

# The Clock Riddle: The Failure of Einstein's Lorentz Transformation

Robert J. Buenker\*

Fachbereich C-Mathematik und Naturwissenschaften,  
Bergische Universität Wuppertal,  
Gaussstr. 20, 42119 Wuppertal, Germany

The present Note calls attention to an aspect of the special theory of relativity (SR) that has heretofore been ignored. An example of two observers approaching one another at uniform velocity is considered. It is shown that opposite predictions are obtained when different methods of applying the theory are used. It is pointed out that this contradiction necessarily invalidates the Lorentz transformation (LT) of SR, but in no way affects the viability of the corresponding velocity transformation (VT).

*Keywords:* Fitzgerald-Lorentz length contraction (FLC), time dilation, postulates of relativity, Lorentz transformation (LT), velocity transformation (VT)

---

\* E-mail: [buenker@uni-wuppertal.de](mailto:buenker@uni-wuppertal.de);

Homepage: [alternativelorentztransformation.blogspot.com](http://alternativelorentztransformation.blogspot.com)

## I. Introduction

The central result of Einstein's special theory of relativity (SR) is the Lorentz transformation (LT) [1]. Two of its most famous predictions are Fitzgerald-Lorentz length contraction (FLC) and time dilation. There have been numerous experimental observations of time dilation [2-5], but the FLC remains a question of some dispute due to the lack of an explicit empirical verification for this phenomenon. Indeed, it has often been argued that it is completely impossible to do this because of the need to determine the current locations of both ends of the object of the measurement at exactly the same time. Yet in the last half-century, length measurements have become routinely possible by using atomic clocks to measure the elapsed time it takes for a light pulse to traverse the distance between two such end points. This development makes it possible to test the internal consistency of SR without actually carrying out experiments, as will be shown below.

## II. The Clock Riddle

Consider a standard situation in discussions of relativity in which two inertial systems  $S$  and  $S'$  approach each other at *uniform velocity*  $\mathbf{v}$  along the mutual  $x, x'$  axis of their coordinate systems. The goal is to measure the distance ( $\Delta y$  and  $\Delta y'$ ) between two points A and B from the vantage point of respective observers O and O' who are at rest in  $S$  and  $S'$ , respectively, whereby the line joining A and B is aligned along a direction which is *perpendicular* to  $\mathbf{v}$ . According to the FLC [1], O and O' must agree on the value of this distance, hence

$$\Delta y = \Delta y' . \quad (1)$$

To check this prediction, each observer determines the value of the above distance by measuring the elapsed time ( $\Delta t$  and  $\Delta t'$ ) for a

light pulse to pass between A and B. However, because of time dilation [1], it is known that the clocks in their respective rest frames run at different rates, hence:

$$\Delta t \neq \Delta t' . \quad (2)$$

O and O' compute the value of this distance by using the fact that the speed of light is equal to c for both of them, in accord with Einstein's second postulate [1]:

$$\Delta y = c\Delta t , \text{ and} \quad (3)$$

$$\Delta y' = c\Delta t' . \quad (4)$$

Because of eq. (2), it therefore follows that

$$\Delta y \neq \Delta y' \quad (5)$$

when using this method to determine the respective distance values, in direct contradiction to the prediction of the FLC in eq. (1).

One question that inevitably arises in evaluating these arguments is the possible role that the simultaneity of events or lack thereof in SR [1] might play in resolving the above contradiction. It is easy to show that such concerns are unwarranted by considering the following special case. Let S be the rest frame of the earth and take points A and B to be stationary in it. It is obvious that an observer O in S can measure the distance between the two points repeatedly over an arbitrarily long period of time, in which case the result of the measurement will always be the same ( $\Delta y$ ). Assume further that S' was initially not moving relative to S before being accelerated to its current uniform velocity v. As a result, the proper clocks at rest in S' have a slower rate than those at rest in S by a factor

$\gamma(v) = \left(1 - \frac{v^2}{c^2}\right)^{-0.5}$ . The observer O' in S' can measure the location of each of the points over a long period of time as well. By

construction, the positions of the two points along the  $x'$  axis ( $x'_A$  and  $x'_B$ ) will vary with time, whereas the corresponding positions ( $y'_A$  and  $y'_B$ ) in the perpendicular direction will always be the same.

As a result, the distance between the two points  $\Delta y' = |y'_A - y'_B|$  will also be constant, so that the key differential quantity,  $\Delta y - \Delta y'$ , will always have the same value independent of when the measurements by either observer were actually carried out. It is clear that any theory that gives opposite results about whether  $\Delta y$  is equal to  $\Delta y'$  or not is unacceptable and needs to be modified.

It might appear that the contradiction in eqs. (1-5) can be overcome on the grounds of there not being a clear means for  $O'$  to carry out the measurements of the current positions of the two points at rest in  $S$ . This line of approach misses the whole point of the present argument, however, namely that each of these equations is based directly on assumptions or deductions in Einstein's theory. It is assumed implicitly that each of the quantities  $\Delta y$ ,  $\Delta y'$ ,  $\Delta t$  and  $\Delta t'$  in these equations is knowable by some means. The only valid way to dispute any one of these relationships is therefore to show that it does not actually follow directly from SR [1].

The underlying problem with SR that is revealed by the above considerations is that the theory fails to recognize that the speed of an object, the distance traveled by it, and the corresponding elapsed time to do so are not independent quantities. Once any two of them are known, the third is completely specified. This observation also holds for relationships involving these quantities. It therefore defies logic to assert that two observers agree on the speed of light but disagree on the elapsed time of its travel between two points, and then go on to

claim that the distance traveled by the light is somehow the same for both.

It is well known that of the three relationships above, only two of them have received substantial confirmation experimentally, namely the constancy of the speed of light and time dilation. Measurement of the Doppler effect for light emitted from a moving source gives a clear affirmation that the speed of light is the same for observers in different states of motion by virtue of simultaneous wavelength and frequency determinations. Quantitative predictions of the effects of time dilation have also been verified in numerous experiments [2-5]. At the same time, it is clear that the measurement of distance at a remote location *can never be direct*, so one clearly must approach this goal using assumptions that are both well-founded and can be verified independently. This strategy is best exemplified in the operation of the Global Positioning System (GPS). The key objective in this technology is to measure the distance between a given satellite and a position on the ground. In order to do this, it is necessary to measure the elapsed time required for a light pulse to travel between these two points, whereby it is assumed that the light travels with speed  $c$  along the way. The respective times of emission and absorption of the pulse are measured in two different rest frames, so it is necessary to account for the effects of time dilation and also the gravitational red shift on the respective clocks that are used for this purpose. Theory is used to estimate the ratio of the rates of the satellite and ground clocks [6], and experience with GPS demonstrates that this *indirect approach* is quite effective.

The goal in the GPS methodology is to measure distances from the vantage point of an observer on the earth's surface (S). However, it is clear that the same theoretical considerations can be used to measure distances from the vantage point of an observer on the satellite [7], i.e. by  $O'$  in the above example. It can be safely assumed that  $O'$  also

measures the light pulse to travel between A and B with the same speed  $c$  as O does. It is important to note that this result can only be obtained if proper clocks and measuring rods are used to carry out the measurement, since otherwise it is incorrect to assume that O' would actually measure the light speed to have a value of  $c$ . One knows from experiment [4, 6] how to compute the ratio of the proper clock rates used by O and O' in the present case. This ratio can be looked upon as the *conversion factor* between the respective units of time employed in the two rest frames. In practice, this means that the elapsed time  $\Delta t'$  can be deduced from the corresponding value  $\Delta t$  measured by O, thereby allowing for an accurate determination of  $\Delta y'$  via eq. (4).

The above considerations also have consequences for the modern definition of the meter [8] as the distance traveled by light in free space in  $c^{-1}$  s. Since proper clocks in different rest frames do not run at the same rate, it follows that the length of a meter also varies from one rest frame to another. The period of a proper clock has replaced the wavelength standard previously used to define the meter. Thus, the slower the clock, the farther light travels in  $c^{-1}$  s. The length of the meter changes by the same fraction as the duration of one second as a given rest frame undergoes acceleration. A local observer cannot measure such changes on the basis of exclusively *in situ* measurements because every object that is stationary in his rest frame has its dimensions changed in a perfectly uniform manner in exactly the same ratio as the corresponding rates of proper clocks. The constancy of light speed in free space for all observers, Einstein's second postulate [1], demands that this be so.

The above arguments are not restricted to length measurements in a perpendicular direction. The relationship between the measured

values for a distance that is aligned parallel to  $\mathbf{v}$  also depends on the ratio of the rates of proper clocks in  $S$  and  $S'$ , namely as:

$$\Delta x' = \gamma^{-1} \Delta x . \quad (6)$$

This result again stands in contradiction to the FLC [1], which states that

$$\Delta x' = \gamma \Delta x , \quad (7)$$

i.e., that lengths in the rest frame of  $S'$  contract along a parallel direction from the standpoint of observer  $O$  in  $S$ . Based on the experience with GPS technology, the opposite is seen to be the case because the unit of distance increases with the periods of local proper clocks. Thus  $O'$  must measure *smaller* values for all distances than his counterpart in  $S$ , not because the object of the measurement is different, but rather because the length of the meter that  $O'$  employs to make the measurement is *larger* than that used by  $O$ .

In summary, it needs to be emphasized that no actual experiments are involved in the original argumentation based on eqs. (1-5). Each of the latter results exclusively from assumptions of SR [1]. The lack of internal consistency obviously means that at least one of these relationships is incorrect. That in turn is unequivocal proof that the LT is not a valid physical space-time transformation since both the FLC and Einstein's light-speed postulate are inextricably connected with it. However, it also should be clear that the above contradiction has no consequences for the corresponding velocity transformation (VT) which is derived from the LT. This is a critical observation since there have been numerous experimental verifications of the VT such as the aberration of star light from the zenith [9] and the Fresnel light drag effect [10].

### III. The Relativistic Velocity Transformation

It might appear strange that the VT given below can be a valid set of equations even though it is derived from the LT:

$$u_x' = \left(1 - \frac{vu_x}{c^2}\right)^{-1} (u_x - v) = \eta(u_x - v) \quad (8a)$$

$$u_y' = \gamma^{-1} \left(1 - \frac{vu_x}{c^2}\right)^{-1} u_y = \eta\gamma^{-1} u_y \quad (8b)$$

$$u_z' = \gamma^{-1} \left(1 - \frac{vu_x}{c^2}\right)^{-1} u_z = \eta\gamma^{-1} u_z, \quad (8c)$$

where  $u_x' = \frac{x'}{t'}$  and  $u_x = \frac{x}{t}$  etc. are the velocity components of an

object measured by  $O'$  and  $O$ , respectively [ $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-0.5}$ ]. The

reason, as shown by Lorentz [11] as early as 1899, is that there is a normalization function that needs to be specified before the transformation equations can be fully determined. The latter function cancels out when ratios of space and time variables are computed in order to obtain the velocity components, so its value is inconsequential for that purpose. Einstein [1] made an assumption for the normalization function that ultimately leads to the  $y = y'$  relation of the LT and eq. (1) of the FLC.

It is a simple matter to eliminate the contradiction in eqs. (1-5), namely to make another assumption for the above normalization function that requires that time measurements of the two observers in  $S$  and  $S'$  satisfy a condition of *strictly proportional rates* of the corresponding clocks:

$$t' = tQ^{-1}. \quad (9)$$

The above relationship is clearly consistent with the inequality caused by time dilation in eq. (2), but it also serves to eliminate eq. (1) of the FLC. The alternative Lorentz transformation (ALT [7, 12]) can then be defined by combining eq. (7) through multiplication with each of the VT equations. Instead of eq. (1), the corresponding relation obtained from the ALT is therefore [see eq. (8b)]:

$$\Delta y' = (Q\gamma)^{-1} \left( 1 - \frac{vu_x}{c^2} \right)^{-1} \Delta y = \eta (Q\gamma)^{-1} \Delta y, \quad (10)$$

which is clearly consistent with eq. (5) as well as with the VT and Einstein's second postulate.

## IV. Conclusion

The example discussed in Sect. II shows unequivocally that one obtains contradictory predictions from the LT of SR depending on whether one assumes the FLC or the light-speed postulate to predict the relationship between the distance measurements of two moving observers. When the FLC is used, the distance values must be equal as in eq. (1), whereas in the latter case, the values must differ by arbitrarily large amounts *depending on the extent of time dilation* for each of the observers' clocks [see eqs. (2, 9)]. The LT is subject to the same rules of logic as any other physical theory, hence it is shown to be invalid on the basis of this contradiction.

One should be careful not to confuse a “contradiction” with a “paradox.” The latter is a *true* statement that requires explanation because of its possibly counterintuitive character. A contradiction, on the other hand, is a proof that the premises of a logical argument lead to a demonstrably false conclusion. In the present example, one set of premises leads to an equality ( $\Delta y = \Delta y'$ ), while another finds that the

same two quantities are not equal. The discovery of a contradiction that results from a physical theory has irreversible consequences: *the theory must be amended in such a way as to remove the contradiction, while still remaining consistent with all other observations for which it has hitherto provided a satisfactory explanation.*

Fortunately, there is a clear path to achieving the desired amended theory because of the fact that the contradiction within the LT has no effect on the corresponding relativistic velocity transformation (VT). Einstein made an assumption in his derivation of the LT [1] which forced the offending  $y = y'$  and  $z = z'$  equations of the LT. Replacing this with the assumption of strictly proportional rates of clocks in motion in eq. (9) while retaining the VT in its original form leads to an alternative Lorentz transformation (ALT). The latter removes the aforementioned contradiction in the LT by allowing observers to agree on the value of the speed of light in free space while disagreeing on the values of distances that are aligned perpendicularly to their relative velocity. In this way, it is ultimately the effects of time dilation on their respective clocks that determine the ratio of their measured distances.

There are also other theoretical advantages of eliminating the LT in relativity. For example, there is no longer any need to deny the possibility that neutrinos and photons [13] can move faster than  $u = c$ , despite the recent observations that clearly indicate that such speeds are attainable in the laboratory. Space and time are not mixed in the ALT because of eq. (9), so there is no connection in the amended theory between Einstein causality and superluminal motion. It also eliminates the conclusion that measurement is *subjective*, so that two clocks can be running slower than each other and two rods can each be shorter than one another at the same time. The latter characteristic of SR caused Einstein to predict [1] that only red shifts

should be observed in the transverse Doppler effect. Subsequent experiments with rotors have demonstrated instead that blue shifts are found [3, 14] whenever the Mössbauer detector moves faster than the x-ray source. Experiments with atomic clocks on circumnavigating airplanes [4] also disprove the theory that an observer on the ground must always find the airplane clocks to be running slower. Other experimental results such as the aberration of starlight at the zenith [9] do not require the LT and can be successfully explained on the basis of the VT alone. Still other successful predictions of SR are not affected by any change in the space-time transformation because they involve other variables such as momentum and energy.

In summary, despite the long-held belief in the LT, critical analysis of its internal consistency shows that it does not qualify as a scientifically viable theory. Once one submits to the latter conclusion, it is possible to explain the results of relativistic experiments in a completely straightforward manner by putting emphasis on the VT and eliminating the claim of the inextricable mixing of space and time in the description of the relative motion of objects.

## Acknowledgement

The author wishes to thank Dr. Thomas E. Phipps, Jr. for his incisive comments and suggestions during the course of the present work.

## References

- [1] A. Einstein, “Zur Elektrodynamik bewegter Körper”, *Annalen der Physik* **322(10)** (1905) 891-921.
- [2] H. E. Ives and G. R. Stilwell, “An Experimental Study of the Rate of a Moving Atomic Clock”, *Journal of the Optical Society of America* **28(7)** (1938) 215-219 (contd. in **31(5)** (1941) 369-374).
- [3] H. J. Hay, J. P. Schiffer, T. E. Cranshaw, and P. A. Egelstaff, “Measurement of the Red Shift in an Accelerated System Using the Mössbauer Effect in  $\text{Fe}^{57}$ ” *Physical Review Letters* **4(4)**, (1960) 165-166.

- [4] J. C. Hafele and R. E. Keating, "Around-the-World Atomic Clocks: Predicted Relativistic Time Gains", *Science* **177** (1972) 166-168.
- [5] B. Rossi, N. H. Hilberry and J. B. Hoag, "The Disintegration of Mesotrons", *Physical Review* **56(8)** (1939) 837-838, "The Variation of the Hard Component of Cosmic Rays with Height and the Disintegration of Mesotrons", *Physical Review* **57(6)** (1940) 461-469; B. Rossi and D. B. Hall, "Variation of the Rate of Decay of Mesotrons with Momentum", *Physical Review* **59(3)** (1941) 223-228; B. Rossi, K. Greisen, J. C. Stearns, D. K. Froman and P. G. Koontz, "Further Measurements of the Mesotron Lifetime", *Physical Review* **61(11-12)** (1942) 675-679.
- [6] T. Van Flandern, in: *Open Questions in Relativistic Physics*, ed. F. Selleri, Apeiron (Montreal, 1998), p. 81.
- [7] R. J. Buenker, "The Global Positioning System and the Lorentz Transformation", *Apeiron* **15(3)** (2008) 254-269.
- [8] T. Wilkie, "Time to remeasure the metre", *New Scientist* **27** (27 October 1983), 258-263.
- [9] A. Pais, 'Subtle is the Lord...' *The Science and Life of Albert Einstein*, Oxford University Press (1982), p. 144.
- [10] M. von Laue, "Die Mitführung des Lichtes durch bewegte Körper nach dem Relativitätsprinzip", *Annalen der Physik* **328(10)** (1907) 989-990.
- [11] H. A. Lorentz, "Electromagnetic phenomena in a system moving with any velocity smaller than that of light", *Proceedings of the Royal Netherlands Academy of Arts and Sciences* **6** (1904) 809-831; P. Zeeman and A. D. Fokker, eds., *Collected Papers*, Vol. 5, Nijhoff (The Hague, 1935-1939) p. 172.
- [12] R. J. Buenker, "Simultaneity and the Constancy of the Speed of Light: Normalization of Space-time Vectors in the Lorentz Transformation", *Apeiron* **16(1)** (2009) 96-146.
- [13] A. M. Steinberg, P. G. Kwiat and R. Y. Chiao, "Measurement of the single-photon tunneling time", *Physical Review Letters* **71(5)** (1993) 708-711.
- [14] C. W. Sherwin, "Some Recent Experimental Tests of the 'Clock Paradox'", *Physical Review* **120(1)** (1960) 17-21.