The Myth of FitzGerald-Lorentz Length Contraction and the Reality of Einstein's Velocity Transformation

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The origin of the FitzGerald-Lorentz length contraction (FLC) hypothesis is discussed and attempts to verify it empirically are analyzed in detail. It is pointed out that the Lorentz transformation (LT) and the FLC not only rest on Einstein's two postulates of relativity, but also depend in a crucial way upon an additional assumption that was untested experimentally at the time. It is shown that replacing the latter assumption with another based on direct tests of the time-dilation phenomenon also explains the Michelson-Morley null result without invoking either the LT or the FLC but while still remaining consistent with the relativistic velocity transformation (VT). Moreover, an example is presented ("clock riddle") which demonstrates a lack of internal consistency in the LT formulation of relativity theory. Finally, it is shown that all confirmed experimental results that have hitherto been ascribed to the LT can be predicted equally well on the basis of an alternative Lorentz transformation (ALT) that avoids the above inconsistency. This discovery leads to a restatement of the relativity principle (RP) which not only recognizes that physical laws are the same in all inertial systems but also that the units in which they are expressed may differ in a systematic manner from one rest frame to another as a consequence of acceleration.

Keywords: postulates of special relativity, degree of freedom in the Lorentz transformation, velocity transformation (VT), alternative Lorentz transformation (ALT), amended relativity principle (ARP)

I. Introduction

The theoretical possibility of relativistic length contraction obtained credence in the latter part of the 19th century [1]. Experiments that were carried out in this period demonstrated that key assumptions of classical theory were in need of revision. FitzGerald [2] was the first to suggest that the negative result of the Michelson-Morley experiment [3] could be explained by assuming that the lengths of objects change as they move through the air. A few years later Lorentz [4] independently came to the same conclusion. Both authors believed that it was necessary to assume that there is an interaction with an "aether" that is responsible for the decrease in length of an object along the parallel direction. A key aspect of their speculation was that no change was expected in the perpendicular direction. One of the main objectives of Einstein's relativity theory in 1905 [5] was to show that an aether is not required to explain the phenomena in question, but he nonetheless concluded on the basis of the Lorentz transformation (LT) that the FitzGerald-Lorentz length contraction

(FLC) hypothesis was correct without any need for modification. It has proven quite difficult to verify this prediction of Einstein's theory on a definitive basis, but there have been some observations which have been claimed to at least provide convincing indirect confirmation of its validity.

The original deduction of the FLC is based squarely on the classical Galilean velocity transformation (GVT). It assumes that velocities are simply additive. The GVT by itself is unable to explain the experimental observation that the speed of light in free space is independent of the state of motion of the source. This was the result of the Michelson-Morley experiment [3], for example, at least if one neglects the effects of the air through which the light moved. Einstein's theory [5] replaces the GVT with the more complicated relativistic velocity transformation (VT). It reduces to the GVT in the limit of slowly moving objects, but is nonetheless able to account for the non-additivity observed in the case of the speed of light. The VT is closely related to the LT and the mathematical relationship between them is the key to understanding how the FLC took hold in Einstein's theory, as will be discussed below.

II. The Non-uniqueness of the Lorentz Transformation

Application of the GVT to the Michelson-Morley experiment is quite straightforward. It is assumed that light travels at speed c relative to the observer when the source is stationary in the laboratory. It is further assumed that there exists a medium/aether traveling at speed v along the positive x axis. When the light is dragged along by the aether, the resultant speed of the light is dependent on its direction. It has a maximum value of c + v if the direction is parallel to that of the aether, but only c – v if the direction is anti-parallel. Due to Fizeau's experimental verification of the Fresnel light-drag effect in 1851, it was already known to physicists [6] that this view of the kinematics of light was oversimplified if the medium is water or some other transparent liquid. Nonetheless, it was concluded on the basis of the GVT that the time Δt_x required for a light beam to pass a distance L between the source and a mirror and then return to its starting point along the x axis is:

$$\Delta t_x = L\,(c + v)^{-1} + L\,(c - v)^{-1} = 2Lc^{-1}\,(1 - v^2/c^2)^{-1} \tag{1}$$

It was further assumed that the speed of light was not affected by the aether if it moves in a perpendicular direction. Use of the Pythagorean Theorem then gives the following result for the return-trip elapsed time Δt along the y axis [7]:

$$\Delta t_y = 2Lc^{-1}\,(1 - v^2/c^2)^{-0.5}\,. \tag{2}$$

The conclusion was directly applicable to the Michelson-Morley experimental arrangement [3]. If two light beams left the origin at the same time, traveling exactly the same distance L to their respective mirrors, they would not return at the same time if the speed of the aether was not equal to zero. The FLC assumption is intended to remove any such discrepancy in the elapsed times. Accordingly, the distance L in eq. (1) was predicted to be contracted by a factor of $(1 - v^2/c^2)^{0.5}$ when the light moves along the x axis because of an interaction with the aether, whereas the distance was unchanged in eq. (2) when the light moves along a perpendicular direction. This correction succeeds in making $\Delta t_x = \Delta t_y$, in *ad hoc* agreement with the experimental result obtained by Michelson and Morley in their interference experiment [3]. Because of the relativity principle (RP) it had to be assumed that an observer co-moving with the aether would not notice any contraction in his rest frame, but FitzGerald nonetheless concluded [1,2] "that the molecular forces are affected by the motion, and that the size of a body alters consequently." One can be forgiven for asking what constitutes the body in the example of the light beams moving a distance L through space, but the above conclusion for changes in the sizes of actual material objects is suitably concrete and has maintained credence up until the present day.

Einstein [5] changed the narrative in two ways with his theory. He removed the aether as a possible influence and he simply postulated that the speed of light in free space is independent of the state of motion of the source and is therefore independent of direction in the example under discussion. He nonetheless maintained belief in the FLC as a consequence of his overall goal of developing a new kinematic theory. To see how this situation came about, it is helpful to take a close look at the relativistic velocity transformation (VT) that was introduced as a replacement for the classical GVT:

$$u_x' = (1 - vu_x c^{-2})^{-1}(u_x - v) = \eta\,(u_x - v) \quad (3a)$$

$$u_y' = \gamma^{-1}\,(1 - vu_x c^{-2})^{-1}\, u_y = \eta\,\gamma^{-1}\, u_y \quad (3b)$$

$$u_z' = \gamma^{-1}\,(1 - vu_x c^{-2})^{-1}\, u_z = \eta\,\gamma^{-1}\, u_z. \quad (3c)$$

The variables u_x, u_x' etc. are the components of the velocity of an object as measured by respective observers in two inertial systems S and S' which are moving with relative speed v along their common x,x' axis [$\gamma = (1 - v^2c^{-2})^{-0.5}$] (actually, only eq. (3a) is given explicitly in Einstein's paper [5]). In the limit of $v = 0$, both γ and η approach values of unity and the equations become identical with those of the GVT ($u_x' = u_x - v$, $u_y' = u_y$ and $u_z' = u_z$). However, the VT satisfies Einstein's light-speed postulate, that is, if the magnitude/speed of the object's velocity **u** is c in S, then the transformed velocity **u'** observed in S' will also have the same magnitude, although generally with a different direction than **u**. With respect to the discussion of the Michelson-Morley experiment [3], the VT achieves the seemingly counter-intuitive result of "c' = c – v = c." This was the very result that FitzGerald and Lorentz tried to achieve with their length-contraction assumption and the intervention of an aether.

The centerpiece of Einstein's relativity theory is not the VT, however, but rather the Lorentz transformation (LT) of space (x, y, z and x', y' and z') and time (t and t') coordinates from which it is derived. One of its main characteristics is the Lorentz invariance condition:

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = x^2 + y^2 + z^2 - c^2t^2. \tag{4}$$

A particularly simple way to derive Lorentz invariance is to start with the following equations for the motion of a light pulse, in which case the x, y, z, t and corresponding primed variables need to be replaced by intervals $\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$ etc. for two locations at different times t_2 and t_1 in order for the results to be applicable to speeds, i.e ratios of intervals of space and time:

$$\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^{2\Delta}t'^2 = 0 \tag{5}$$

$$\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2 = 0. \tag{6}$$

These two relations clearly satisfy Einstein's light-speed postulate for observers in S and S' and they can be used to arrive at the interval version of eq. (4) by simply equating their respective left-hand sides. The implicit assumption is then that Lorentz invariance holds quite generally for the motion of all objects and not just for the special case of a light pulse. However, there is an obvious mathematical objection, or at least a point of interest, to be raised about the way eqs. (5-6) have been used in the derivation of eq. (4). Since they are both homogenous equations, it follows that one can just as well conclude that the two left-hand sides are simply related by an arbitrarily chosen proportionality constant ε, thereby leading to the much less specific condition of invariance:

$$\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2\Delta t'^2 = \varepsilon^2 (\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2). \tag{7}$$

This degree of freedom was first pointed out by Lorentz in 1898 [8] and it was also known to Einstein at the time that he gave his derivation of the LT [5]. It led Lorentz to write down the following general space-time transformation:

$$\Delta t' = \gamma\, \varepsilon\, (\Delta t - v\Delta x c^{-2}) \tag{8a}$$

$$\Delta x' = \gamma\, \varepsilon\, (\Delta x - v\Delta t) \tag{8b}$$

$$\Delta y' = \varepsilon\, \Delta y \tag{8c}$$

$$\Delta z' = \varepsilon\, \Delta z, \tag{8d}$$

in which the proportionality constant ε of eq. (7) appears in each of the four relations. Squaring and adding leads directly to eq. (7). Division of the respective equations for Δx',Δy'andΔ z' by Δt' also leads directly to the VT of eqs. (3a-c), with the velocity components $u_x = \Delta x \Delta t^{-1}$, $u_x' = \Delta x' \Delta t'^{-1}$, etc.

Einstein nonetheless insisted that the only acceptable value for the proportionality constant was $\varepsilon = 1$ (he referred to it as φ in his work; see p. 900 of Ref. 5). He did this by first asserting that φ/ε is a "temporarily unknown function of v." He then gave a symmetry argument that proved that under this condition the function could only have a constant value of unity. Both the LT and the Lorentz invariance condition of eq. (4) therefore rest solidly on the (undeclared and untested) assumption that φ cannot depend on any other variable than the relative speed v of S and S'. He also showed that application of the LT, using eqs. (8b-d), to differences of spatial coordinates (with $\varepsilon = 1$) leads to the FLC, i.e. $\Delta x' = \gamma\Delta x$, $\Delta y' = \Delta y$ and $\Delta z' = \Delta z$.

The discovery that the FLC fits in perfectly with Einstein's theory raises the question of how this applies to the Michelson-Morley experiment [3]. It is an interesting fact of history that Einstein never mentioned this work in his 1905 paper, claiming to have never heard of it prior to that time and therefore to not have been influenced by it in formulating his theory of relativity [9]. Yet the FLC originally came about because of an attempt to understand the null result of the Michelson-Morley experiment within the context of the classical GVT. How can it be that one still needs the FLC for this purpose even after a new velocity transformation, the VT of eqs. (3a-c), has been introduced to give a less *ad hoc* interpretation of the kinematics of light propagation than was previously possible? First of all, it is clear that the VT satisfies the light-speed postulate and therefore guarantees that light travels at the same speed across both arms of the Michelson-Morley apparatus regardless of the state of motion of the observer. That is sufficient in itself to guarantee that the time of travel will be the same as long as the length L of the arms is the same. As soon as one claims in addition, because of the FLC, that the two lengths *will differ in other inertial systems*, however, it is clear that the travel times will no longer be equal for observers moving relative to the laboratory in which the apparatus is at rest. Consequently, they should not find a null result in the interference pattern, in contradiction to what is always observed. In short, the VT and the FLC do not mesh theoretically when applied to the Michelson-Morely experiment. This raises the question of whether Einstein's value of $\varepsilon = 1$ in the general Lorentz transformation of eqs. (8a-d), which is not essential for the VT but is for the FLC and the LT, is actually correct. Before pursuing this possibility, it is well to review other experiments that have been claimed to be supportive of the FLC.

III. Attempts to Verify the FLC

It is fundamentally impossible to directly measure the length of an object when it is moving relative to the observer. Thus, one has to rely on some hopefully reliable assumptions to deduce the way in which distances vary with relative velocity **v**. The remarks of Kennedy and Thorndike in 1932 [10] are instructive as to how such investigations have tended to proceed: "In fact, it seems that the only experiment heretofore reported that permits any definite interpretation is that of Michelson and Morley; and the null result of this experiment is completely explained if we suppose that space dimensions in the direction of motion are contracted by an amount depending upon a suitable function of velocity." Careful reading of their statement shows that they don't actually assume that the FLC is established fact, but they nonetheless used it as the basis of their experiment "to test directly whether time satisfies the requirements of relativity." Kennedy and Thorndike expand upon the FLC assumption one page later by stating that "the Michelson-Morley experiment indicates that a system moving with uniform velocity v with respect to such a system has dimensions in the direction of motion contracted in the ratio $[1 - v^2/c^2]^{0.5}$ as compared to dimensions in the fixed system, while dimensions perpendicular to this direction are unchanged. This is in part assumption, for although there can be little doubt that the experiment yields a strictly null result, nevertheless it actually shows only that dimensions in the direction of and perpendicular to the motion are in the ratio mentioned."

But what stands in the way of assuming that the real cause of the null result in the Michelson-Morley experiment is simply that the speed of light is the same in all directions in all inertial systems, exactly as Einstein's second postulate of relativity demands? Kennedy and Thorndike certainly don't deny this possibility. Quite the contrary, they accept it as fact when they conclude that the LT is satisfied in both their experiment and in Michelson-Morley's because this assumption guarantees that the VT is valid as well. The null result in the latter experiment can just as surely be explained by assuming that the distances traveled by the light *are exactly equal in the two* *arms*. That distance can still depend on v, but the assumption of equal speeds in the two arms simply demands that the respective times of travel vary in the same way so that the ratio of distance travelled to elapsed times is completely independent of v. In the Kennedy-Thorndike experiment [10] it was found that a null result also occurs when the two arms are not of the same length. These authors conclude that the LT is also valid in this case, which means that the VT is satisfied as well. Since they start out with the assumption that the FLC is also valid, they have forced a value of $\varepsilon = 1$ in the general Lorentz transformation of eqs. 8a-d. Every other characteristic of the LT must also be satisfied in their analysis because of this choice, including time dilation. If they had made a different choice for the value of ε, the VT would still be satisfied by their experimental data, but the FLC would not. It is still possible to have time dilation in the theory when the VT is assumed to be valid, but it must have different characteristics than the version implied by the LT, as we shall show in the following section.

The main point of the present discussion should be obvious. *It is impossible to prove the validity of the FLC on the basis of a logical argument that assumes it.* It only takes one assumption to fix the value of ε in eqs. (8a-d) and therefore eliminate the degree of freedom inherent in the general Lorentz transformation. There are a large number of equivalent assumptions that will lead to the LT as long as no error in logic is committed along the way. One can use Einstein's assumption about the restricted functional dependence of ε/φ [5] or make a similar assumption about coefficients in the Lorentz boost matrix [11-13], or start out by assuming the validity of the FLC, as Kennedy and Thorndike have done [10], or insist on the Lorentz invariance condition of eq. (4) or demand that $y' = y$ and $z' = z$ because the motion of S,S' is perpendicular thereto [14] or take any other of the LT equations as a necessary fact of nature. One can also argue that the Lorentz matrices must satisfy certain group properties, at least in the direction of relative motion. In each case the train leads unerringly to the same place, the LT [15]. But all of the above assumptions are purely theoretical. The best one can say is that they lead to an aesthetically pleasing appearance for the space-time transformation, but they do not constitute proof in themselves that the resulting theory is correct. Even new confirmatory evidence doesn't prove this if one adheres to the strict rules of logical argumentation, but of course one is encouraged when indisputable information of this character becomes available. The fact is that experiment can only have a decisive effect when it *contradicts* a clear prediction of a theory. However, it is also possible to invalidate a theory by showing that it is not internally consistent. When either of these possibilities occurs, a new "covering" theory must be found that removes the contradiction while still maintaining consistency with all the previous successful predictions/interpretations of its precursor.

In this spirit of the above remarks it is interesting to consider more recent independent evidence of the validity of the FLC which is based on experiments studying the Josephson effect. Laub et al. [16] found that magnetic flux quanta in Josephson tunnel junctions undergo Lorentz contraction. In annular junctions pairs of vortices and anti-vortices are created that move in opposite directions and "collide" with each other. The collision region was visualized in their experiments with the aid of low-temperature electron microscopy. It was assumed that the length of the collision region is proportional to the length of the vortices. The collision region was found to contract with increasing vortex velocity. A decrease in the length of this region with increasing voltage, which is proportional to the vortex speed, is clearly visible in the authors' Fig. 4 [16]. The experiments are not able to quantitatively verify that the collision length varies in direct proportion to $\gamma^{-1}(v)$, however, as prescribed by the FLC.

In analyzing the Laub et al. data, it is important to keep in mind that the length of the collision region is a measure of the density of electrons moving in opposite directions through the Josephson apparatus. Consequently, *there is another reason for the decrease in length that has nothing whatsoever to do with Lorentz contraction.* The distribution of such large numbers of electrons is characterized by a de Broglie wavelength λ [17] which is inversely proportional to their momentum p in the laboratory ($\lambda p = h$, Planck's constant). That this effect is distinct from Lorentz contraction is seen from the fact that the velocity dependence of the FLC is the same for all types of particles, whereas the de Broglie wavelength contraction will be far greater for collections of protons than for electrons moving at the same speed. Without doing more quantitative studies, it is impossible to know which of the two effects is actually responsible for the observed contraction, but there seems to be no reason to discount at least a partial effect caused by de Broglie's law [17]. This possibility has been ignored in the discussion given by Laub et al. [16]. Similar remarks hold for more recent experiments [18] demonstrating an increase in the frequency of synchrotron radiation when electrons are accelerated. Increased ionization is also expected when the de Broglie wavelength of a distribution of nuclei is shortened by virtue of an increase in their speed relative to the laboratory.

There is another fundamental experiment that has a bearing on the question of relativistic length variations that has largely escaped attention, however. The transverse Doppler effect was first demonstrated by Ives and Stilwell in 1938 [19]. It was predicted by Einstein in his original work [5] and the results of their experiment are universally accepted as proof of the existence of time dilation. After elimination of the directional (non-relativistic) portion of the Doppler effect, it is found that the frequency of radiation of a moving source is smaller than that of an identical source at rest in the laboratory, and by the predicted factor of γ (v). The actual measurements were not of the frequency of the emitted radiation, however, but rather of its wavelength [19]. It was shown that there is a shift to the red, i.e. to larger wavelength, when absorption lines for the parallel and anti-parallel directions were recorded on the same photographic plate as the corresponding standard value when the source is stationary. The conclusion that the frequency decreases with the speed of the source is based on the thoroughly realistic assumption that it is inversely proportional to the wavelength of the radiation, i.e. the speed of light is independent of v. This is clear evidence of *length expansion accompanying time dilation*, not length contraction. Moreover, the increase in wavelength is the same in all directions since the transverse Doppler frequency is independent of orientation. The RP states that the increase in wavelength will go unnoticed for the observer co-moving with the light source, *which indicates that the diffraction grating used to measure it must have increased in dimension by exactly the same fraction in all directions.*

Nothing that has been found in the discussion of the transverse Doppler effect contradicts the VT in any way, but it does conflict directly with the FLC, and consequently, with a definite prediction of the LT. In the following section it will be shown that the contradiction can be removed without affecting any of the well-known successes of Einstein's theory by simply making a different assumption about the normalization function ε in the general Lorentz transformation of eqs. (8a-d).

IV. The Universal Time-Dilation Law and the New Lorentz Transformation

The Ives-Stilwell [19] experiment still left an important question open about time dilation. According to the LT, the measurement process obeys a relativistic symmetry principle which causes two observers in relative motion to each find that it is the other's proper clocks that are running slower. The transverse Doppler effect provides a potentially definitive means of testing this principle since the theory predicts that a red shift must be observed by *both* observers when they exchange light signals. The Ives-Stillwell results verified that a red shift is found when a light source is accelerated relative to an observer at rest in the laboratory, but it was impossible for a second observer co-moving with the light source to perform the reverse measurement. Several decades later, Hay et al. [20] used the Mössbauer effect to measure the frequency of x-rays from a source that was mounted close to the axis of a high-speed rotor. Since the absorber was located near the rim of the rotor, their experiment allowed them to determine the transverse Doppler frequency shift ($\Delta\nu/\nu$) for the opposite case when the observer was moving faster in the laboratory than the light source. They summarized their findings in the following empirical formula:

$$\Delta\nu/\nu = (R_a^{\,2} - R_s^{\,2})\,\omega^2/2c^2, \qquad (9)$$

where ω is the circular frequency of the rotor and R_a, R_s are the respective distances of the absorber and x-ray source from the rotor axis. Since $R_a > R_s$ in their experimental arrangement, it is clear that a *blue shift* was observed, in clear contradiction to the LT prediction, but the authors preferred to emphasize the fact that the *magnitude* of the effect is in agreement with theoretical expectations. The aforementioned symmetry principle would only be quantitatively verified if their result satisfied a different empirical formula, namely:

$$\Delta\nu/\nu = \gamma^{-1}(|R_a - R_s|\,\omega) - 1 \approx -(R_a - R_s)^2\,\omega^2/2c^2, \qquad (10)$$

This expression is invariant to an exchange of the positions of the absorber and light source, whereas the actual results are anti-symmetric in this regard.

Sherwin [21] took a more critical view of the rotor experiment when writing a few months after it was reported. He pointed out that it was completely *unambiguous* which clock was running slower in the experiment and attributed the observed asymmetry in eq. (9) to the fact that the absorber was subject to acceleration. He asserted that the symmetry expected from Einstein's relativity theory in eq. (10) only occurs under conditions of *uniform translation*, but he gave no experimental evidence for his position that the order of clock rates is completely ambiguous in this case [22].

His conclusion was tested a decade later by Hafele and Keating when they carried out experiments with atomic clocks carried onboard circumnavigating airplanes [23-24]. They found that the elapsed times τ_i on the various clocks could be ordered on the basis of their speeds v_{i0} relative to the Earth's non-rotating axis (center of mass). It was necessary to also take into account the known effects of gravity on the clock rates to separate out the effects of Einsteinean time dilation. Their timing results could be fit to the following empirical formula:

$$\tau_1\gamma(v_{10}) = \tau_2\gamma(v_{20}). \tag{11}$$

It agrees quantitatively with eq. (9) when ν_i^{-1} is substituted for τ_i and the speeds v_{i0} to be inserted in the γ quantities are taken relative to the rotor axis. As such, eq. (11) can be looked upon as the Universal Time-Dilation Law. It gives correct results for all known time-dilation experiments, and it stands in direct contradiction to the LT symmetry principle. What is left out of Sherwin's analysis [21] is the unavoidable conclusion that *all other predictions of the LT also lose their credibility for these cases where asymmetry is observed*. This consequence of the experimental data for time dilation and length variations is far more relevant to the general discussion of relativistic effects than the mere search for a single example for which the symmetry principle of the LT is actually fulfilled.

A survey of the literature shows that all other experimental tests of the LT space-time relationships *only involve ratios of these quantities*. As such they do not require the LT at all and thus cannot be properly cited as verifications of this version of the general Lorentz space-time transformation of eqs. (8a-d). For example, von Laue's derivation of the Fresnel light-drag effect [25] is based exclusively on the VT. Similarly, the aberration of starlight at the zenith can be explained entirely by comparing the parallel and perpendicular velocity components of light in the rest frames of the star and the Earth [26]. The Thomas precession effect [27] for atomic spins also involves the *ratio* of two quantities (a differential angle $d\varphi$ and elapsed time dt) that depend on the normalization factor in eqs. (8a-d) in the same way and thus does not require the LT for its derivation [28].

The Sagnac effect can be explained entirely on the basis of Einstein's light-speed postulate and the VT. Two light beams travelling in opposite directions on a circular platform of radius r rotating with frequency ω must travel different distances before interfering. Beam A must travel completely around to reach this point on the platform during one full revolution. The length travelled is therefore assumed on the basis of the light-speed postulate to be $l_A = ct_A = 2\pi r + r\omega t_A$, where t_A is the corresponding time of travel. The other beam (B) does not make it all the way around, so its length traveled during one full revolution of the wheel before reaching the point of interference is $l_B = ct_B = 2\pi r - r\omega t_B$. Solving for the respective elapsed times gives $t_A = 2\pi r\, (c - r\omega)^{-1}$ and $t_B = 2\pi r\, (c + r\omega)^{-1}$.

The difference is thus

$$\Delta t = t_A - t_B = 2\pi r(2r\omega)(c^2 - r^2\omega^2)^{-1} \approx 4\pi r^2\omega c^{-2} = 4A\omega c^{-2},$$

which is the observed value in the laboratory (A is the area of the platform). An observer in another inertial system simply measures a different value for Δt because his proper clock runs at a different rate than that at rest in the laboratory, but the same value for the light speed is measured in both cases according to the light-speed postulate.

There is another aspect of the GPS technology that should be mentioned in the context of the Sagnac effect. Once ΔT has been determined by comparison of the clock readings on the satellite and the ground, its value is not generally multiplied by c but rather must take into account the relative speed v of the two clocks. This adjustment is referred to as the linear Sagnac effect, and is often argued to be in violation of the light-speed postulate. It can be explained as follows. Assume the satellite is moving directly away from the earth at the time the light signal is sent. At that time the distance to the satellite receiver is D and it is this quantity that needs to be determined. The signal must travel farther to reach this receiver, however, namely $D + v\Delta T$. The extra term takes the motion of the receiver into account, whereby ΔT is the actual elapsed time for the signal to arrive there. In order to determine D from ΔT, it is necessary to divide this total distance the light travels by c, i.e. $\Delta T = (D + v \Delta T)/c$. Solving then gives: $D = (c - v)\ \Delta T$. It only appears that the speed of light is $c - v$ from this equation. In reality, the success of the adjustment procedure is due entirely to the assumption of the light-speed postulate.

The above discussion can be summarized quite succinctly: *experiment has always supported the VT but it has also always contradicted the symmetry principle demanded by the LT.* The way to proceed is therefore clear. It is necessary to eliminate Einstein's assumption regarding the alleged limited functional dependence of Lorentz's ε in the general space-time transformation of eqs. (8a-d) and replace it with another that recognizes the fundamental asymmetric relationships in the Universal Time-Dilation Law of eq. (11). It is a simple matter to bring the latter result into a form that is compatible with the notation in eqs. (8a-d), namely as:

$$\Delta t' = \Delta t Q^{-1}, \tag{12}$$

where Q is a ratio of the elapsed times in eq. (11) and/or the corresponding γ (v_{i0}) values therein. Combining this result with eq. (8a) allows one to solve for ε:

$$\Delta t' = \gamma\, \varepsilon\, (\Delta t - v\Delta x c^{-2}) = \Delta t Q^{-1}, \tag{13}$$

$$\varepsilon = [(1 - v\Delta x\, \Delta t^{-1} c^{-2})\gamma Q]^{-1} = \eta(\gamma Q)^{-1}. \tag{14}$$

The resulting space-time transformation is thus obtained by substituting this value for ε in eqs. (8a-d):

$$\Delta t' = Q^{-1}\Delta t \tag{15a}$$

$$\Delta x' = \eta\, Q^{-1}\, (\Delta x - v\Delta t) \tag{15b}$$

$$\Delta y' = \eta\, (\gamma\, Q)^{-1}\, \Delta y \tag{15c}$$

$$\Delta z' = \eta\, (\gamma\, Q)^{-1}\, \Delta z, \tag{15d}$$

whereby the first of these equations is identical to eq. (12) by construction. Note that the proportionality factor Q also appears in eqs. (15b-d). This is clearly necessary in order to ensure that the new transformation is consistent with the VT of eqs. (3a-c) (note that η also appears in the VT). The resulting set of space-time equations has been referred to in previous work [30-32] as the alternative Lorentz transformation (ALT). The Lorentz invariance condition of the LT is replaced by the two symmetrically related equations below [(see eq. (7)]:

$$\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2\Delta t'^2 = \eta^2 (\gamma\, Q)^{-2} (\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t^2), \tag{16}$$

$$\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2\Delta t'^2 = \eta'^2 (\gamma\, Q')^{-2} (\Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2\, \Delta t'^2). \tag{17}$$

In the second of these equations η' must be obtained from $\eta = (1 - vu_x c^{-2})^{-1} = (1 - v\Delta x \Delta t^{-1} c^{-2})^{-1}$ in the standard way by exchanging corresponding primed and unprimed values and changing v to –v, i.e. $\eta' = (1 + vu_x' c^{-2})^{-1} = (1 + v\Delta x' \Delta t'^{-1} c^{-2})^{-1}$. The value of γ remains the same because it is a function of v^2. The value of $Q' = Q^{-1}$ is fixed by forming the inverse of eq. (15a), i.e. $\Delta t = Q'^{-1}\ \Delta t' = Q\ \Delta t'$. Both equations must be equivalent in order to satisfy the RP, hence $\eta^2 (\gamma Q)^{-2}$ in eq. (16) must be equal to the reciprocal of $\eta'^2 (\gamma Q')^{-2}$ in eq. (18). The primed variables in the definition of η' can be eliminated by using the ALT, whereupon the following identity is obtained:

$$\eta\, \eta' = \gamma^2. \tag{18}$$

Consequently, eqs. (16) and (17) are seen to be equivalent since $Q' = Q^{-1}$. From the definition in eq. (14) it can be seen that $\varepsilon' = \varepsilon^{-1}$, i.e. by making the usual operation of interchanging primed and unprimed values and setting v = –v. This result satisfies the necessary relationship that produces an identity when the transformation from S to S' is followed by the corresponding one from S' to S.

How are the results of the Michelson-Morley experiment described in the revised version of the theory based on the ALT? Assume that the experiment is first performed in S. The observer there finds that the elapsed round-trip time in both arms of the apparatus is equal to $T = 2Lc^{-1}$ since the length of each arm is L and the speed of light is equal to c. Now imagine that the apparatus is moved away from its original location to inertial system S' where the clocks run $Q = \gamma$ times slower than in S. The observer in S' finds no change in the results of the experiment. He measures the elapsed time to be $\Delta t' = T$ in both arms of the apparatus, the lengths of which are still $\Delta l' = L$ in each case. The speed of light also has not changed for him. All of this is consistent with the RP since both S' and S are inertial systems. The results are different for the observer in S, however, even though he continues to measure the speed of light in the apparatus to have a value of c in all directions. He now finds that the elapsed round-trip time is $\Delta t = \gamma T$ in each arm because of the time dilation in S'. He therefore observes no change in the interference pattern, consistent with experiment. The length of each arm also has changed for him, however. It has increased to $\Delta l = 0.5c\Delta t = 0.5c\gamma T = 0.5c\gamma(2Lc^{-1}) = \gamma L$. Thus the observer in S finds that there is *both time dilation and length expansion in all directions in S'*. His results therefore stand in contradiction to the FLC predicted by the LT, but they still agree with the null result for the time difference observed in the Michelson-Morley experiment itself [3].

There is another way to prove that the LT is inconsistent and therefore invalid. As discussed elsewhere [33], the LT predicts that observers in different inertial systems S and S' must obtain the same value for distances that are aligned in a perpendicular direction to their relative velocity ($\Delta y = \Delta y'$). Yet their clocks run at different rates because of time dilation ($\Delta t \neq \Delta t'$). Since the speed of light must be the same for both, they can perform the above distance measurement in a different way according to the LT, namely by multiplying their respective elapsed times with c. The result in this case is: $\Delta y = c\ \Delta t \neq c\ \Delta t' = \Delta y'$, which stands in direct contradiction to the corresponding FLC prediction mentioned first. Any theory that gives different answers for the same question when it is simply applied in different ways is invalid and must be rejected. This example has been referred to as the "clock riddle" in previous work [33], in contrast to the longstanding "clock paradox." Note that the latter is completely explained on the basis of the Universal Time-Dilation Law of eq. (11) and the corresponding relation of eq. (15a) in the ALT. The standard way to measure Δy and $\Delta y'$ is with a clock, consistent with the modern definition of the meter [34] as the distance traveled by a light pulse in c^{-1} s, and thus it is the contradictory result of equal values for these quantities predicted by the FLC that must be discarded.

The reason that the observer in S finds the length of each arm to be γL whereas his counterpart in S' finds it to be L in the above discussion of the Michelson-Morley experiment is *because the unit of length in S' has increased from 1 m to a value of γ m*. The unit of time is now γ s in S', whereas the unit of velocity remains unchanged since it is the ratio of the latter two quantities. The observer in S' does not notice that his units of time and distance have increased because all changes are uniform relative to their original values in S. This state of affairs is exactly what Galileo was talking about when he enunciated his relativity principle in 1632. He did not claim that the passengers and the objects locked below deck on their ship had not undergone changes as a result of their uniform translation, only that there was no way they could distinguish between their current state of motion and that when they were docked at the seashore. The symmetry principle of the LT *does not permit such a simple formulation in terms of rational units of length and time*. This is not possible when the observers in different inertial systems cannot agree on whose proper clock runs slower or whose meter stick is shorter. For this reason it is advisable to restate the relativity principle as follows:

> *The laws of physics are the same in all inertial systems but the units in which they are expressed can and do vary in a systematic manner from one rest frame to another.*

Physical laws are mathematical equations. They must retain their validity when the system of units is changed, whether this involves simply expressing results in feet instead of meters or pounds instead of Newtons, *or because the standards used to define these quantities have undergone changes themselves*. Experiment tells us that the clock rates, lengths and inertial masses of standard objects change when they are accelerated, and so this must be taken into account when comparing measured values obtained for the same quantity by observers who are stationary in different inertial systems.

The constant Q in the ALT of eqs. (8a-d) is a *conversion factor* between these units and therefore must appear explicitly in the space-time transformation relating measured values in the two participating inertial systems. In a companion article [35], it has been shown that the conversion factors for the units of all physical properties are always powers of Q. Moreover, a different constant determines the corresponding conversion factors required to account for the effects of changes in gravitational potential. Together these values allow one to quantitatively predict how the measurements of the same property will differ for observers located at different positions in the gravitational field and/or in different states of motion.

V. Conclusion

The consensus view among physicists is that Einstein's Lorentz transformation (LT) is the only way to satisfy his two postulates of relativity. However, there are actually an infinite number of ways to do this because of a degree of freedom in the general space-time transformation introduced by Lorentz in 1898. Einstein eliminated this uncertainty in his original derivation of the LT by making an assertion about the functional dependence of a normalization function that appears in each of the space-time equations in Lorentz's formulation. Many of the most famous predictions of Einstein's relativity theory such as the Lorentz invariance condition, Fitzgerald-Lorentz length contraction (FLC) and the symmetry principle governing the respective measurements of observers in relative motion, are dependent on the latter assumption every bit as much as they are on the aforementioned two postulates. The experiments that have been carried out in the preceding century have invariably found that measurement has an asymmetric quality which stands in direct contradiction to the LT. That two clocks can both be running slower than the other at the same time is an idea that has never gone beyond the realm of pure fantasy. The relative rates of atomic clocks onboard airplanes and satellites have been found to depend on their speeds relative to a definite reference frame rather than on their speed relative to one another. Demonstrations of the transverse Doppler effect using high-speed rotors have shown that there is no ambiguity as to whether the clocks associated with the absorber or the x-ray source run slower. It is purely a matter of knowing which one is farthest from the axis of the rotor in the experiment. Claims from the authors that their empirical findings agree with Einstein's theory of time dilation ignore the fact that the sign of the effect is different than predicted and emphasize instead that it simply has the expected magnitude.

The LT is also seen to lack internal consistency when attention is turned to the measurement of the distance between two points that lie along a line that is *perpendicular* to the relative velocity of two inertial systems (clock riddle [33]). According to the FLC, two observers that are stationary in S and S' respectively must agree on the value of this distance ($\Delta y = \Delta y'$). Yet the LT also predicts that the proper clocks used by these observers will run at different rates ($\Delta t \neq \Delta t'$) even though the speed of light is the same for both. Making use of the latter two characteristics of the theory shows that the two observers must *disagree* on the magnitude of the above distance if they carry out the measurements by sending a light pulse between the two points since $\Delta y = c\ \Delta t \neq c\ \Delta t' = \Delta y'$. Since the modern definition of the meter [34] is the distance traveled by a light pulse in

c^{-1} s, it is clear that the latter measurement procedure is standard and thus the contradictory result predicted by the FLC must be discarded.

The above considerations show definitively that the LT is not a physically valid space-time transformation, but they in no way speak against Einstein's postulates and his velocity transformation (VT). Instead, they show that Einstein erred with his assumption regarding the normalization constant in the general Lorentz transformation. Any value of ε in eqs. (8a-d) leads to the same VT and that is the key to correcting the problems that the LT imposes on relativity theory. The solution is to find a different value of ε that is consistent with the asymmetry observed in time-dilation experiments, specifically with the Universal Time-Dilation Law of eq. (11). The resulting set of equations is referred to as the alternative Lorentz transformation (ALT [30-32]). It does not affect the fundamental predictions of the VT such as the aberration of starlight at the zenith and the Fresnel light-drag effect, but it does remove a number of aspects of the theory such as the FLC that have never received direct experimental confirmation. It eliminates the concept of space-time mixing, for example, as well as remote non-simultaneity. Instead, it foresees a strict proportionality between the values of measured times in different inertial systems ($\Delta t' = Q^{-1}\Delta t$). The theory of measurement implied by the ALT is completely objective as a result, unlike the case for the LT, which claims that it is just a matter of perspective which of two elapsed times or lengths or inertial masses is greater. Employing the ALT instead makes possible the use of a different set of rational units in each inertial system. The conversion factors connecting them are powers of the constant Q appearing in all four of the ALT equations. The laws of physics are the same in all inertial systems as a result, but the relativity principle has been restated in Sect. IV (amended relativity principle or ARP) to take account of the basic fact that the units on which they are defined vary with the state of motion of the observer. In this way, Galileo's vision is upheld for passengers locked up below deck on a ship moving on a calm sea, without giving up on either of the later discoveries of the constancy of the speed of light and the slowing down of proper clocks as they are accelerated.

(April 3, 2013)

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