# On the Advance of Mercury's Perihelion due to Inertial Induction and the Possibility of Solar Oblateness

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The model of inertial induction [1-4] proposed by the author has been able to demonstrate the exact equivalence of inertial and gravitational masses, and also reveal the existence of a very small cosmic drag on objects moving with uniform speed in the quasistatic universe, resulting in the observed cosmological redshift. When the model is applied to solar system dynamics, a number of interesting results are obtained. In this paper the model has been used to estimate its effect on the advance of Mercury's perihelion. The unaccounted 43″ per century is explained by general relativity with the stipulation that the sun is spherically symmetric, as the oblateness of the sun will also produce an advance. Such a strict condition makes many researchers uncomfortable as the sun is a spinning body, and it is expected to possess oblateness, like planets. The magnitude of $J_2$ due to solar oblateness suggested by researchers, varies from $1.25 \times 10^{-7}$ to $7.96 \times 10^{-5}$. It is shown that inertial induction produces about –7.16″ of perihelion advance per century. This is an interesting result suggesting that solar oblateness effect can be accommodated without any conflict with the GR prediction. $J_2$ can be up to $2.79 \times 10^{-5}$. However, the possibility exists that a major part of the excess advance of the perihelion could be generated by the sun's oblateness. A very accurate direct detection of oblateness is doubtful based on photosphere observations.

## Introduction

The advance of the perihelion of the planet Mercury has been a subject of study for more than a century and a half. According to Newtonian mechanics, perturbations by all the planets can explain the total shift except for a tiny amount of 43″ per century. Einstein showed that general relativity produces an advance of the perihelion position of about the same magnitude, *i.e.*, 43″ per century. The advance of Mercury's perihelion could be also explained if the Sun were slightly oblate (due to its spinning motion) instead of being a perfect sphere. However, the matter of solar oblateness has continued to be subjected to dispute. A very large number of studies based on modeling and experimental observations have been conducted, and the suggested values of the second zonal spherical harmonic coefficient, $J_2$, due to the oblateness of the Sun, vary from $1. \times 10^{-7}$[5] to $7.96 \times 10^{-5}$[6]. A detailed account of all results for $J_2$ can be found in [7]. The main difficulty has been to avoid any conflict with GR and allow the Sun to possess an oblateness of meaningful magnitude. In consideration of the above, any treatment of the excess advance of Mercury's perihelion as a proof of GR appears to be questionable.

The model of inertial induction based on a dynamic gravitational interaction has yielded a number of interesting results [1-4]. In this model the total gravitational force between two bodies depends not only on their separation but also on their relative velocity and acceleration. Figure 1 shows two particles A and B with gravitational

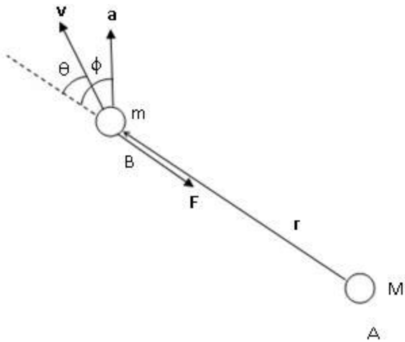


Fig 1.Inertial induction between two particles in the mean rest frame of the universe

masses *M* and *m,* respectively and ***r*** is the position of B with respect to A. if ***v*** and ***a*** be the velocity and acceleration of B with respect to A in the mean rest frame of the universe, then the total gravitational force on B due to its interaction with A is given by

$$\boldsymbol{F} = -\frac{G\,Mm}{r^2}\,\hat{\boldsymbol{u}}_r - \frac{G\,M\,m}{c^2 r^2} v^2 f\left(\theta\right)\,\hat{\boldsymbol{u}}_r - \frac{G\,M\,m}{c^2 r} af\left(\varphi\right)\,\hat{\boldsymbol{u}}_r \quad (1)$$

where *G* is the gravitational constant, *c* is the speed of light, $\hat{\boldsymbol{u}}_{\mathbf{r}}$ is the unit vector along ***r*****,** and *f(θ)* and *f(ϕ)* represent the inclination effects. We take $f(\theta) = \hat{\boldsymbol{u}}_{\mathbf{r}}\hat{\boldsymbol{u}}_{\mathbf{v}}\,|\hat{\boldsymbol{u}}_{\mathbf{r}}\ \hat{\boldsymbol{u}}_{\mathbf{v}}|$ and $f(\phi) = \hat{\boldsymbol{u}}_{\mathbf{r}}\hat{\boldsymbol{u}}_{\boldsymbol{a}}\,|\hat{\boldsymbol{u}}_{\mathbf{r}}\ \hat{\boldsymbol{u}}_{\boldsymbol{a}}|$, where $\hat{\boldsymbol{u}}_{\boldsymbol{v}}$ and $\hat{\boldsymbol{u}}_{\boldsymbol{a}}$ are the unit vectors along ***v*** and ***a***, respectively. It has been shown [1-4] that when this interaction of a particle of gravitational mass *m* with the matter present in the rest of the universe is calculated, we get

$$\boldsymbol{F} = -k\frac{m\,m^2}{c}\,\hat{\boldsymbol{u}}_{\boldsymbol{v}} - m\mathbf{a} \quad (2)$$

with $k = \left(\pi\,G\,\rho\right)^{\frac{1}{2}}$, where $\rho$ is the average mean density of the universe and ***v*** is the velocity of the particle with respect to the mean rest frame of the universe assumed to be infinite and quasi-static.

Equation 2 indicates exact equivalence of gravitational and inertial masses. The first term on the R.H.S. of (2) is very small cosmic drag force whose action on photons produces the observed cosmological redshift without any Hubble expansion. This model has been applied to a number of phenomena of on the solar system, galactic and extragalactic scales. In each case the predicted effects have been found to be present. In what follows the model has been applied to the case of Mercury's orbital motion to examine the effect on its perihelion advance.

## Perturbing Forces due to Inertial Induction

Before we attempt to estimate the effect of inertial induction on the motion of the perihelion position of the Mercury's orbit it is necessary to determine the perturbing force due to inertial induction. Since the mass of the Sun is much larger than the planets and the distance is comparatively less we ignore the effect of the planets. The 7º inclination of the orbital plane of Mercury with the ecliptic is also ignored to keep the analysis simple without introducing any significant error.

Figure 2 shows Mercury's orbit with the rotating Sun at a focus, as seen in the mean rest frame of the universe. The frame of reference that carries the sun along with the rest of the solar system moves with respect to the mean reset frame of the universe with negligible acceleration and can be treated as an inertial frame in the Newtonian sense. Mercury's location is at P. There will be primarily two types of perturbing forces on Mercury due to inertial induction. Forces on Mercury due to its relative velocity and relative acceleration with respect to the Sun, $\boldsymbol{F}_r$, will act along the radial direction. A transverse force, $\boldsymbol{F}_\psi$ will act on Mercury due to the rotation of the Sun, $\Omega$ [4, 8]. We can write

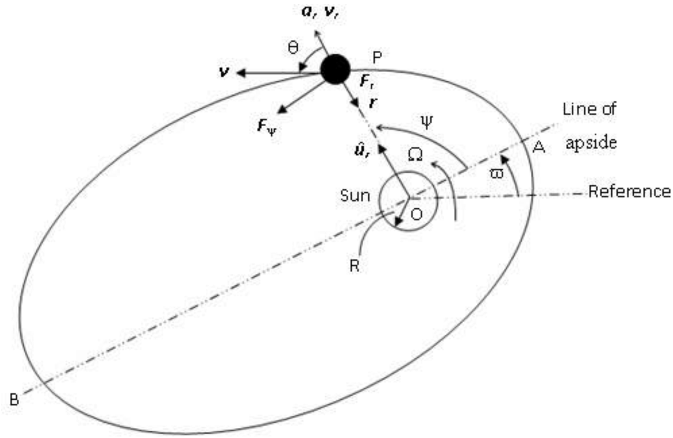


Fig. 2. Mercury's orbit and forces due to inertial induction in the mean rest frame of the universe

$$\boldsymbol{F}_r = -\frac{G\,M\,m}{c^2\,r^2} v^2 \cos\theta \left|\cos\theta\right| \hat{\boldsymbol{u}}_{\boldsymbol{r}} - \frac{G\,M\,m}{c^2\,r}\; a_{\mathrm{r}}\; \hat{\boldsymbol{u}}_{\boldsymbol{r}} \tag{3}$$

where $M$ is the mass of the Sun, $m$ is the mass of mercury, $v$ is the velocity of Mercury, $a_r$ is the radial component of acceleration of Mercury. Since the Sun is much more massive compared to Mercury we assume the Sun to be fixed in its location (i.e., the center of mass of the Sun-Mercury system coincides with the center of the Sun). From orbital mechanics we know

$$r = \frac{h^2}{G\,M\left(1 + e\cos\psi\right)} \tag{4}$$

and

$$\dot{\psi} = \frac{h}{r^2} \tag{5}$$

where $h$ is the angular momentum of Mercury about O (the center of the Sun) per unit mass, $e$ is the eccentricity of the orbit and $\psi$ is the angle made by the line OP with the line of perihelion. Again from Newton's law of gravitation we know

$$\boldsymbol{a}_r = -\frac{GM}{r^2}\ \hat{\boldsymbol{u}}_{\boldsymbol{r}} \tag{6}$$

Differentiating (4) and using (5)

$$\dot{r} = \frac{eGM}{h}\sin\psi \tag{7}$$

We should note from (7) that during the half orbit from A to B, $\psi$ varies from 0 to π and $\dot{r} > 0$. So, in the first term of the R.H.S. of (3) $v^2 \cos\theta\,|\cos\theta|$ can be written as $\dot{r}^2$ and the velocity dependent inertial induction force acts toward O. Using this, along with (6) and (7) in (3) the radial component of the perturbing force due to velocity and acceleration is obtained as follows:

$$F_r = \frac{GMm}{c^2 r}\cdot\frac{GM}{r^2} - \frac{GMm}{c^2 r^2}\left(\frac{eGM}{h}\right)^2 \sin^2\psi$$

Simplifying

$$F_r = \frac{(GM)^2 m}{c^2 r^3} - \frac{(GM)^3 e^2 m}{c^2 r^2 h^2}\sin^2\psi$$

Using the expression for $r$ in the above equation we get

$$F_r = \frac{(GM)^5 m(1+e\cos\psi)^3}{c^2 h^6} - \frac{(GM)^5 e^2 m}{c^2 h^6}(1+e\cos\psi)^2\sin^2\psi$$

or,

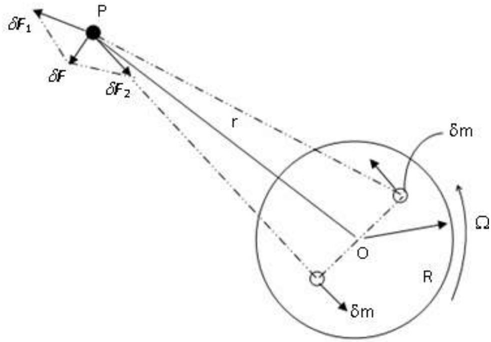


Fig. 3. Force on Mercury due to the Sun's rotation

$$F_r = \frac{(G\,M)^5\, m\,(1 + e\,\cos\psi)^2}{c^2\, h^6}\left(1 + e\,\cos\psi - e^2 \sin^2\psi\right) \qquad (8)$$

To determine the transverse component of the perturbing force due to inertial induction we have to consider Fig. 3. Two mass elements of the rotating Sun are shown along with their directions of motion. It is seen that Mercury at the position P is acted upon by two forces, $\delta\boldsymbol{F}_1$ and $\delta\boldsymbol{F}_2$, due to the two moving mass elements of the spinning Sun because of velocity dependent inertial induction. The resultant of these two forces, $\delta\boldsymbol{F}$, acts on Mercury in a direction perpendicular to the line OP. The total transverse force $F_\psi$ can be found by summing up the effects due to all mass elements of the Sun. This has been done [4,8]. The resultant transverse force due to velocity dependent inertial induction of the material of the rotating Sun with Mercury can be written as follows:

$$F_\psi \approx 6\times 10^{-3}\,\frac{G\,Mm}{c^2\, r^3}{}^2\, \Omega^2\, R^3 \ \text{(in SI units)} \qquad (9)$$

where $\Omega$ is the angular speed of the Sun's rotation and $R$ is the radius of the Sun. Using (4) in (9) we get

$$F_\psi \approx 6\times10^{-3}\frac{(GM)^4\,\Omega^2 R^3 m}{c^2 h^6}(1+e\cos\psi)^3 \tag{10}$$

Substituting numerical values it can be shown that

$$F_r / F_\psi \sim 10^7$$

It can be further shown that the advance of the perihelion due to $F_\psi$ is of the order of $10^{-5}{}''$ per century. Therefore, it is prudent to ignore $F_\psi$ without introducing any appreciable error in the result.

## Advance of Perihelion due to $F_r$

The position of the line of apsides from the reference line is denoted by $\varpi$, as shown in Fig. 2. The position of Mercury from the perihelion is given by $\psi$. From standard texts on orbital mechanics it is known that the rate of advance of the perihelion due to a small perturbing radial force $F_r$ can be expressed as follows:

$$\dot{\varpi} = -\frac{h}{GM}\cdot\frac{F_r}{m}\cdot\frac{\cos\psi}{e}$$

Substituting the expression for $F_r$ from (8) in the above equation we get

$$\dot{\varpi} = -\frac{h}{eGM}\frac{(GM)^5(1+e\cos\psi)^2}{c^2h^6}\cos\psi\left(1+e\cos\psi-e^2\sin^2\psi\right) \tag{11}$$

Now we can write

$$\dot{\varpi} = \frac{d\varpi}{dt} = \frac{d\varpi}{d\psi}\cdot\dot{\psi} = \frac{d\varpi}{d\psi}\cdot\frac{h}{r^2}$$

Using the expression for $r$

$$\dot{\varpi} = \frac{d\varpi}{d\psi} \cdot \frac{(GM)^2 (1 + e\cos\psi)^2}{h^3} \tag{12}$$

Substituting the expression for $\dot{\varpi}$ in the L.H.S of (11) we obtain the following equation after simplification:

$$\frac{d\varpi}{d\psi} = -\frac{1}{e}\left(\frac{GM}{ch}\right)^2 \left(1 + e\cos\psi - e^2\sin^2\psi\right)\cos\psi$$

The amount of advance per orbital motion of Mercury is just

$$\delta\varpi = -\int_0^{2\pi} \frac{1}{e}\left(\frac{GM}{ch}\right)^2 \left(1 + e\cos\psi - e^2\sin^2\psi\right)\cos\psi \, d\,\psi$$

$$= -\frac{1}{e}\left(\frac{GM}{ch}\right)^2 \left[\int_0^{2\pi} \cos\psi\, d\psi + e\int_0^{2\pi} \cos^2\psi\, d\psi - e^2\int_0^{2\pi} \sin^2\psi\cos\psi\, d\psi\right]$$

It is clear that the contributions of the first and the third terms inside the third bracket are zero. Thus

$$\delta\varpi = -4\left(\frac{GM}{ch}\right)^2 \int_0^{\pi/2} \cos^2\psi\, d\psi$$

$$= -4\left(\frac{GM}{ch}\right)^2 \frac{\pi}{4}$$

Finally

$$\delta\varpi = -\pi\left(\frac{GM}{ch}\right)^2 \tag{13}$$

Since Mercury takes about 88 days to complete one orbit the advance of the perihelion per century $\Delta\varpi$ is as follows:

$$\varDelta\varpi = -\pi\left(\frac{GM}{ch}\right)^2 \cdot \frac{365}{88} \times 100 \times \frac{180}{\pi} \times 60 \times 60$$

$$= -7.16'' \text{ per century}$$

This is an interesting result. The currently observed excess advance of the perihelion is about 43″ per century. At the same time general Relativity predicts 43″ per century advance without leaving any room for other effects including oblateness of the Sun. This negative advance can create room for other still unaccounted effects on the advance without bringing any serious conflict with the GR prediction.

## Effect of Solar Oblateness

The advance of the perihelion of Mercury due to solar oblateness per orbit is given by

$$\delta\varpi \approx \frac{6\pi J_2 R^2 (GM)^2}{h^4} \tag{14}$$

where $J_2$ is the second zonal spherical harmonic coefficient due to the oblateness of the Sun. Currently there is no definite knowledge about the value of $J_2$. Suggestions by researchers vary from $1.25 \times 10^{-7}$ to $7.96 \times 10^{-5}$! The main concern among researchers in having an appreciable value of $J_2$ is its conflict with the GR theory. But if the above result of negative advance is correct, then (14) yields that $J_2$ can be up to $2.79 \times 10^{-5}$ without creating any conflict with the GR prediction.

## Concluding Remarks

It is interesting to note that the order of magnitude of perihelion due to inertial induction is so close to the observed excess. This may not be a coincidence if we recall the excellent agreement of inertial induction effects with a number of different phenomena [4]. This effect helps to provide some scope for other effects (including solar oblateness) without producing any serious conflict with the GR prediction. It should be noted that only acceleration-dependent inertial induction produces some significant advance of perihelion. Velocity-dependent inertial induction due to the radial motion of Mercury does not produce any secular change. The velocity-dependent inertial induction effect due to the rotation of the Sun results in a very small value of perihelion advance.

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