# Bringing Simultaneity Back to Life

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Einstein's deduction of the relativity of simultaneity rested crucially upon the Maxwell picture of light propagation. Maxwell's theory, however, has several potential vulnerabilities to criticism, beginning with a failure noted during the nineteenth century to exhibit invariance under the Galilean transformation, then considered to describe physical inertial motions. We show, by redoing Einstein's train example with Galilean invariant mathematics, that the imposition of this type of invariance suffices to restore distant simultaneity to physical validity. A crucial experiment is suggested.

## 1. Introduction

The physical invalidity of distant simultaneity, known as "the relativity of simultaneity," was established in the public mind most dramatically by means of Einstein's Gedanken experiments, notably the famous one involving two lightning strikes at the ends of a moving train. Recently an up-dated variant of that Gedanken proof has been given by Elisha Huggins[1], for the benefit of modern physics teachers, in terms of moving Martians and Venusians. However, it must be said that the proofs of failure of distant simultaneity rest solely (as far as I know) upon Gedanken evidence. No matter how up-to-date the thinking, it is still ratiocination, not observation, that rules. For this reason, physicists should cling to reservations – until observational evidence confers its definitive blessing on their thoughts; *i.e.*, until a crucial experiment can be done.

Are there any theoretical reasons to tread lightly in this area? Yes, indeed. A simple analysis of Einstein's train example shows it to be critically dependent on the validity of Maxwell's picture of light "propagation." If there were any reason to question that picture, the Gedanken foundations supporting the relativity of simultaneity could fail catastrophically. But, some time after Maxwell's equations were given their modern form, quantum mechanics made it clear that "propagation" is not the simple, causal process that people of Einstein's generation assumed. With reference to light, *propagation* refers to the existence of a photon in a quantum *pure state*. A pure state of any "particle" is known to possess aspects of acausality; for such a state designates a mode of existence that is about as far from any supported by classical intuition as the mind can stretch.

Einstein chose to take Maxwell exactly at his word, that light propagates in an entirely classical causal way, proceeding from point A of emission to point B of absorption at a fixed speed *c*, uninfluenced by any proper or relative motions of points A and B. The mind is not stretched a bit. Nothing could be more classical or more ploddingly causal … or less like what the notion of a pure state suggests. Such an observation in itself carries no conviction, because we do not know exactly what a pure state does suggest. (Feynman: "Nobody understands quantum mechanics.") The thought merely serves to open our minds a tiny smidgen.

Let us look at the problem from another angle: In the nineteenth century the failure of Maxwell's equations to exhibit invariance under the Galilean transformation threatened to bring down that whole house of cards, since experiments (*e.g.*, by Mascart[2]) showed relativity to be an empirical fact at first order in *v/c*. (Michelson-Morley showed it to be a fact also at second order.) This forced upon physicists a momentous choice: Either (1) Maxwell's equations were wrong and had to be changed, or (2) the Galilean transformation had to be discarded. Einstein chose the second of these options, and in so doing took the whole kit and caboodle of what purported to be "physics" right along with him.

But rather than accepting that choice without question as the basis for all our subsequent teachings and Gedanken elaborations, let us exercise our present Gedanken capabilities by examining the first approach. In that case we choose to view the failure of invariance of Maxwell's equations as just what it seems to be, a failure. We need, then, to find an invariant (under the Galilean transformation) version of Maxwell's equations. But that is the easy part – in fact it was already done by Heinrich Hertz (Chapter 14 of his *Electric Waves*[3]), and in more recent times by numerous others. The trick is to recognize that the failure of invariance results from the presence in Maxwell's equations of the non-invariant partial time derivative operator, $\partial / \partial t$. When this is replaced by the invariant total time derivative operator $d / dt = \partial / \partial t + \vec{v}_d \cdot \nabla$, and some tinkering is done with one of the source terms, then such modified Maxwell's equations prove to be Galilean invariant, hence in agreement with a relativity principle.

It is important to understand that the new convective velocity parameter $\vec{v}_d$, which incidentally proves to be the same as the "test charge velocity" appearing in the Lorentz force law, is best viewed as the velocity of the radiation detector or absorber (relative to the observer's field point). This is the case because the test charge acts as a field detector. On this matter Hertz's physical interpretation went fatally astray. He understood his new $v_d$ parameter to represent an *ether velocity*. He further assumed a Stokesian ether that co-moved with observable matter. So, his ether became "observable," hence subject to falsification (disproof). Thus, when experiments were done, his theory was observationally disproven[4], and his version of electromagnetic theory was dropped like a hotcake. But in fact there was nothing wrong with his mathematics; the only thing wrong was his ether interpretation. Since Maxwell also relied on an ether interpretation, the only superiority (?) of the Maxwell theory lay in its non-falsifiability, consequent upon Maxwell's failure to specify any criterion of ether observability. This is a good example of how to win by not parting with too much information.

How do we know that $d/dt$ is invariant under the Galilean transformation? The proof is immediate, given the Galilean velocity addition law, $\vec{v}'_d = \vec{v}_d - \vec{v}$, where $\vec{v}$ denotes the constant velocity of the primed inertial system with respect to the unprimed one, and the *d*-subscripted quantities are arbitrary detector velocities measured with respect to the two systems. Also needed is the easily proven fact that $\vec{\nabla}' = \vec{\nabla}$ under the Galilean transformation [$\vec{r}' = \vec{r} - \vec{v}t,\ t' = t$]. Then

$$\left(\frac{d}{dt}\right)' = \left(\frac{\partial}{\partial t} + \vec{v}_d \cdot \vec{\nabla}\right)' = \frac{\partial}{\partial t'} + \vec{v}'_d \cdot \vec{\nabla}'$$

$$= \left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}\right) + \left(\vec{v}_d - \vec{v}\right) \cdot \vec{\nabla} = \frac{\partial}{\partial t} + \vec{v}_d \cdot \vec{\nabla} = \left(\frac{d}{dt}\right)$$

*Q.E.D.* This makes use of the non-invariance of $\partial/\partial t' = \partial/\partial t + \vec{v} \cdot \vec{\nabla} \neq \partial/\partial t$, which is easily proved from the Galilean transformation by applying the chain rule of partial differentiation.

With *d/dt* replacing $\partial / \partial t$ in Maxwell's equations, we get the Galilean invariant form of the free-space electromagnetic field equations as

$$\vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{d\vec{E}}{dt} = \frac{4\pi}{c}\vec{j}_m$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho$$

Here the source current $\vec{j}_m$ (measured by a detector moving with velocity $\vec{v}_d$ with respect to the field point) is related to the Maxwellian source current $\vec{j}_s$ (measured by a detector at rest at the field point) by $\vec{j}_m = \vec{j}_s - \rho\vec{v}_d$. By this interpretation of $v_d$ as field detector velocity, $v_d = 0$ restores the above Hertzian equations identically to Maxwell's equations and restores the detector to the Maxwell condition of immobility at the field point; thus showing Hertz's theory to be a covering theory of Maxwell's. To get a wave equation, we take the curl of the first of these field equations (assuming the source-free case, $\vec{j}_m = 0$) and the total time derivative of the second equation, and apply a vector identity to obtain

$$\vec{\nabla}^2\vec{B} - \frac{1}{c^2}\frac{d^2\vec{B}}{dt^2} = 0.$$

Similarly, taking the curl of the second field equation and the total time derivative of the first (with $\vec{j}_m = 0$) we get

$$\vec{\nabla}^2\vec{E}-\frac{1}{c^2}\frac{d^2\vec{E}}{dt^2}=0\,.$$

These are the Galilean-invariant wave equations that will be used in what follows. If Einstein is right (and I think he is), that for unqualified *invariance* we should be using invariant proper time $\tau$ (of the field detector) instead of frame time *t*, then the wave equations just given are valid only at first order. They would need correction at higher orders through replacement of *t* by $\tau$. Fortunately, the issue addressed here, that of the validity of distant simultaneity, can be settled at first order, so we shall not need higher-order refinements.

## 2. Invariant wave equation solution.

We shall seek a solution of the electric field wave equation of the form $\vec{E}=\vec{E}\left(p\right)$, where

$$p\equiv\vec{k}\cdot\vec{r}-\omega t=xk_x+yk_y+zk_z-\omega t\quad.$$

We then calculate

$$\begin{aligned}\nabla^2\vec{E}\left(p\right)&=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\vec{E}\left(p\right)\\&=\left(k_x^2+k_y^2+k_z^2\right)\vec{E}\left(p\right)=k^2\vec{E}\left(p\right)\end{aligned}$$

and

$$\frac{d^2}{dt^2}\vec{E}(p)=\left(\frac{\partial}{\partial t}+\vec{v}_d\cdot\vec{\nabla}\right)^2\vec{E}(p)$$
$$=\left(\frac{\partial^2}{\partial t^2}+2\frac{\partial}{\partial t}\left(\vec{v}_d\cdot\vec{\nabla}\right)+\left(\vec{v}_d\cdot\vec{\nabla}\right)^2\right)\vec{E}(p)$$
$$=\left(\omega^2-2\omega\left(\vec{v}_d\cdot\vec{k}\right)+\left(\vec{v}_d\cdot\vec{k}\right)^2\right)\vec{E}(p)=\left(\omega-\left(\vec{v}_d\cdot\vec{k}\right)\right)^2\vec{E}(p)$$

Putting these results into the wave equation for $\vec{E}$, we get

$$\left[-k^2+\frac{1}{c^2}\left(\omega-\left(\vec{v}_d\cdot\vec{k}\right)\right)^2\right]\vec{E}=0\,.$$

From the necessary vanishing of the coefficient of $\vec{E}$ it follows that

$$ck=\left|\omega-\vec{v}_d\cdot\vec{k}\right| \quad \text{or} \quad \frac{\omega}{k}=\pm c+\frac{\vec{k}}{k}\cdot\vec{v}_d\,.$$

Here $k=\sqrt{\left(\vec{k}\cdot\vec{k}\right)}$. It is customary to recognize $\omega/k$ as a wave or phase propagation speed *u*; *viz.*,

$$u=\frac{\omega}{k}=\pm c+\frac{\vec{k}}{k}\cdot\vec{v}_d\,.$$

The corresponding result in Maxwell's theory is $u=\pm c$. We see that in the invariant theory there is at first order a convective term affecting the propagation speed of light. This is not always observable, because of a nineteenth-century theorem known as *Potier's principle*[5]. This states that the spatial path taken by light (as in interference and diffraction experiments) is unaltered by changes in the velocity parameter $\vec{v}_d$ in the additive term $\left(\vec{k}/k\right)\cdot\vec{v}_d$. Thus what is observable by this class of experiments is the same as if $\vec{v}_d=0$.

This fact, adjoined to the physical interpretation of the velocity parameter as ether wind velocity, was instrumental during the nineteenth century in establishing the relativity principle as empirically valid[2] at first order. That is, any effect of ether wind on light paths in space was in principle unobservable. However, according to Potier's principle, light transit times *were* affected by the additive phase velocity term. However, in the nineteenth century time intervals were not measurable with sufficient accuracy to verify that aspect of the principle – and the principle has subsequently been largely forgotten; so it cannot be said that experiment either supports or refutes the above phase velocity prediction of the Galilean invariant theory. Here we shall treat it as true that phase velocity $u$ obeys the above-derived law, different from Maxwell's result.

## 3. Einstein's train analyzed by means of the Galilean invariant field equations

Since I decline to associate with Martians or Venusians[1], I shall revert to Einstein's original classic analysis. Recall that a train of rest length $2L$ passes a station at constant speed $v_d$. (Since we consider only effects of first order in $v_d / c$, rest length and moving length are the same.) Just at the instant the middle of the train comes opposite the stationmaster S, lightning bolts strike at front and rear of the train, simultaneously as judged by S. That is, the wave front of each lightning flash, having traveled distance $L$, is detected at the same time by the eye of S. For a train rider R (a differently-moving inertial observer), located at the train's midpoint, however, as Einstein showed, the two flashes are detected non-simultaneously (because of the train's forward motion). Hence the "relativity of simultaneity" was established. Let us analyze this in some detail, continuing to restrict our attention to the first order.

We consider the viewpoint of S. If the lightning bolts strike at his time $t = 0$, then the flashes will reach his eye simultaneously at S-frame time $t_1 = L/c$. Let the train move to the right at speed $v_d$. Then the rear-originating flash, with a wave front that moves rightward, may be thought to start at time $t = 0$ when R is at $x = 0$, opposite S. It reaches the eye of R and is detected there at a later S-frame time $t_{rR}$ and up-track (rightward) distance $x_{rR}$, obeying $x_{rR} = v_d t_{rR}$. Also, to describe the lightning flash, we have $x_{rR} + L = v_r t_{rR}$, where $v_r$ is the S-measured speed of the right-going flash. Solving these two equations, we get

$$t_{rR} = \frac{L}{v_r - v_d}$$

and

$$x_{rR} = v_d t_{rR} = \frac{v_d L}{v_r - v_d}$$

Similarly, let the front-originating (leftward-propagating) flash reach R at S-measured time $t_{fR}$ and position $x_{fR} = v_d x_{fR}$. Adjoining the relation $L - x_{fR} = v_f t_{fR}$, where $v_f$ is the speed of the left-going flash from the front of the train, as measured by S, and solving, we get

$$t_{fR} = \frac{L}{v_f + v_d}$$

and

$$x_{fR} = v_d t_{fR} = \frac{v_d L}{v_f + v_d} \quad .$$

These four equations are the general formulas we shall need to analyze the simultaneity issue.

First, consider the case in which light speed is determined by the Maxwell theory, $v_r = v_f = c.$ The above general formulas then yield

$$t_{rR} = \frac{L}{c - v_d} = \frac{t_1}{1 - v_d / c} \quad \text{and} \quad x_{rR} = v_d t_{rR} = \frac{v_d t_1}{1 - v_d / c} \ .$$

Also

$$t_{fR} = \frac{L}{c + v_d} = \frac{t_1}{1 + v_d / c} \quad \text{and} \quad x_{fR} = v_d t_{fR} = \frac{v_d t_1}{1 + v_d / c} \ .$$

At a glance, then, we see that Einstein was right, provided Maxwell's equations correctly describe the propagation of light: The times of flash detection by the train rider R, $t_{rR}$ and $t_{fR}$, as judged by S, are indeed not the same, even though each flash has in both S's and R's view traveled the same distance *L*. That can only mean (as S reasons) that the lightning strikes occurred non-simultaneously in R's commoving frame, although they were simultaneous by hypothesis in S's frame.

But suppose the physical propagation of the flashes was governed instead by the Galilean invariant (Hertzian) theory. In that case the light propagation velocity is not $u = c$, as we are accustomed to think, but instead is $u = c + \left(\vec{k} / k\right) \cdot \vec{v}_d$, as shown above. (Here we treat the eye of R as our light detector, and recognize that this detector is in motion at velocity $\vec{v}_d$ relative to the S-observer, to whose viewpoint we are confining attention.) Then for the right-going wave from the rear we get $u = v_r = c + v_d$, and for the left-going wave from the front $u = v_f = c - v_d$. Putting these values in our four general equations, we obtain

$$t_{rR} = \frac{L}{c + v_d - v_d} = \frac{L}{c} = t_1 \quad \text{and} \quad x_{rR} = v_d t_1 \ .$$

Also

$$t_{fR} = \frac{L}{c - v_d + v_d} = \frac{L}{c} = t_1 \quad \text{and} \quad x_{fR} = v_d t_1 \ .$$

Thus we derive from Einstein's train example exactly the opposite conclusion from the one he reached. Namely, we have shown that according to the Galilean invariant version of electromagnetic theory S deduces from his own observations that $t_{rR} = t_{fR} = t_1$, as well as $x_{rR} = x_{fR} = \vec{v}_d t_1$; so that R receives the flash signals from front and rear of the train simultaneously (and of course at the same train location along the track). S also perceived the same flashes as simultaneous. So, as this instance illustrates, *simultaneity is absolute*, if you make the right assumptions about light propagation. Because of the symmetry of relative motion, what R deduces from his observations will agree with what S deduces from his. In a Galilean invariant formulation of electromagnetism simultaneity is therefore a physical fact not altered by changes of inertial system viewpoint. Note that this conclusion rests primarily upon description of inertial transformations via (Galilean) invariant, instead of (Lorentz) covariant, mathematics. Our analysis here is valid only at first order, but the same conclusion is reached via a higher-order analysis ($t \to \tau$), as has been shown elsewhere[6].

# 4. Discussion

Are the assumptions we have made plausible? We have replaced covariance with invariance. Which is physics? Both involve form preservation under rival candidates to represent physical inertial transformations; thus both fit with a relativity principle. This circumstance brings out in starkly graphic relief the amazing fact that during a century nobody has felt dissatisfied enough with Einstein's assumptions to look seriously into available alternatives. That is not the way real physics progresses. Historically, it is the way unreal physics progressed, for instance, during the period of the "dark ages" millennium in which scientists backed Ptolemaic assumptions by unanimous consensus. (The science was settled.) Are we entering a new dark age in physics?

***Crucial Experiment:*** Maxwell's non-invariant equations have here been identified as the underlying point of contention. Instead of being taken for granted, those equations deserve to be searchingly tested. Elsewhere I have shown[6] that a simple test of covariant Maxwell theory against invariant Hertzian theory can be accomplished by using the existing Very Long-Based Interferometry (VLBI) system (given validity of its claims to astrometric precision) to measure to second-order accuracy the figure of *stellar aberration*. Maxwell-Einstein covariant theory predicts one thing, Hertzian invariant theory (in its higher-order form[6]) predicts another. The experiment is crucial. It should have been done long ago, if for no other reason than to check Einstein's second-order aberration formula. Why has this not been done? Perhaps, since it uses apparatus already paid for, it is too cheap? Meanwhile, numerous physics experiments in the billion-dollar range have been performed. To judge from their actions, physicists favor expensive experiments over cheap ones – to such an extent that in certain instances they are capable of overlooking the cheap ones entirely.

## References

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