Gravito-electromagnetism-another step towards a unified theory of the origin of mass

Brian Hills brian.hills@ifr.ac.uk

Norwich Research Park, IFR, Colney Lane, Norwich, NR47UA, UK

The field equations of gravito-electromagnetism are derived from first principles and shown to lead to the phenomenon of "mass induction" which provides a bridge between quantum field theories of particle mass and the older cosmological ideas of Mach. Incorporating mass induction into the equivalence principle leads to the exponential metric derived by Puthoff (2002) and used by Yilmaz (1976) in his modification of general relativity. The analysis strengthens the case that gravito-electromagnetism is not just a weak-field, linearised approximation of general relativity, but a viable alternative gravitational theory that provides a natural theoretical basis for pseudo-static cosmologies.

Keywords: Gravitation, inertial mass, equivalence principle, pseudostatic cosmology, exponential metric, GEM, gravitomagnetism

1. Introduction

This paper shows how the field equations of gravito-electromagnetism (GEM) can be derived from mass-energy conservation and how the resulting formalism provides a bridge between two very different theoretical approaches to the origin of mass. The first approach uses quantum field theory and the best known example is probably that of Higgs, who introduced the field of scalar, spin-zero bosons into the standard model to give mass to quarks and charged leptons (Djouadi, 2008). The recent report (Aczel, 2011) that the Large Hadron Collider (LHC) has so far failed to detect the Higgs boson in the expected energy range means that the search must now progress to other, less probable, energies, but it also raises doubts as to whether the Higgs boson exists at all. Indeed, a recent paper by Carlos Quimbay and John Morales (2011) presents an alternative theory where the standard model lacks the Higgs field and in which fermions acquire mass by interaction with the quantum vacuum and guage bosons from interaction with charge fluctuations of the vacuum. This and other theories (Rueda and Haisch, 2005; Cameron, 2011) focus on the role of the quantum vacuum in providing the intrinsic energy, e_{int}, of elementary particles. Alternatively, the elementary particles are described using lattice theory so that their internal energy is contained in the lattices themselves (Koshmieder, 2003, 2008, 2011). The transition to macroscopic inertial effects is contained in the equation, $m_{int} = e_{int}/c^2$ describing the equivalence of mass and energy together with the usual relativistic definition of momentum describing particle dynamics (Rindler, 2001).

These quantum vacuum and lattice approaches are highly topical but they leave open the question as to how they relate to much older cosmological ideas about the origin of inertial mass, especially those of Mach, whereby inertial mass finds its origin in gravitational interactions with all the distant matter in the Universe. Are Mach's ideas obsolete, wrong or merely in need of modification? Relational Mechanics (Assis, 1999; Ghosh, 2000) has successfully derived Mach's idea but the theory is based on an empirical force law proposed by Weber that requires instantaneous action at a distance and makes no reference to field theory. In this paper we take the opposite "field theoretical" viewpoint and show how the field equations of gravito-electromagnetism together with the Lorentz force law also provide a cosmological explanation of inertial mass in agreement with Mach's conjectures and in a way that complements the vacuum quantum-field and lattice derivations. It is further shown how gravito-electromagnetism leads to an exponential metric that agrees with that derived using the polarisable vacuum model (Puthoff, 2002) and with the exponential metric used in the modified form of general relativity proposed by Yilmaz (1976). The cosmological implications of the new approach are considered and the case is presented that gravito-electromagnetism is a viable alternative theory of gravity.

2. Gravito-electromagnetism and the origin of mass

According to Mach's ideas in the late 19th century, the inertial mass of a particle has its origin in its gravitational interaction with all the distant matter in the Universe. Little progress was made in quantifying this idea until the 1950's when Sciama (1952) showed how it could emerge not from general relativity but from the analogy between gravitation and electromagnetism. This analogy can be traced back to Oliver Heaviside (1893) who first suggested that a gravitational theory could be developed along the same lines as electromagnetism and, over a century later this idea was revived by Oleg Jefimenko (2000) in his book "Causality, Electromagnetic Induction and Gravitation". Gravito-electromagnetism (GEM), as the theory came to be called, therefore appears to be the key to quantifying Mach's ideas, and is the theoretical framework used in this paper. However as long as GEM was merely "by analogy" it had no firm theoretical basis. This appeared to be provided when it was shown that the Maxwell-like field equations of gravito-electromagnetism could be derived by linearising the field equations of general relativity in the limit of weak fields and flat space-time. However there is no agreement in the literature about the linearization process and the resulting field equations are not usually exact analogues of the Maxwell equations. An entirely different way of deriving gravito-electromagnetism was suggested by the recent work of José Heras (2007). Heras showed that the retarded field equations of classical electromagnetism could be derived, without approximation, from the continuity equation expressing charge conservation. Although Heras did not consider gravitation, his work implies that the equations of "gravito-electromagnetism" can also be derived, without approximation, from the continuity equation for mass-energy conservation. This derivation from first principles, which is summarised in the next section, is the first indication that gravito-electromagnetism is more than just an analogy or a linearised approximation to general relativity but a self-consistent alternative theory of gravity.

2.1 Deriving gravito-electromagnetism from first principles

Heras (2007) based his derivation of electromagnetism on the continuity equation,

$$\frac{\partial \rho}{\partial t} = -\nabla . \mathbf{J} \qquad (1)$$

expressing charge conservation. This suggests that gravito-electromagnetism can also be derived from the continuity equation expressing the conservation of mass in which case ρ becomes the source mass density and **J** the mass flux density. Starting with the continuity equation, Heras derived a mathematical identity theorem showing that it implies the existence of two retarded fields, $\mathbf{E_g}$ and $\mathbf{B_g}$ satisfying the Jefimenko equations (Jefimenko, 2004),

$$\mathbf{E_g}(\mathbf{r},t)=\frac{\alpha}{4\pi}\int d^3x'\left(\frac{\mathbf{r}}{r^3}\rho(x',t')+\frac{\mathbf{r}}{cr^2}\frac{\partial\rho(x',t')}{\partial t}-\frac{1}{c^2r}\frac{\partial\mathbf{J}(x',t')}{\partial t}\right) \quad (2)$$

$$\mathbf{B_g}(\mathbf{r},t)=\frac{\beta}{4\pi}\int d^3x'\left(\mathbf{J}(x',t')\times\frac{\mathbf{r}}{r^3}+\frac{\partial\mathbf{J}(x',t')}{\partial t}\times\frac{\mathbf{r}}{cr^2}\right) \quad (3)$$

where

$$\mathbf{r}=(\mathbf{x}-\mathbf{x}') \quad \text{and} \quad r=|\mathbf{x}-\mathbf{x}'| \quad (4)$$

Here $\mathbf{x}'$ refers to the position of the source and $\mathbf{x}$ refers to the position where the fields are measured and we note that the sources, $\rho(x',t')$ and $\mathbf{J}(x',t')$, are to be evaluated at the retarded time, $t' = t - r/c$, where c is the speed of propagation of the field. If we start from mass continuity then $\mathbf{E_g}(\mathbf{r},t)$ is the gravitoelectric field and $\mathbf{B_g}(\mathbf{r},t)$ is the gravitomagnetic field and the proportionality constants satisfy the condition, $\alpha = \beta\gamma/c^2$, where c is the speed of gravity, taken to be the speed of light. The first term in equation 2 decreases as $1/r^2$ and gives the retarded version of Newton's law of gravitation. But the Jefimenko equations show that there are, in addition, two types of gravitational field with five independent sources of those fields. In general, the five terms in equations 2 and 3 create gravitational forces that act in different directions and have subtly different effects although little work has been done to research them. Heras further showed that the two retarded fields, $\mathbf{E_g}(\mathbf{r},t)$ and $\mathbf{B_g}(\mathbf{r},t)$ satisfy Maxwell-type field equations such that,

$$\nabla.\mathbf{E_g} = \alpha\rho \tag{5}$$

$$\nabla.\mathbf{B_g} = 0 \tag{6}$$

$$\nabla\times\mathbf{E_g} = -\gamma\frac{\partial\mathbf{B_g}}{\partial t} \tag{7}$$

$$\nabla\times\mathbf{B_g} = \beta\mathbf{J}+\left(\frac{\beta}{\alpha}\right)\frac{\partial\mathbf{E_g}}{\partial t} \tag{8}$$

Equation 5 is the differential form of Newton's law of gravity, which immediately identifies the coefficient α, in SI units, with –4πG. If we assume that the vacuum speed of gravity is the same as the vacuum light speed, c, and is constant, then the product βγ will also be constant, but there is no agreement in the literature about the individual values of β and γ. If, following Heaviside and Jefimenko, we assume a complete analogy with electromagnetism, then $\gamma = 1$ and $\beta = -4\pi G/c^2$. However, as already mentioned, Maxwell-like field equations can also be derived by linearising Einstein's field equations in the weak field and flat space-time limits, but, unfortunately, there appears to be no agreement about the linearization process. For example, the field equations derived by Agop and co-workers (1999) in SI units are the same as equations 5-8, corresponding to $\gamma = 1$ and $\beta = -4\pi G/c^2$. On the other hand, in SI units, the field equations derived by Mashboon (2008) correspond to $\gamma = ½$ and $\beta = -8\pi G/c^2$. Because γβ is unchanged, Mashboon's field equations still conform to equations 5-8, but other authors (Chashchina et al., 2009; Forrester, 2010) have derived different forms of the linearised field equations. To avoid confusion, in this paper the field equations of gravito-electromagnetism, equations 2-8, will be left in their "native" form with the coefficients α, β and γ in place and it is then clear how they affect the physics. Of course, the fields express themselves as forces acting on particles according to the Lorentz force law,

$$\mathbf{F} = \rho\left(\mathbf{E}_g + \mathbf{v} \times \gamma \mathbf{B}_g\right) \tag{9}$$

As we shall see, the factor of γ in equation 9 is required for the consistent derivation of Newtonian mechanics. Equations (1) to (9) imply that all the mathematical apparatus of classical electromagnetism can be carried over into gravito-electromagnetism and this includes the scalar and vector potentials needed to derive the cosmological origin of mass.

2.2 The cosmological approach to the origin of mass

Mach's principle states, in his own words, that

"The inertia of a body must increase when ponderable masses are piled up in its neighbourhood"
and
"A body in an otherwise empty universe should have no inertial mass".
Mach did not provide a derivation of these statements but, following the ideas of Sciama (1952) it will now be shown how they emerge naturally from gravito-electromagnetism. We begin with the expressions for the retarded scalar and vector potentials,

$$\emptyset_g(r,t) = \frac{\alpha}{4\pi}\int d^3r' \frac{\rho_g(r',t_r)}{|r-r'|} \tag{10}$$

$$\mathbf{A}_g(r,t) = \frac{\beta}{4\pi}\int d^3r' \frac{J_i(r',t_r)}{|r-r'|} \tag{11}$$

In terms of these potentials, the gravitoelectric and gravitomagnetic fields assume the familiar form,

$$\mathbf{E}_g = -\nabla\emptyset_g - \gamma\frac{\partial \mathbf{A}_g}{\partial t} \tag{12}$$

$$\mathbf{B}_g = \nabla\times\mathbf{A}_g \tag{13}$$

Now consider the hypothetical situation of a single test particle in the vacuum of space with only the distant galaxies as an absolute reference frame. In this special case, **v**, in the Lorentz force expression is the instantaneous velocity of the test particle relative to the distant galaxies and the test particle interacts only with the scalar and vector potentials arising from the distant matter in the universe. Of course, this cosmological scalar potential is, on average, homogeneous and isotropic and can be treated merely as a constant background potential, φ_{gb}, which acts *instantaneously* on the test particle simply because it is "already there". Retardation effects in this background field can therefore be ignored. By symmetry, the background vector potential, $\mathbf{A_g}$, experienced by a test particle that is stationary with respect to the distant galaxies is zero. Centering the inertial frame of reference on the single test particle, equation 10 therefore gives the background potential as,

$$\varphi_{gb} = -\int_V dr^3 \frac{G(r)\rho(r)}{r} \tag{14}$$

For generality the gravitational constant has been given a weak dependence on cosmological distances such that it is essentially constant over intra-galactic distances but can vary over inter-galactic distances, a point that will be justified in section 4. We could have replaced ρ(r) by an average radial-independent term, ρ_{av}, but we eventually wish to include the effects of a local stationary mass, M, so retain the more general form. A similar simplification applies to the vector potential which becomes

$$\mathbf{A_g} = n\int_V dr^3 \frac{G(r)\rho(r)\mathbf{v}}{c^2 r} = \frac{n\mathbf{v}}{c^2}\ \varphi_{gb} \tag{15}$$

where n is either 1 or 2 depending on the choice of the coefficient β, being 1 if β is $-4\pi G/c^2$ or 2 if β is $-8\pi G/c^2$. From the viewpoint of an inertial frame centred on the test particle it is the rest of the Universe that is moving with velocity –**v** and creating an instantaneous vector potential at the test particle which vanishes when the particle is stationary. The field, $\mathbf{A_g}$, is instantaneously created because the background scalar potential, ϕ_{gb}, is already present, so, once again, no retardation effect need be considered. Substituting eqn.15 into eqns. 12 and 9, the Lorentz force acting on the test particle becomes,

$$\mathbf{F}_g = m\mathbf{E_g} = m\nabla\emptyset_{gb} - \left(\frac{\gamma n m\mathbf{a}}{c^2}\right)\int_V dr^3 \frac{G(r)\rho(r)}{r} \tag{16}$$

The first term on the right is zero because the background gravitational potential is uniform, but the acceleration-dependent term is not zero because the spherical symmetry is broken by the acceleration vector. We proceed by analysing this acceleration term and note that the product γn in equation 16 is unity regardless of the choice of coefficients γ and β, provided we assume the speed of gravity is a constant. Mach's principle and the derivation of inertial mass are therefore independent of the choice of parameters in section 2.1. The Machian effect of "piling up local mass" can now be investigated by writing,

$$\rho(r)= \rho_{av}+ M\delta\left(r-r'\right) \tag{17}$$

which corresponds to introducing a thin spherical shell of total mass, M, at a distance r' from the test particle; whereas ρ_{av} is the average mass density of all the galaxies in the distant Universe. Again, we will ignore retardation effects in this local interaction and assume that M does not move relative to the distant galaxies. Then the acceleration term in eqn.16 becomes,

$$\mathbf{F} = -\left(\frac{m\mathbf{a}}{c^2}\right)\int d^3r\frac{G(r)\,\rho_{av}}{r} - \left(\frac{m\,\mathbf{a}}{c^2}\right)\left(\frac{GM}{r'}\right) \tag{18}$$

The significance of these terms becomes transparent if they are written in the form,

$$\mathbf{F} = -(m\mathbf{a})I - \mathbf{a}\Delta \tag{19}$$

where

$$I = \left(\frac{1}{c^2}\right)\int_V dr^3\,\frac{G(r)\rho_{av}}{r} \tag{20}$$

and

$$\Delta m = \frac{GMm}{c^2 r'} \quad (21)$$

The first term in eqn.19 is none other than Newton's second law multiplied by the factor I. The negative sign shows that the inertial mass, m, acts to oppose the effect of an external force. Furthermore we know that Newton's second law is locally valid which means that the integral, I, must be unity. But if I is unity it follows that,

$$\varphi_{gb} = \int_V dr^3 \frac{G(r)\rho_{av}}{r} = c^2 \quad (22)$$

The background scalar potential field therefore assumes the constant value c^2. With this value, the background gravitational vector potential, $\mathbf{A}_{gb}$, in equation 15 turns out to be n times the test particles velocity:

$$\mathbf{A}_{gb} = \frac{n\mathbf{v}}{c^2}\,\varphi_{gb} = n\mathbf{v} \quad (23)$$

This is consistent with the identification of the vector potential with the "potential field momentum" (Konopinski, 1978), whereby the particles momentum is $\gamma m\mathbf{A}_{gb}$, or $n\gamma m\mathbf{v}$, which is just the Newtonian value, $m\mathbf{v}$, because $n\gamma$ is unity. A particles momentum therefore arises because its uniform motion creates its own gravitational vector potential, $n\mathbf{v}$, which, relative to the particle, arises from the "backward" motion of the distant galaxies! Newton's second law follows immediately because force is the rate of change of momentum, but now we see how it emerges naturally once the existence of the distant galaxies is acknowledged in the theoretical framework. Indeed, equation 23 is a quantitative statement of Mach's principle which can be seen by hypothetically considering the test particle with velocity **v** and mass m then annihilating the distant galaxies (while ignoring retardation effects) so that φ_{gb} becomes zero. In this thought experiment $\mathbf{A_{gb}}$ also becomes zero, so the test particles momentum, $\gamma m\mathbf{A_{gb}}$, is zero. But if the velocity is not zero the only way this can happen is if the test particles inertial mass, m, vanishes. In other words, annihilating the distant matter in the Universe sets the inertial mass to zero, which is Mach's principle.

Let us now put the distant galaxies back in place (!) and investigate the effect of "pilling up local mass" by examining equation 21. We see that pilling up the local mass, M, has the effect of inducing more mass, Δm, in the test particle, which is precisely what is required by Mach's principle. Indeed, if we use the equality $\varphi_{gb} = c^2$, equation 21 can be rewritten,

$$\frac{\Delta m}{m} = \frac{\emptyset_{Loc}(r')}{\emptyset_{gb}} \tag{24}$$

where the local scalar gravitoelectric potential, ϕ_{loc}, from the spherical shell has been defined in the usual way as GM/r'. Equation 21 or 24 can be written in various ways. If we note that an increment dM, in M causes an increment dm in m then it can be expressed as,

$$\frac{dm}{dM} = \frac{Gm}{rc^2} \tag{25}$$

whose solution is,

$$\int_{m_{int}}^{m} \frac{dm}{m} = \int_0^M dM \frac{G}{rc^2} \quad \text{which gives} \quad \frac{m}{m_{int}} = e^{\frac{GM}{rc^2}} \quad (26)$$

where m_{int} is the particles "internal" mass in the absence of a local gravitational potential. It is significant that an exponential dependence of mass on the local gravitational potential emerges in this way, because it will be shown that other physical properties also have an exponential dependence on local gravitational potentials. In the limit where the spherical shell of mass, M, becomes the mass of the whole Universe the potential, GM/r', becomes ϕ_{gb}, which is c^2, and equation 21 shows that $\Delta m = m$. In other words, the whole of a particle's mass results from interaction with all the distant matter in the Universe, which is again Mach's principle. A straightforward application of gravito-electromagnetism has therefore provided the quantitative statement of Mach's ideas about the origin of mass.

So far we have ignored the contribution of the gravitomagnetic field, $\mathbf{B_g}$, in the Lorenz force expression. Incorporating this term, we obtain,

$$\mathbf{F} = m\left[\mathbf{E_g} + \gamma \mathbf{v} \times \mathbf{B_g}\right] = m\left[-\gamma \frac{d\mathbf{A_g}}{dt} + \gamma \mathbf{v} \times \left(\nabla \times \mathbf{A_g}\right)\right] \quad (27)$$

Substituting n$\mathbf{v}$ for $\mathbf{A_g}$ and noting that $n\gamma$ is one, gives,

$$\mathbf{F} = m[-\mathbf{a} + \mathbf{v} \times (\nabla \times \mathbf{v})] \quad (28)$$

This is Newton's equation of motion for curvilinear trajectories. Consider, the special case where the particle undergoes uniform circular motion in a frame of reference (such as the Earth) rotating with angular velocity, $\boldsymbol{\omega}$, with respect to the distant stars. For this situation it is a matter of Euclidean geometry to show that $\mathbf{a} = \mathbf{a_e} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})$, and $\boldsymbol{\nabla}\times\mathbf{v} = \mathbf{2}\boldsymbol{\omega}$ where $\mathbf{a_e}$ is the particle acceleration in the Earth's rotating frame. Then equation 28 becomes,

$$\mathbf{F} - m\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) - 2m\boldsymbol{\omega}\times\mathbf{v} = m\mathbf{a_e} \tag{29}$$

The second and third terms on the left hand side will be recognised as the fictitious centripetal and coriolis forces of Newtonian mechanics, respectively, and we now see how, in agreement with Mach's ideas, these forces arise within the framework of gravito-electromagnetism from rotational motion relative to the distant galaxies. The famous rotating bucket experiment (Assis, 1999) whereby the water in a spinning bucket on the Earth's surface develops a curvature, and which persuaded Newton to introduce the ether as an absolute frame of reference for all rotation, is now resolved in the simple concept that the rotation is relative not to the "ether" but to the distant galaxies and the centripetal force term in equation 29 left when **v** is zero (so the bucket has no linear velocity with respect to the Earth) causes the rotating water inside the bucket to have a curved surface. Newton could not have deduced this because his force law lacked the velocity and acceleration terms and, of course, the term arising from the gravitomagnetic field. If we use an Earth frame of reference by putting $\boldsymbol{\omega}$ to zero and define the moment of the force **F** acting on a rigid body as the torque, **N** = **r** x **F**, where **F** is m$\mathbf{a_e}$ (or md**v**/dt) then **N** = d/dt(**r** x **p**) = d**L**/dt where **L** is the angular momentum. From this equation all of rigid body rotational mechanics can be derived (Goldstein, 1980) so classical mechanics emerges naturally from gravito-electromagnetism and the Lorentz force law. It is interesting to note, in passing, that it was not only Newton, but also Einstein who was perplexed by the simple bucket experiment because, surprisingly, General Relativity has, so far, failed to derive equation 29, (Assis, 1999) despite numerous attempts (Brill and Cohen, 1966; Reinhardt, 1973). This is another piece of evidence that gravito-electromagnetism is more than just a linearised approximation to general relativity, which is a theme that will be taken up again in section 7.

2.3 Unifying the cosmological and quantum field approaches to mass.

If, according to equation 26, mass can be induced by other local masses we are led to reconsider our current understanding of the nature of mass and its relationship to energy. In the quantum field and lattice approaches to particle mass, the internal energy of a fundamental particle, e_{int}, arises from its lattice or from its interaction with the quantum vacuum (Quimbay and Morales, 2011) or, possibly, with the Higgs boson (Djouadi, 2008) or through photon impedance (Cameron, 2011). This internal energy is then identified with the particle's rest energy and its "rest mass" would be given as e_{int}/c^2. However we have just seen that a static gravitational potential can induce mass in the particle so that the rest mass must vary depending on the gravitational potential it experiences. This suggests that the total mass should, instead, be defined as $m = (m_{int} + \Delta m)$ where Δm is the extra mass induced by the particles interaction with external local gravitational potentials and the internal mass, m_{int}, is the mass equivalent of all the fundamental particles comprising the test particle (and their interactions) in the absence of any external local gravitational potential. The total energy, e_T, of the particle is then the sum of its total potential energy and kinetic energy, where the total gravitational energy is the product of the total mass and the total gravitational potential acting on the mass, just as the electrical potential energy is the product of the total charge and the electric potential,

$$e_T = m(\emptyset_{gb} + \emptyset_{local}) + q\emptyset_e + e_k = (m_{int} + \Delta m)(\emptyset_{gb} + \emptyset_{local}) + q\emptyset_e + e_k \quad (30)$$

Here the kinetic energy, e_k, has been included and the particle has an electric charge, q, in an electric potential, ϕ_e. The rest energy, e_0, can now be defined as that residual energy when the kinetic energy is zero:

$$e_0 = (m_{int} + \Delta m)(\emptyset_{gb} + \emptyset_{local}) + q\emptyset_e \quad (31)$$

which shows that the rest energy is no longer simply the internal energy, e_{int}, but includes terms from the induced mass, as well as the static local gravitational and external electrical interactions. The internal energy is that energy remaining when there are no local gravitational or electric potentials so the induced mass is zero and there is no kinetic energy:

$$e_{int} = m_{int}\emptyset_{gb} = m_{int}c^2 \quad (32)$$

In gravito-electromagnetism, equation 32 is the formal expression of Einstein's famous equation and we note that it has been deduced without reference to the postulates of Special Relativity. Moreover we see that, in gravito-electromagnetism, it pertains only to the internal energy of the particle and shows that the background gravitational potential of all the distant matter in the universe, ϕ_{gb}, provides the scaling factor between the internal mass, m_{int}, and the internal energy. Of course, gravito-electromagnetism (GEM) does not provide values for e_{int} for the elementary particles which is the role of quantum field theory of the vacuum but we now see how GEM complements quantum field theory by incorporating the mass induction process.

It is sometimes, incorrectly, assumed that one can write $e_T = mc^2$ as though this relationship applies to the total energy and total mass, but equation 30 shows that this is only approximately true for stationary particles in the absence of electric potentials and when $\phi_{loc} << \phi_{gb}$ so that local gravitational potentials can be neglected.

The induced mass effect implies that gravitational or electric field potential energy is not some abstract concept introduced to balance energy equations but is actually energy stored as induced mass in the particle. Increasing the gravitational or electrical potential of the particle causes a real increase in its total mass, m. If the particle undergoes free fall in a gravitational potential it gains kinetic energy at the expense of its mass, m, so that its total energy, e_T, is conserved. If it is heated so that its internal energy, U, increases by an amount dU, then the particles internal energy, e_{int}, increases by an amount, dU, and its internal mass, m_{int}, increases by an amount dU/c^2. It seems that conventional physics has been somewhat schizophrenic in its treatment of mass, because, while it happily acknowledges that energy and mass can be interconverted via the relationship "$e = mc^2$", it also, tacitly assumes that the mass is a constant of the motion and cannot vary as a particle interchanges potential energy and kinetic energy in its motion through potential energy fields! Indeed the very name "potential energy" means that it is stored as mass in the particle which is "potentially" available for conversion into kinetic energy. A more explicit definition of mass, consistent with gravito-electromagnetism and Mach's principle has therefore been introduced.

3. Mass induction and the Equivalence Principle

The exponential dependence of the induced mass on the local gravitational potential (equation 26) suggests that similar exponential relationships may exist for all other physical quantities. These relationships emerge when the induced mass concept is incorporated into Einstein's equivalence principle. To do this, consider a test particle of mass, m, in a freely-falling inertial frame coming from infinity towards a mass, M. For non-relativistic speeds when u << c, we have

$$\frac{d(mu)}{dt} = -\frac{GMm}{r^2} \tag{33}$$

or

$$umdu + u^2dm = -\frac{GMm}{r^2}dr \tag{34}$$

Noting that, according to equation 31, the induced mass, dm, is -GMmdr/c^2r^2, this becomes

$$umdu - \frac{u^2GMm}{c^2r^2}dr = -\frac{GMm}{r^2}dr \tag{35}$$

This can be integrated assuming the particle has an initial velocity, v, at r = ∞,

$$\int_{v}^{u} du \frac{u}{[1-\frac{u^2}{c^2}]} = -GM \int_{\infty}^{r} \frac{dr}{r^2} \tag{36}$$

whose solution is

$$u^2 = c^2 \left\{1-\left[1-\frac{v^2}{c^2}\right] e^{-\frac{2GM}{c^2 r}}\right\} \tag{37}$$

This is the required equivalence between an inertial frame moving at velocity, u, with an inertial frame under free-fall in a gravitational field. In other words, it associates a gravitational field at each point, r, with an equivalent free-fall, inertial velocity field, u(r). In the limit v = 0 and small M, it reduces to the Newtonian limit $u^2 = 2GM/r$ and in the limit v = 0 and M = ∞, we find $u^2 = c^2$, which was a result derived earlier. The equivalence principle in the form of equation 37 shows that for static gravitational interactions the Lorentz relativistic factor, γ_r, involving the factor, u^2/c^2, changes to γ_{gem} which is obtained by substituting for u^2/c^2 with equation 37. In other words,

$$\gamma_r = \frac{1}{[1-u^2/c^2]^{\frac{1}{2}}} \quad \text{becomes} \quad \gamma_{gem}(v,r) = \frac{e^{\frac{GM}{c^2 r}}}{\left[1-\frac{v^2}{c^2}\right]^{\frac{1}{2}}} \tag{38}$$

In the absence of an external gravitational potential (M = 0), $\gamma_{gem}(v)$ reduces to the usual Lorentz relativistic factor where v is the relative velocity of the test particle as seen by a stationary observer outside the gravitational potential. Conversely, in the v = 0 limit it reduces to the same exponential factor derived by Puthoff using a Lagrangian approach and treating the quantum vacuum as a polarisable medium (Puthoff, 2002). In the following we will specialise to the v = 0 limit corresponding to stationary particles in a static gravitational potential as observed from outside the gravitational potential. The mass induction equation 26 in this v = 0 limit can then be written more concisely as

$$\frac{m(r)}{m_{int}} = \gamma_{gem} \tag{39}$$

where γ_{gem} denotes $\gamma_{gem}(v=0)$. In an analogous way the usual relativistic expressions for time dilation and length contraction become $T(r)/T = \gamma_{gem}$ and $L(r)/L = \gamma_{gem}^{-1}$ which is none other than the exponential metric proposed by Yilmaz (1976). Moreover this metric corresponds to a null geodesic such that $c(r)/c = \gamma_{gem}^{-2}$ which has been independently derived using the polarisable vacuum model by Puthoff (2002). We see therefore, that gravito-electromagnetism is consistent with the polarisable vacuum model and the implications of this will be discussed in future work on light propagation. The above analysis suggests that an exponential metric should also be used in the quantum vacuum model recently presented by Urban and co-workers (2011). They showed that the vacuum permittivity, ε_0, is proportional to the ratio, $e_e^2/\hbar c$, and the vacuum permeability, μ_0, is proportional to $\hbar/ce_e^2$ where e_e is the charge on the electron. This immediately implies that $\varepsilon_0(r)/\varepsilon_0 = \gamma_{gem}^2$, and this, in turn, means that the internal mass, m_{int}, of all normal matter such as atoms, molecules and plasma also varies in a local gravitational potential as $m_{int}(r)/m_{int} = \gamma_{gem}^2$. To see this consider the internal energy, e_{int}, of a simple parallel-plate capacitor of plate area, A, and plate separation, d, as the permittivity, $\varepsilon_0(r)$, of the vacuum varies through interaction with a gravitational potential. Its capacitance, C(r), is $\varepsilon_0(r)A/d$ and its internal energy is $e_{int}(r) = C(r)V^2/2$. This means that the internal energy of the capacitor varies as,

$$e_{int}(r) = \frac{\epsilon_0(r)AV^2}{2d} \tag{40}$$

If an atom is now modelled as a tiny capacitor we deduce that the internal energy of normal matter is directly proportional to the vacuum permittivity, $\varepsilon_0(r)$, so that

$$\frac{e_{int}(r)}{e_{int}} = \frac{\epsilon_0(r)}{\epsilon_0} = \gamma_{gem}^2 \tag{41}$$

Dividing through by c^2 gives

$$\frac{m_{int}(r)}{m_{int}} = \gamma_{gem}^2 \tag{42}$$

Substituting (42) into (39) we find that the total mass, m, varies with position in a gravitational potential as $\gamma_{gem}{}^3$:

$$m(r) = m_{int}(r)\gamma_{gem} = m_{int}\,\gamma_{gem}^3 \tag{43}$$

This dependence of the total mass on its position, r, in a local gravitational potential allows a similar expression to be derived for the total particle energy. According to equation 30, the total energy of a stationary particle in a gravitational potential, in the absence of electric potentials, is given as,

$$e_T(r) = m(r)[c(r)^2 + \emptyset_{loc}] \tag{44}$$

Provided we can neglect the local gravitational potential because it is small compared to $c(r)^2$ we can substitute for m(r) and c(r) to obtain, as an approximation,

$$\frac{e_T(r)}{e_{int}} = \gamma_{gem}^{-1} \tag{45}$$

This can be taken further by noting that if a test particle of energy, e_{int}, in the absence of a local gravitational potential, is converted into a photon it will have an equivalent energy, $\hbar\omega_0$, but the same test particle in a gravitational potential will convert into a photon of frequency, $\hbar\omega(r)$, such that

$$\frac{\omega(r)}{\omega_0} = \frac{e_T(r)}{e_{int}} = \gamma_{gem}^{-1} \tag{46}$$

This describes the familiar gravitationally-induced redshift observed in the Pound-Rebka experiments. Note that this does not predict that the frequency of light varies in its passage through a gravitational potential as is sometimes wrongly claimed. Such a frequency shift would violate energy conservation. Instead the well-known gravitationally-induced redshift (or blueshift) of light is caused by time dilation of the transmitting source in the gravitational potential. This can be seen by inverting equation 46 to give the time dilation of a stationary radiating quantum source in a gravitational potential:

$$\frac{T(r)}{T} = \gamma_{gem} \tag{47}$$

But this time dilation will exist whether or not the system is radiating, so equation 47 describes the universal gravitationally-induced time dilation of all quantum systems, and therefore of all matter. The same relationship can be derived from the Heisenberg uncertainty principle according to which the lifetime, T, of a stationary, transient energy state is $\hbar/2e_T$, so that $T(r)/T = e_{int}/e_T(r) = \gamma_{gem}$. A universal gravitationally-induced Lorentz contraction also follows from the Compton expression relating wavelength and mass in a quantum system. The Compton wavelength is defined as the wavelength of light that has the equivalent energy as the total mass of a stationary particle. In other words,

$$m_{int}c^2 = \frac{hc}{\lambda_c} \quad \text{or} \quad \lambda_c = \frac{h}{m_{int}c} \tag{48}$$

In a gravitational potential this becomes,

$$\lambda_c(r) = \frac{h}{m(r)c(r)} \tag{49}$$

Substituting for m(r) and c(r) this gives,

$$\frac{\lambda_c(r)}{\lambda_c} = \gamma_{gem}^{-1} \tag{50}$$

But the “size” of a quantum system is directly proportional to the Compton wavelength. For example, the radius of the ground state Bohr orbit of a hydrogen atom is the Compton wavelength divided by the fine structure constant. So taking the Compton wavelength of a quantum system in the vacuum as a measure of the quantum length scale we see that a gravitational potential causes length contraction such that,

$$\frac{L(r)}{L} = \gamma_{gem}^{-1} \qquad (51)$$

The metric equations (47) and (51) are, of course, the same as the more heuristic derivation from the equivalence principle presented earlier and show, as they must, that the force in Newton's second law, m(r)**a**(r), is invariant to the effect of local gravitational potentials.

The exponential form of the expressions for mass, energy, frequency, length and time expressed in powers of the relativistic factor, γ_{gem}, has one very important consequence, namely that in gravito-electromagnetism there are no event horizons and therefore no black holes! In general relativity a black hole exists and is characterised by an event horizon located at a radial distance, R_s, equal to the Shwarzschild radius, $2GM/c^2$. General relativity with the Shwarzschild metric therefore predicts that no light or matter can escape through this event horizon and to an outside stationary observer the period of an Einstein clock moving towards the event horizon shows infinite time dilation (infinite slowing down) as it reaches R_s. However, these pathological divergences are done away with in gravito-electromagnetism because at all non-zero values of r, γ_{gem} remains finite. The same effect is, of course, found in the modification of general relativity proposed by Yilmaz which uses the same exponential metric (Yilmaz, 1976) and means that (supermassive) neutron stars would need to assume the role conventionally assigned to (supermassive) black holes. As mentioned, the same exponential factors have been independently derived by Puthoff using a polarisable vacuum model (Puthoff,

2002) and shown to give correct values for the perihelion precession of Mercury, the bending of light and the Shapiro time delay and for the other PPN tests of general relativity. This is a second hint that gravito-electromagnetism may be more than just an approximate linearised form of general relativity.

The effect of local gravitational potentials on the vacuum permittivity (equation 41) and permeability suggests that local gravitational potentials can be incorporated into the Maxwell and GEM field equations by modifying the coefficients α and β. In electromagnetism equations 5 to 8 correspond to Maxwell's equations with the coefficients α and β equal to $1/\varepsilon_0$ and μ_0 respectively with $\gamma = 1$, and, of course, **E** and **B** then refer to the electric and magnetic fields. So in the presence of a gravitational potential these coefficients will be modified such that,

$$\frac{\alpha(r)}{\alpha} = \frac{\epsilon_0}{\epsilon_0(r)} = \gamma_{gem}{}^{-2} \tag{52}$$

$$\frac{\beta(r)}{\beta} = \frac{\mu(r)}{\mu_0} = \gamma_{gem}{}^{2} \tag{53}$$

where the electromagnetic coefficient, γ, is assumed to remain independent of gravitational potentials. The GEM field equations will also be modified by local gravitational potentials. If we assume that the gravitational constant, G, is unaffected by local gravitational potentials then gravitational coefficient α (= $-4\pi G$) is also invariant. However the gravitational coefficient, β, (= $-4\pi G/c^2$) will vary as $\gamma_{gem}{}^4$, assuming the gravitational coefficient γ is unity and invariant.

In principle, these gravitationally modified coefficients suggest how it may be possible to correct a fundamental asymmetry in the Lorentz force expression whereby the source mass creates gravitoelectric and gravitomagnetic fields that, via the Lorentz force equation, act on a “target” mass but the effects of the fields created by the target mass on the source mass are neglected, or have to be added as an afterthought. Instead, the above analysis suggests that the retarded fields created by the target mass should be incorporated as a local gravitational potential in the field equations using the modified coefficients, β(r) and c(r). The resulting modified gravitational fields from the source can then be incorporated into the Lorentz force equation to calculate the effect on the target mass, although, in practice, this procedure may well need to be done using an iterative numerical perturbation method. Of course, Relational Mechanics (Assis, 1999) solves this asymmetry problem directly by invoking an empirical Weber force law based only on relative inter-particle distances, velocities and accelerations. But the price that must be paid to do this is the assumption that forces propagate at infinite speed so there is instantaneous action at a distance, in violation of the causality relationship in special relativity. Whether this relational approach can be shown to be the $c = \infty$ limit of the covariant retarded field approach of gravito-electromagnetism and the Lorentz force law remains to be researched.

4. The radial dependence of the gravitational constant, G.

For Newton’s second law to be valid we found that the integral, I, in equation 20 had to be unity. But this presents another conceptual difficulty because a straightforward evaluation of I with a constant, G, and ρ_{av}, over the volume of the Universe, does not give unity, but rather infinity!

$$I = \left(\frac{G\rho_{av}}{c^2}\right)\int_V dr^3 \frac{1}{r} \;\rightarrow\infty \tag{54}$$

This divergence presented a serious paradox for Newton and has been discussed at length by Assis (1999). Laplace suggested that the divergence could be avoided if the gravitational constant was given a slight attenuation with distance such that

$$G(r) = Ge^{-\xi r} \tag{55}$$

but he was unable to provide any value or theory for the exponential decay coefficient, ξ. In gravito-electromagnetism the unit value of the integral I provides this missing theory because we can write,

$$I = \left(\frac{4\pi}{c^2}\right)G\rho_{av}\int_0^\infty dre^{-\xi r}r = \left(\frac{4\pi}{c^2\xi^2}\right)G\,\rho_{av} = 1 \tag{56}$$

so that

$$\xi^2 = \frac{4\pi G\rho_{av}}{c^2} \tag{57}$$

This shows that the decay constant, ξ, depends on the average mass density of the Universe and is so small, numerically, that the gravitational constant will only vary slightly over large cosmological distances and is locally constant. There is, however a subtlety hiding in equation 57 because it has already been shown that the speed of gravity, c, is not a constant, but is reduced in the presence of a local gravitational potential according to the relation $c(r) = c\gamma_{gem}^{-2}$. If we assume that G, and ρ_{av} are constants, then spatial variations in c(r) imply that there are also local variations in ξ such that $\xi(r) = \xi\, \gamma_{gem}^2$. However, because c(r) and $\xi(r)$ enter the integral, I, as a product, the integral remains invariant to local gravitational potentials, so that Newton's second law holds even in local gravitational potentials, as indeed, it must.

5. Induced mass and dark matter

The mass induction effect, equation 41, shows that the total mass of an object depends on its location in a gravitational potential so we are led to ask what effect this has on the gravitational force between two masses. The general case of two moving, interacting masses can be treated within the framework of gravito-electromagnetism by combining the induced mass effect with the Lorentz force equation and using the five types of gravitational forces associated with each term in the Jefimenko equations, 2 and 3. However, this lengthy calculation is outside the scope of this paper so, instead, the analysis will be limited to the effect of induced mass on the interaction between two *stationary* masses so that only the first term in equation 2, corresponding to the unretarded form of Newton's law remains. With this caveat, consider two stationary particles, of mass $m_1(r)$ and $m_2(r)$ separated by a distance, r. The induced mass effect, equation 41, means that the Newtonian gravitational force, F, between them now has an additional exponential factor,

$$F = \frac{G(r)m_1(r)m_2(r)}{r^2} = \frac{Gm_{1int}m_{2int}}{r^2}\, e^{\frac{3G(m_{1int}+m_{2int})}{rc^2}} \qquad (58)$$

where we have assumed a local interaction such that ξ is zero. This force law corresponds to a gravitational potential, $\Phi(r)$,

$$\Phi(r) = -\int_r^\infty drF(r) = -\frac{Gm_{1int}m_{2int}}{r^2}\int_r^\infty dr\, e^{\frac{3G(m_{1int}+m_{2int})}{rc^2}} \quad (59)$$

Evaluating the integral gives

$$\Phi(r) = \frac{m_{1int}m_{2int}c^2}{3(m_{1int}+m_{2int})}\left[1\text{-}e^{\frac{3G(m_{1int}+m_{2int})}{rc^2}}\right] \quad (60)$$

The Newtonian limit is obtained by expanding the exponential in 60 and taking the $c \to \infty$ limit, when $\Phi(r) = -Gm_{1int}m_{2int}/r$ as required. The fact that the exponential in equation 60 involves the factor G/c^2 ensures that the extra force created by the induced mass effect is a minor perturbation on the conventional Newtonian inverse square law, though it may become significant in large assemblies of massive objects, such as globular star clusters, galaxies and galactic clusters and may partly explain the origin of the "hidden mass" (or "dark matter") introduced to explain the dynamics of these objects. Indeed it seems premature to introduce mysterious 'dark matter' before the collective dynamics has been analysed with the induced mass effect and the five retarded force terms described by the Jefimenko equations, 2 and 3 so this should be a priority for future work.

6. Mass induction and expansion cosmology

It appears, therefore, that the calculation of the energy, e_{int}, of an elementary particle using the quantum field theory of the vacuum (e.g. by Quimbay and Morales, 2011) together with the revised definitions of the relationship between e_{int} and total mass from gravito-electromagnetism (GEM) provides a description of the origin of mass consistent with Mach's ideas. But this success also introduces a cosmological problem because gravito-electromagnetism operates in a flat space-time metric which is devoid of the expansion predicted by general relativity. What then is the origin of cosmological expansion? Does cosmological expansion exist at all? Or should we return to older and simpler "pseudostatic" cosmological models where the universe is non-expanding and of indefinite age and indeterminate extent? The only way to answer these questions is to re-examine the observational evidence for cosmological expansion which is not as strong as one might be led to believe (Ratcliffe, 2010; Harnett, 2011; Crawford, 2011). To see this we examine five key observations starting with galactic structure.

6.1 Galactic structure

In 2004, the Hubble Space Telescope completed a deep sky study of a small region of space below the constellation of Orion and revealed more than 10,000 galaxies at an estimated distance of 13.3 billion light years. The images contained galaxies having mature spiral arms containing countless ancient (red-giant) stars. According to big-bang inflation cosmology the universe is about 13.7 billion years old, so how, one has to ask, could such galaxies have evolved in just 400 million years from the diffuse cloud of high-energy quark-gluon plasma believed to have been created in the early stages of the big bang? Just the formation of a spiral arm galaxy from an elliptic galaxy requires billions of years as the flattened elliptic galaxy needs to rotate dozens of times to form the spiral arms and average galaxy rotational periods are of the order of 300 million years. Even the red-giant stars in elliptic galaxies are believed to be more than 10 billion years old. Instead these data would seem to support a much simpler pseudo-static cosmology in which there is a non-expanding distribution of galaxies at all stages of their life cycle that extends as far as our observations allow.

6.2 Cosmological redshifts

It is well known that expansion cosmologies find their support in Hubble's relationship between the increasing remoteness of galaxies and their recessional velocity. But it is very difficult to test this relationship because there is no independent way to measure galactic recession velocities at cosmological distances. Moreover there are many other possible explanations of the observed redshift data that point to a correlation between redshift with distance rather than with recession velocity (Ratcliffe, 2010). For example, a seminal paper by Ari Brynjolfsson (2005) shows that light interacting with dilute intergalactic plasma looses energy in the form of microwave quanta, resulting in a redshift-distance correlation and a thermalised background of microwave radiation which contributes to the Cosmic Microwave Background, which, in pseudo-static cosmology is just radiation from all the luminous matter in the universe that has been thermalised to a temperature of 3.5 K by absorption and re-radiation by the plasma, dust and molecules in intergalactic and interstellar space over hundreds of billions of years.

6.3 The angular size of radiogalaxies

The angular size of radiogalaxies up to a redshift, z, of 2 shows a simple inverse relationship with redshift, which is consistent with a static Universe, but not with expansion cosmologies. A fit with the big-bang inflation model can only be obtained if it is assumed that there is rapid expansion in size with epoch. In fact a galaxy at $z = 3.2$ would need to be about 6 times smaller in its linear dimension than at $z = 0$ to fit the data, but there is no evidence for such large size changes. A comparison of five of the brightest cluster galaxies (BCG's) at $0.8 < z < 1.3$ with a group of BCG's at $z = 0.2$ showed no more than a 30% decrease in size, indicating little or no size evolution (Harnett, 2011).

6.4 The Tolman Surface Brightness test of expansion cosmology

In 1935 Hubble and Tolman proposed a test of cosmological expansion based on the brightness of galaxies as a function of redshift. In a static universe the brightness of the same object varies with distance as $(1+z)^{-1}$; but in expansion cosmology it varies as $(1+z)^{-4}$. The extra factor of $(1+z)^{-3}$ arises because there is a factor of $(1+z)^{-1}$ from cosmological time dilation; another factor of $(1+z)^{-1}$ from the frequency redshift of photons due to cosmological expansion; and another factor of $(1+z)^{-1}$ because the object was closer to us when it emitted the photons. To apply the Tolman test it is, of course, necessary to find an object that is the same regardless of redshift, and, for various theoretical reasons, the maximum brightness of a galaxy in the ultraviolet has an upper limit that cannot be exceeded (Lerner, 2009). This limit was first measured for nearby galaxies and it was found that no, more distant galaxy exceeded this limit provided a non-expanding Universe model was assumed, such that redshifts depended linearly on distance. However if an expanding model of the Universe was assumed then galaxies had to be up to six times brighter than this limiting maximum (Harnett, 2011).

6.5 Cosmological time dilation

Cosmological expansion predicts that objects experience time dilation (slowing of rate processes) as a function of increasing recession distance (1+z). Such is not, of course, the case with the pseudo-static universe, so this provides another critical cosmological test. Quasars have luminosities that vary over times of weeks to years but a comparison of groups of quasars at low (z < 1) and high (z >1) redshifts showed no significant difference in their statistically averaged luminosity-time curves (Hawkins, 2001), consistent with the non-existence of cosmological time dilation. A similar result was found for the luminosity-time curves of Gamma Ray Bursts (GRB's) (Crawford, 2011). The weight of evidence is therefore, arguably, in favour of static rather than expansion cosmologies and this supports the gravito-electromagnetic analysis in a flat space-time metric presented earlier.

7. Discussion

The previous sections have provided several hints that gravito-electromagnetism (GEM) may be more than merely the weak field, linearised approximation of general relativity (GR). The fact that GEM can be independently derived from mass-energy conservation and gives an exponential relativistic factor, γ_{gem}, that passes the usual PPN tests without reference to the postulates of general relativity is an important consideration. Moreover, GEM, together with the quantum vacuum concept appears to provide a self-consistent theory of mass and rotational dynamics (equation 29) lacking in general relativity. The evidence in favour of a pseudostatic cosmology summarized in section 6 also hints that GEM in a flat space-time may be a better description of gravitational interactions on the large scale than GR and expansion. If so, there would be no need for the 'fixes' to cosmological expansion theory such as inflation theory and dark energy. However it remains to be seen whether a proper application of mass induction, the Lorentz force law and the retarded Jefimenko equations provides an accurate description of galactic rotational dynamics. If it does then there would also be no need for mysterious dark matter.

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