Radius of electron, magnetic moment and helical motion of the charge of electron

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Depending on different electromagnetic phenomenon, several models of electron are described by the scientists for more than a century. Electromagnetic phenomenon revealed eight different electron radii, which are related with each other in -quantized way. Leading from one -quantized relation amongst classical electron radius and Compton radius of electron, composite radius is defined. Higher order corrections to magnetic moment and g-factor are used to describe more accurate and a generalised form of composite radius. Depending on the generalised composite radius the helical model of electron is developed which is a modified relativistic spinning sphere model but with slightly aspheric nature.

Keywords: Electron-model, Magnetic moment, Electron radius

Introduction

Electron was discovered in 1897 by J. J. Thomson. After the discovery of electron several models of the electron have been proposed [1], [2]. The proposals are based on the properties of the electron, which are indeed enigmatic. They are roughly divided into three classes, in which the electron is regarded as: a) A strictly point-like particle; b) An actual extended particle; c) An extended-like particle in which the position of the point-like charge is distinct from the particle center-of-mass [2].

As electron is a charged lepton, its properties involve electromagnetic phenomena. Different electromagnetic phenomena revealed eight different radii of electron [3],[4]. The models of electron are also related to the size of the particle or the radii and hence with electromagnetic phenomenon as those different electromagnetic phenomena are the origin of the radii. Relativistic spinning sphere model of electron introduces the spectroscopic way to treat electron model in a semi-classical manner which involves a spherical structure of the particle with tiny charge and mass without violating QED theory [3].

It is noteworthy that the strictly point-like models face the problem with classical formalism and the velocity goes beyond c. Again the extended model in which the charge is glued over the entire body violates QED. Hence the extended body with a point like charge is more approachable. Relativistic spiining sphere model [3] is of that type. Here we are introducing the way to corelate relativistic spinning sphere model with the semi-classical helical motion of charge, which in other words can be said as type of zitterbewegung motion. Zitterbewegung model of electron was [5], [6], [7] and [8], originally proposed by Schrodinger [5] is also carrying the feature of an extended-like particle with a point-like charge that is distinct from the behaviour of its center-of-charge and center-of-mass.

Indeed the hypothesis of spinning electron or a fast rotating particle incorporates an angular momentum and a magnetic moment to the electron [9], [10]. This magnetic moment was originally introduced due to Dirac equation and calculated of one Bohr magneton [11]. Again the g-factor coming out from magnetic moment is related to the fine structure constant, which is claimed to be one of the most accurately measured constants. These leave impact on the facts and figures of the spectroscopic properties of electron. Hence starting with a semi-classical model of electron we can proceed to the QED-corrected region of particles to describe the electromagnetic phenomenon with the help of different electron radii and also in connection with the fine-structure constant.

In the framework of relativistic spinning sphere model we have incorporated the helical motion of point-like charge of the electron with the help of the fine structure constant and the recent measurements of anomalous magnetic moment of the electron.

Magnetic self-energy and composite radius of electron

Four different kinds of mass (or equivalently energy) are attributed to electron. They are electrostatic self-energy (W_E), magnetic self-energy (W_H), mechanical mass (W_M) and gravitational mass (W_G). Magnetic self-energy is about only 0.1% of the total energy of electron [3].

Magnetic self-energy of an electron is the energy contained in the magnetic field, associated with the magnetic moment [3].

Therefore using this concept we develop the electromagnetic part of desired model. According to RSS model of electron [3], which is in close approximation with the calculation of Rasetti and Fermi [3], the total magnetic self-energy of the electron comes out as

$$W_H = \frac{2\mu^2}{3{R_H}^3}, \tag{1}$$

where $\mu(= \frac{e\hbar}{2mc})$ is the magnetic moment and R_H is the magnetic field radius of electron [3]. Magnetic field radius is closer to Compton radius in size. To match the theoretical and experimental values of magnetic moment of electron, in 1948 J. Schwinger introduced a correction term, which is known as Schwinger-corrected mass term [12]. Schwinger-correction can be expressed in terms of energy as

$$W_H \simeq m.\frac{\alpha}{2\pi}c^2. \tag{2}$$

Equating the expressions (1) and (2) for magnetic self-energy, we have

$${R_H}^3 \simeq {R_C}^3(1 + \frac{\alpha}{2\pi})^2 \tag{3}$$

Re-arranging and re-combining the terms of equation (3) we resolve a composition (only addition in length) of classical electron radius and Compton electron radius as

$${R_H}^3 = R_C {R_{C0}}^2, \tag{4}$$

where

$$R_{C0} = (R_C + \frac{R_0}{2\pi}) \tag{5}$$

As R_{C0} is defined basically with the classical electron radius and Compton radius, we say this as Composite radius of electron. Fine structure constant, $\alpha = \frac{e^2}{\hbar c}$ relates the classical electron radius and Compton radius [3],[13] in the way

$$R_0 = \alpha R_C. \tag{6}$$

Equation (6) indicates α-quantizedrelation among the two radii of electron. In fact RSS (relativistic spinning sphere) model, given by M. H. MacGregor [3] correlates the spectroscopic properties of the electron accurately to first order in α. Results of some other properties of the electron are also observed with α-quantization in some recent works [14], [15] and [16].

Using the relation (6) in equation (5) we can define composite radius in another way as

$$R_{C0} = R_C(1 + \frac{\alpha}{2\pi}). \tag{7}$$

Magnetic moment of electron, g-factor and composite radius

Magnitude of the fundamental intrinsic magnetic moment of electron without the radiative corrections is defined as $\frac{e\hbar}{2mc}$ [17]. Hence this is also being known as zeroth-order value for electron magnetic moment [3]. In QED the measurement of magnetic moment of electron states the interaction of electron with the fluctuating vacuum. This also ensures of substructure of electron [18], [19] and [20]. This zeroth-order of electron magnetic moment was given by Uhlenbeck and Goudsmit [3]. Later it was realised that the actual magnetic moment for electron is approximately 0.01% larger than this value. This concludes in a corrected form of magnetic moment as [3]

$$\mu = \frac{e\hbar}{2mc}(1 + \frac{\alpha}{2\pi}), \tag{8}$$

where, α is the fine structure constant and $\frac{\alpha}{2\pi}$ is the famous Schwinger correction [12]. Combining equations (7) and (8), we have the magnetic moment of electron as

$$\mu = \frac{eR_{C0}}{2}. \tag{9}$$

Hence one can say that

$$\mu = \frac{eR_C}{2} + \frac{eR_0}{4\pi}. \tag{10}$$

The factor $(1 + \frac{\alpha}{2\pi})$ made it possible to express the magnetic moment with R_C and R_0 . The factor $(1+\frac{\alpha}{2\pi})$ is also connecting the g-factor and the fine structure constant as [3]

$$\frac{g}{2} = 1 + \frac{\alpha}{2\pi} \tag{11}$$

and equation (11) states that about the dependence of g-factor on α [21].

In fact with recent result, a more accurate g is expressed as [22]

$$\frac{g}{2} = 1 + (\frac{\alpha}{2\pi}) - 0.3284790(\frac{\alpha}{\pi})^2 + 1.1765(\frac{\alpha}{\pi})^3 - 0.8(\frac{\alpha}{\pi})^4. \tag{12}$$

It is to be mentioned that more accurate value of g means the change in the value of magnetic moment also. Hence the structure of this composite radius also changes and exactly this is expressed from both equations (11) and (12) in equation (13)

$$R_{C0} = \frac{g}{2} R_C. \tag{13}$$

With the help of g factor and equation (13) the ratio of classical electron radius and Compton radius can be concluded as

$$\frac{R_0}{R_C} = 2\pi(\frac{g}{2} - 1). \tag{14}$$

Again the factor $(\frac{g}{2} - 1)$ is related with the anomalous magnetic moment of electron and the Bohr magneton as [23]

$$a = \frac{\mu}{\mu_B} - 1 = \frac{g-2}{2}. \tag{15}$$

where,a is the anomalous magnetic moment of electron.

Using equations (13) and (15) together, one can conclude that,

$$R_{C0} = (1 + a) R_C. \tag{16}$$

Electromagnetic mass of electron is defined as

$$\Delta m = m.\frac{\alpha}{2\pi}. \tag{17}$$

Combination of equations (11), (15) and (17) produce the expression of electromagnetic mass in terms of anomalous magnetic moment as

$$\Delta m = m(\frac{g}{2} - 1) = ma. \tag{18}$$

At the present situation anomalous magnetic moment is expressed from experimental facts up to higher order of fine structure constant as [24], [25]

$$a_e(QED) = C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ..., \quad (19)$$

where, $C_e^{(i)}$ s are the co-efficients and the first one was calculated by Schwinger in 1948 [12], [24].

Hence the recent measurement of g-factor is also got affected with these values, which in turn leaves impact on the electromagnetic mass, Δm too. This in fact offers us not only to measure the electromagnetic mass of electron more accurately, also ensures the more accurate measurement of mechanical mass of electron and the ultimately the more prcised values of spin too.

Therefore the electromagnetic mass can be calculated with the corrected higher order terms as

$$\Delta m = m[C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...], \quad (20)$$

which is exactly identical with equation (17), only with more accurate measurement. The corresponding energy is then expressed in with the help of equation (20)

$$W_H \simeq mc^2[C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]. \quad (21)$$

In the same way one can re-write the magnetic moment as

$$\mu = \frac{e\hbar}{2mc}[C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]. \quad (22)$$

Using equations (22) in equation (1) and equating with equation (21) we have

$$R_H{}^3 = \frac{2}{3}\alpha R_C{}^3 \frac{[1 + C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]^2}{[C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]}. \quad (23)$$

Using equation (6) in equation (23) again we have the combination of two radii

$$R_H{}^3 = \frac{2}{3} R_0 R_C{}^2 \frac{[1 + C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]^2}{[C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]} \quad (24)$$

For the convenience of our calculation equation (24) can be written as

$$R_H{}^3 = \frac{2}{3} R_0 R_C{}^2 \frac{[1+\chi]^2}{\chi}, \quad (25)$$

where,

$$\chi = [C_e^{(2)}(\frac{\alpha}{\pi}) + C_e^{(4)}(\frac{\alpha}{\pi})^2 + C_e^{(6)}(\frac{\alpha}{\pi})^3 + C_e^{(8)}(\frac{\alpha}{\pi})^4 + ...]. \quad (26)$$

In a more prcised form we express equation (26) as

$$R_H{}^3 = S R_0 R_C{}^2 (1+\chi)^2 = S R_0 [R_C(1+\chi)]^2 = S R_0 R_{C0\chi}{}^2, \quad (27)$$

where,

$$S = \frac{2}{3}\chi^{-1} \quad (28)$$

and

$$R_{C0\chi} = R_C(1+\chi). \quad (29)$$

Equation (29) reveals here the new expression of composite radius of electron. The first term in the left hand side of equation (29) is only Compton radius part, but the second term χR_C involves α -quantized terms of electron radii. Hence the nature of helical motion can be invariant like the preliminary version of composite radius and this χR_C part will take care of the distance between two successive turns.

We developed the nature of helical motion of charge and electron model with the composite radius in a companion paper. Continuing the similar effect for the corrected pattern of composite radius we can have the total time required for the motion of the charge as

$$T = \frac{R_E + 2n\pi R_{C0\chi}}{v}. \tag{30}$$

The corresponding current therefore can be written as

$$I = \frac{ev}{R_E + 2n\pi R_{C0\chi}}. \tag{31}$$

Current and magnetic moment are related as [26]

$$\mu = \frac{IA}{c}, \tag{32}$$

where, A is the corresponding area. The corresponding magnetic moment comes out to be

$$\mu = \frac{2(n-1)\pi ev\chi {R_C}^2}{c(R_E + 2n\pi R_{C0\chi})}. \tag{33}$$

Using the approximation of infinite long current carrying wire (in Gaussian) the magnetic field can be calculated for n-th arbitrary term as

$$B = \frac{2}{R_{C0\chi}}[\frac{ev}{R_E + 2n\pi R_{C0\chi}}], \tag{34}$$

where, v is the corresponding velocity.

Number of turns n and the velocity of the charged particle are chosen here arbitrarily in the way of developing this dynamics. For the existence of the helical motion the lower limit can be chosen for the number of turn as $n = 1$ and the Compton-sized model can have a maximum length of the helical path as

$$h_{max} = 2R_C. \tag{35}$$

On the other hand the maximum length in terms of χR_C can be written as

$$h = 2(n-1)\pi\chi R_C. \tag{36}$$

Therefore we get the range of n as the upper limit of n comes out by equating the equations (35) and (36) as

$$n = 1 + \frac{1}{\chi}. \tag{37}$$

So n ranges from 1 to $1 + \frac{1}{\chi}$.

At the end of the first turn the magnetic field will be generated as

$$B_1 = \frac{2}{R_{C0\chi}}[\frac{ev_1}{R_E + 2n\pi R_{C0\chi}}], \tag{38}$$

where, v_1 is the primary velocity. Magnetic field will now act on the charge as external magnetic field so that we can consider the sphere as sum of rotating rings.

The generalized angular momentum of the system [26] will be read as

$$L = mR_C v + \frac{eR_{C0}^2 B}{2c}. \quad (39)$$

Hence after the first turn the generalized angular momentum will be

$$L_1 = mR_C v_1 + \frac{eR_{C0\chi}^2 B_1}{2c}. \quad (40)$$

L_1 will initiate the force F_{L1} which will act on the charge in the second turn. The force F_{L1} is

$$F_{L1} = ev_2 B_1. \quad (41)$$

The force for which the charge continues the circulatory motion with the same radius is

$$F_{C1} = \frac{L_1 v_1}{R_C^2} - \frac{eB_1 v_1}{2c}. \quad (42)$$

Equating these two forces from equations (41) and (42) we have

$$v_2 = v_1[\frac{L_1}{eB_1 R_C^2} - \frac{1}{2c}]. \quad (43)$$

Therefore the magnetic field originated at the end of second term, $n = 2$ is

$$B_2 = B_1(\frac{R_E + 2\pi R_{C0\chi}}{R_E + 4\pi R_{C0\chi}})[\frac{L_1}{eB_1 R_C^2} - \frac{1}{2c}]. \quad (44)$$

Hence the generalized angular momentum after $n = 2$ turn will be read as

$$L_2 = mR_C v_2 + \frac{eR_{C0\chi}{}^2 B_2}{2c}. \quad (45)$$

Using v_2 and B_2 from equations (43) and (44) in equation (45) we have

$$L_2 = [\frac{L_1}{eB_1R_C^2} - \frac{1}{2c}][mv_1R_C + \frac{eR_{C0\chi}{}^2B_1}{2c}(\frac{R_E + 2\pi R_{C0\chi}}{R_E + 4\pi R_{C0\chi}})]. \quad (46)$$

In a similar manner we can go up to n-th order and have equations for v_n, B_n and L_n respectively as

$$v_n = v_{n-1}[\frac{L_{n-1}}{eB_{n-1}R_C^2} - \frac{1}{2c}], \quad (47)$$

$$B_n = B_1(\frac{R_E + 2\pi R_{C0\chi}}{R_E + 2n\pi R_{C0\chi}})[\frac{L_1}{eR_C^2B_1} - \frac{1}{2c}][\frac{L_2}{eR_C^2B_2} - \frac{1}{2c}]$$

$$[\frac{L_3}{eB_3R_C^2} - \frac{1}{2c}]........[\frac{L_{n-1}}{eB_{n-1}R_C^2} - \frac{1}{2c}]. \quad (48)$$

and

$$L_n = [\frac{L_1}{eB_1R_C^2} - \frac{1}{2c}][\frac{L_2}{eB_2R_C^2} - \frac{1}{2c}]..$$

$$..[\frac{L_{n-1}}{eB_{n-1}R_C^2} - \frac{1}{2c}][mv_1R_C + \frac{eR_{C0\chi}{}^2B_1}{2c}(\frac{R_E + 2\pi R_{C0\chi}}{R_E + 2n\pi R_{C0\chi}})]. \quad (49)$$

Here with equations (47)-(49) the velocity, magnetic field and the angular momentum for the n-th order are derived. Relativistic spinning sphere model with corrections from anomalous magnetic moment is modified here.

Conclusions

With the introduction of composite radius, the results vary from those of the RSS model. RSS model is developed on Compton radius only. But here classical electron radius is also taking part when magnetic moment is taken with only Schwinger correction. Therefore the results change, but in a regular pattern as fine structure constant, α is controlling the difference between Compton and classical electron radius. Relation of α with the g-factor leads us to connect anomalous magnetic moment with this semi-classical idea of model of electron through the composite radius and Compton radius. The combination of g and α also makes the connection between the QED calculations to the semi-classical approaches. When the anomalous magnetic moment and its recent corrected forms are used, composite radius is also changed and we got a generalised form of the helical motion for relativistic spinning sphere. The structure with radii R_C and $R_{C0\chi}$ is not one exact sphere, rather one can say as aspheric in nature, which is also supported by one recent observation [27]. It is quite interesting that a similar pattern of the electron structure from different sorts of calculations was shown by A. Martin in his article [28].

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