}) + (\vec{v}_d - \vec{v}) \cdot \vec{\nabla} = \partial/\partial t_0 + \vec{v} \cdot \vec{\nabla} = d/dt_0\end{aligned}$$

q.e.d.

[2] *Force on test charge predicted by modified field equations*: In Maxwell's theory the electromagnetic potentials $(\phi, \vec{A})$ obey $\vec{E}_{Maxwell} = -\vec{\nabla}\phi - (1/c)(\partial/\partial t)\vec{A}$. In the proposed alternative invariant theory this is changed to

$$\vec{E} = -\vec{\nabla}\phi - (1/c)(d/dt_0)\vec{A} = -\vec{\nabla}\phi - (1/c)\left[\partial/\partial t_0 + (\vec{v}_d \cdot \vec{\nabla})\right]$$

In the vector identity

$$\vec{\nabla}(\vec{a} \cdot \vec{b}) = (\vec{a} \cdot \vec{\nabla})\vec{b} + (\vec{b} \cdot \vec{\nabla})\vec{a} + \vec{a} \times (\vec{\nabla} \times \vec{b}) + \vec{b} \times (\vec{\nabla} \times \vec{a}),$$

let $\vec{a} \equiv \vec{v}_d(t_0)$ and $\vec{b} \equiv \vec{A}$. [The field detector is considered to be a point particle or small non-rotating rigid body, $\vec{v}_d(t_0)$, not a squirmy "mollusc" or velocity field, $\vec{v}_d(x, y, z, t_0)$.] The identity simplifies to

$$(\vec{v}_d \cdot \vec{\nabla})\vec{A} = -\vec{v}_d \times (\vec{\nabla} \times \vec{A}) + \vec{\nabla}(\vec{\nabla} \cdot \vec{A}).$$

Then $\vec{E} = \vec{E}_{Maxwell} + (\vec{v}_d/c) \times \vec{B} - (1/c)\vec{\nabla}(\vec{v}_d \cdot \vec{A})$, where use has been made of $\vec{B} = \vec{\nabla} \times \vec{A}$. Thus the force on test charge q (absorber or field detector) is $q\vec{E} = \vec{F}_{Lorentz}(q/c)\vec{\nabla}(\vec{v}_d \cdot \vec{A})$, that is to say, the Lorentz force plus an extra gradient term. This force $q\vec{E}$ is purely electric, and is the only force on electric charge.