

Origins of Rest Mass Energy in Einstein's derivations

Ajay Sharma

Fundamental Physics Society

His Mercy Enclave

Post Box 107 GPO Shimla 171001 HP India

email: ajay.pqr@gmail.com

Einstein derived five equations relating to mass (rest mass, mass exchanged) and energy. From the relativistic form of kinetic energy $KE = (M_{\text{motion}} - M_{\text{rest}})c^2$, Einstein derived the classical form of kinetic energy (under the conditions when $v \ll c$). Also writing the relativistic kinetic energy in a typical way Einstein derived Rest Mass Energy, $E_{\text{rest}} = M_{\text{rest}} c^2$, under the condition when a body is at rest $v = 0$, $dx = 0$. The determination of Rest Mass Energy is inconsistent in the sense that it is obtained under this condition, when very first equation ($dK = dW = Fdx$) is zero. Thus the first equation which leads to relativistic form of KE vanishes, and hence other equations are non-existent. So E_{rest} is derived from a NON-EXISTENT equation which is not justified. There is no such example in science. So how can there be a non-zero output without any input? An equation which is mathematically inconsistent, hence cannot be used to explain experimental results. Furthermore, the equation of rest mass energy can be obtained or understood from Einstein's mass energy inter-conversion equation.

Einstein obtained $KE_{rel} = (M_{motion} - M_{rest}) c^2$, when Force Acts in Direction of Displacement

Einstein has derived five types of energies e.g . mass energy inter-conversion equation $E = \Delta mc^2$, light energy mass inter-conversion equation $L = \Delta mc^2$, relativistic energy ($E = M_{motion} c^2$), kinetic energy ($M_{rest} v^2/2$) and Rest mass energy $E_{rest} = M_{rest} c^2$. The units and dimensions of all energies are the same. Each energy is derived under different conditions, and hence have different meanings. For complete understanding the author [1-7] has critically studied Einstein's mass energy inter-conversion equation and the study has led to new findings. Einstein [8] initially derived his relativistic form of kinetic energy and later interpreted the rest mass energy [9, 10-12, 6].

Consider a body moving in the direction of a force; then, according to the Work–Kinetic Energy equation

$$dK = dW = Fdx \quad (1)$$

$$dK = dW = \frac{d}{dt} [M_{motion} v] dx \quad dK = dW = Fdx \quad (2)$$

$$= \left[M_{motion} \frac{dv}{dt} + v \frac{dM_{motion}}{dt} \right] dx \quad (2)$$

$$\begin{aligned} dK &= \{ M_{motion} dv + v d(M_{motion}) \} v [v = dx/dt] \\ &= [M_{motion} v dv + v^2 d(M_{motion})] \end{aligned} \quad (3)$$

The relativistic mass is

$$M_{motion} = \frac{M_{rest}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (4)$$

Eq.(4) is applicable when v is comparable with c , or $M_{\text{motion}}^2 c^2 - M_{\text{rest}}^2 v^2 = c^2 M_{\text{rest}}^2$.

Differentiating w.r.t. time

$$\frac{c^2 d(M_{\text{motion}}^2)}{dt} - \frac{d(M_{\text{motion}}^2 v^2)}{dt} = \frac{d(c^2 M_{\text{rest}}^2)}{dt} \quad (5)$$

Now eq. (5) can be further written as

$$\begin{aligned} c^2 \frac{dM_{\text{motion}}^2}{dt} - \frac{d(M_{\text{motion}}^2 v^2)}{dt} &= \frac{dc^2 M_{\text{rest}}^2}{dt} \\ c^2 \frac{dM_{\text{motion}}^2}{dt} &= \frac{d(M_{\text{motion}}^2 v^2)}{dt} \\ c^2 2M_{\text{motion}} \frac{dM_{\text{motion}}}{dt} &= 2M_{\text{motion}} \frac{dM_{\text{motion}}}{dt} v^2 + M_{\text{motion}}^2 2v \frac{dv}{dt} \end{aligned} \quad (6)$$

Dividing both sides of eq. (6) by $2M_{\text{motion}}$,

$$c^2 dM_{\text{motion}} = v^2 dM_{\text{motion}} + M_{\text{motion}} v dv \quad (7)$$

Now eq.(3) with the help of eq.(7) can be written as

$$dK = dW = c^2 d(M_{\text{motion}}) \quad (8)$$

$$\int dK = \int dW = c^2 \int d(M_{\text{motion}}) \quad (9)$$

$$K = W = c^2 (M_{\text{motion}} - M_{\text{rest}})$$

Or

$$K = W = c^2 (M_{\text{motion}} - M_{\text{rest}}) \quad (10)$$

$$K = W = M_{\text{motion}} c^2 - M_{\text{rest}} c^2$$

Applying the binomial theorem, the classical form of kinetic energy and work done is obtained i.e.

$$K = W = \frac{M_{rest} v^2}{2}$$

2.0 Conditions of derivation of $K = W = M_{motion} c^2 - M_{rest} c^2$

- (i) Eq.(10) is obtained if the force actually displaces the body in its own direction through a displacement dx . If $dx=0$ then $dW = Fdx = 0$
- (ii) Eq.(10) is obtained when the velocity of a body is comparable to speed of light i.e. $v \sim c$; only then eq.(4) give noticeable results.
- (iii) In the case of a considerable amount of force acting on a body and the body does not move ($dx=0$), then eq.(10) is not derivable, as eq. (1) is zero. Now it is obvious that an equation can only be interpreted under those conditions in which it originated. Eq. (10) has been interpreted to obtain $E_{rest} = M_{rest}c^2$, under the condition ($v = 0, dx=0$). If $v=0, dx=0$, then eq.(1) vanishes; hence from a NON-EXISTENT equation non-zero results: $E_{rest} = M_{rest}c^2$ cannot be drawn. In fact no result is feasible from a non-existent equation. This aspect is discussed with its implications. Eq. (10) is interpreted in view of this deduction and interesting results are obtained.

3.0 Interpretation of Eq. (10) in Terms of Kinetic Energy and Work

Now eq. (10) can be physically interpreted as given below. The kinetic energy attained by a body due to the influence of an

external force in accelerated motion when a body moves in the direction of force with velocity v , which is comparable to c , is

$$= c^2 (M_{\text{motion}} - M_{\text{rest}}) = c^2 \left[\frac{M_{\text{rest}}}{\sqrt{1 - \frac{v^2}{c^2}}} - M_{\text{rest}} \right]$$

= [Increase in mass of body due to application of external force in accelerated motion, when the force displaces the body in its own direction with velocity v which is comparable to c] c^2 (11)

Or

The kinetic energy attained by a body due to the influence of an external force in accelerated motion when the force displaces the body in its own direction with velocity v which is comparable to c ,

$$+M_{\text{rest}} c^2 = M_{\text{motion}} c^2 \quad (11)$$

Furthermore, Einstein called $M_{\text{motion}} c^2$ the total energy or relativistic energy [9, 10-12,6]. Then eq. (11) is

$$E_{\text{motion}} = E_T = \text{KE} + M_{\text{rest}} c^2 = M_{\text{motion}} c^2 \quad (12)$$

In terms of work

The work done by a body due to the influence of an external force in accelerated motion when the body moves in the direction of the force with velocity v which is comparable to c , the speed of light

$$= c^2 (M_{\text{motion}} - M_{\text{rest}}) = c^2 \left[\frac{M_{\text{rest}}}{\sqrt{1 - \frac{v^2}{c^2}}} - M_{\text{rest}} \right]$$

Similarly

$$\text{KE} + M_{\text{rest}} c^2 = M_{\text{motion}} c^2 = \text{Relativistic work done} \quad (13)$$

These are relativistic equations i.e. exist when $v \sim c$, as only under this condition relativistic increase in mass is observable.

4.0 The Rest Mass Energy is not derivable when $v = 0, dx = 0$

In 1907, Einstein [9, 10-12, 6] interpreted eq.(12) i.e. $K = c^2 (M_{\text{motion}} - M_{\text{rest}})$ As rest mass energy i.e. $E_{\text{rest}} = M_{\text{rest}}c^2$ or $W_{\text{rest}} = M_{\text{rest}}c^2$. But conceptually and mathematically it is not justified, as when a body is at rest i.e. ($v = 0, dx = 0$) then the very first equation i.e. $dK = dW = Fdx$ vanishes as there is a situation of a mathematical void in the beginning and the rest of the equations cannot be perceived. Thus other equations are NON-EXISTENT. So the origin of rest mass energy lies in the equation which itself has no origin, if this condition is applied ($v = 0, dx = 0$). Therefore the equation for rest mass energy ($E_{\text{rest}} = M_{\text{rest}}c^2$) is obtained from a NON-EXISTENT equation. This conclusion is illogical and arbitrary. An equation can only be interpreted under the conditions it is originated or derived. Hence the above interpretation is unjustified. It can be understood in the following way.

(i) Rest mass energy is not obtained from eq. (10).

Also we have the equation for the relativistic form of kinetic energy (when $v \sim c$) or work done

$$K = W = c^2 (M_{\text{motion}} - M_{\text{rest}}) \quad (10)$$

Applying the condition that the body is at rest, i.e.

$$v = 0, dx = 0, dW = dK = 0,$$

$$0 = c^2 (M_{\text{rest}} - M_{\text{rest}})$$

Or
$$M_{\text{rest}} c^2 = M_{\text{rest}} c^2$$

$$\text{Or} \quad 1=1 \quad (14)$$

which is true. Nevertheless, the result is not the rest mass energy ($E_{\text{rest}} = M_{\text{rest}}c^2$), as obtained by Einstein.

(ii) Rest mass energy not obtained if eq. (10) is written in a slightly different way. Eq. (10) can be written as

$$\text{KE} + M_{\text{rest}} c^2 = M_{\text{motion}} c^2 \quad (15)$$

When a body is at rest, i.e. $v = 0$, $dx = 0$, $dW = dK = 0$, then under this condition eq. (15) becomes,

$$0 + M_{\text{rest}} c^2 = \frac{M_{\text{rest}}}{\sqrt{1 - \frac{0}{c^2}}} c^2 = M_{\text{rest}} c^2$$

$$M_{\text{rest}} c^2 = M_{\text{rest}} c^2$$

$$\text{Or} \quad 1=1 \quad (14)$$

which is true. Hence in no way is the rest mass energy equation ($E_{\text{rest}} = M_{\text{rest}}c^2$) obtained. Similar is the case if this condition is applied to eq. (10). These cases are not discussed by Einstein.

(iii) Also in terms of work. The kinetic energy is obtained from the equation for work; it is evident from eq. (1), if the work done is zero then the energy is also zero. The conclusion can be transparently understood if the results are explained on the basis of work. Or eq. (13) can be written as

$$\text{KE} + M_{\text{rest}}c^2 = M_{\text{motion}}c^2 = \text{Relativistic work done} \quad (13)$$

When a body is at rest, i.e. $v = 0$, $dx = 0$, $dW = dK = 0$, then under this condition eq. (13) becomes

$$0 + M_{\text{rest}} c^2 = M_{\text{rest}} c^2 = \text{Relativistic Work done (when } v = 0) = 0 \quad (16)$$

So $M_{\text{rest}} c^2 = 0$. But neither M_{rest} nor c is zero, thus eq. (13) cannot be interpreted under this condition. This interpretation does not lead to

$E_{\text{rest}} = M_{\text{rest}} c^2$ as in the case of eq. (14). An equation can be only interpreted under the condition it is originated. So it is concluded that an equation cannot be interpreted under the conditions it does not exist. Hence Einstein's deduction of $E_{\text{rest}} = M_{\text{rest}} c^2$ is not justified. Furthermore, an equation which is mathematically inconsistent cannot be used to explain experimental results.

5.0 Einstein's arbitrary way to get Rest Mass Energy by re-writing and interpreting eq.(10).

Einstein wrote eq.(10) in an arbitrary way applying opposite conditions i.e. a body is moving simultaneously with speed comparable to that of light ($v \sim c$) and is at rest ($v=0$). A body cannot be at rest while moving with $v \sim c$ and when a body is at rest it cannot move with $v \sim c$. According to Einstein,

$$M_{\text{motion}} c^2 = \text{Total Energy due to motion when } v \sim c \text{ or } E_T \quad (17)$$

So $M_{\text{motion}} c^2 = \text{Work done due to motion when } v \sim c \text{ or } E_T$.

Thus eq.(15) becomes

$$KE + M_{\text{rest}} c^2 = M_{\text{motion}} c^2 = \text{Total Energy due to motion when } v \sim c \quad (18)$$

or $KE + M_{\text{rest}} c^2 = M_{\text{motion}} c^2 = \text{Work done due to motion when } v \sim c$.

In eq.(15) there is only one sign of equality.

When a body is at rest, i.e. $v = 0$, $dx = 0$, $dW = dK = 0$, then under this condition eq.(18) becomes

$$0 + M_{\text{rest}} c^2 = M_{\text{rest}} c^2 = \text{Total energy due to motion when } v \sim c \text{ (} v = 0 \text{)} \quad (19)$$

$$0 + M_{\text{rest}} c^2 = M_{\text{rest}} c^2 = \text{Work done due to motion when } v \sim c \text{ (} v = 0 \text{)}$$

Einstein wrote,

$$\text{Total energy due to motion defined when } v \sim c \text{ (} v = 0 \text{)} = E_{\text{rest}} \quad (20)$$

$$\text{Total work done due to motion defined when } v \sim c \text{ (} v = 0 \text{)} = W_{\text{rest}} = E_{\text{rest}}$$

But it is not logical to assume velocity at rest ($v=0$) when a body is moving with a speed comparable to that of light. When a body is at rest ($v=0$) then it cannot move with a speed comparable to that of light. So it is an illogical and self contradictory assumption by Einstein; hence bound to give incorrect results. It can be further understood as explained below. When $v = 0$, eq. (1) is zero; the momentum of the body is zero, classical kinetic energy is zero, work done is also zero, but mathematically the total energy or relativistic energy of a body is non-zero.

It is quite arbitrary, as in this case the total energy or relativistic energy ($v \sim c$) is not defined. All the energies follow from eq. (1) which is zero under this condition. Mathematically, Einstein wrote

$$M_{\text{rest}} c^2 = M_{\text{rest}} c^2 = E_{\text{rest}} \quad (21)$$

Furthermore, Einstein interpreted eq. (21) as

$$E_{\text{rest}} = M_{\text{rest}} c^2 \quad (22)$$

If a body is at rest $v = 0$, then eq. (1) i.e. $dK = dW = Fdx=0$, and rest of equations including eq.(10) are NON-EXISTENT; then no conclusions can be derived from a non-existent equation. It is like getting NON-ZERO output from no input; thus it is not justified.

Furthermore, eq. (18) is nothing but another form of eq. (10) and eq. (15) which under similar conditions ($v=0$) don't lead to $E_{\text{rest}} = M_{\text{rest}} c^2$. Under these conditions eq. (10) or eq. (15) gives $1 = 1$, which is in no way rest mass energy.

In a physical example it can be understood in the following way. Consider a 7 story building is being constructed. Now if the first story is demolished then the remaining six floors collapse. Analogously for Einstein's interpretation of mathematical equations (21-22), if the first floor is demolished then the remaining six floors must float in air. The reason is that Einstein has put forth that if the first equation is zero ($W=K=Fdx=0$), even then non-zero results ($E_{\text{rest}} = M_{\text{rest}} c^2$) can

be derived from this. Einstein derived from a non-existent equation, a non-zero result, $E_{\text{rest}} = M_{\text{rest}}c^2$.

Ives [13] has commented in critically analyzing Einstein's work on inter-conversion of the mass and energy equation that Einstein had assumed what he wanted to prove. A similar approach is apparently here also.

6.0 Alternate way to obtain Rest Mass Energy

The equation for rest mass energy can also be obtained [11] with the help of eq.(12) and the equation for relativistic momentum

$$E_T = M_{\text{motion}}c^2 = c^2 \frac{M_{\text{rest}}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (12)$$

$$P = \frac{M_{\text{rest}}}{\sqrt{1 - \frac{v^2}{c^2}}} v \quad (23)$$

Eq. (12) is also derived from eq. (1). Now it can be easily shown that

$$E^2 - P^2c^2 = \frac{c^4 M_{\text{rest}}^2}{\left[1 - \frac{v^2}{c^2}\right]} - \frac{v^2 c^2 M_{\text{rest}}^2}{\left[1 - \frac{v^2}{c^2}\right]} = M_{\text{rest}}^2 c^4$$

or $E^2 = M_{\text{rest}}^2 c^4 + P^2 c^2$

$$E = [M_{\text{rest}}^2 c^4 + P^2 c^2]^{1/2} \quad (24)$$

The rest mass energy can be obtained from eq. (24) by applying the condition when a body is at rest i.e. $v = 0$,

E (Total energy when $v = 0$) = $M_{\text{rest}} c^2$

or $E_{\text{rest}} = M_{\text{rest}} c^2$.

But again the situation is similar when (body is at rest, $v = 0$) then eq. (23) is zero, i.e. it does not exist.

$$P \text{ (when } v = 0) = v \frac{M_{\text{rest}}}{\sqrt{1 - \frac{v^2}{c^2}}} = 0 \quad (25)$$

When a body is at rest $v = 0$ or $dx = 0$, eq. (23) is zero, and the equation of relativistic energy, i.e. eq. (12) is defined when $v \sim c$. The origin of eq. (12) is based upon eq. (10), which is zero under this condition. Further eq. (10) is based upon eq. (1) which is also zero under this condition. Thus this derivation of $E_{\text{rest}} = M_{\text{rest}} c^2$ is also inconsistent. An equation which is mathematically inconsistent cannot be used to explain experimental results.

7.0 Logical and alternate way to obtain rest mass energy

The origin of rest mass energy can also be understood from Einstein's derivation of the mass energy inter-conversion equation. Einstein has derived the mass energy inter-conversion equation [14] as

$$E = (M_b - M_a) c^2 = \Delta mc^2 \quad (26)$$

or

$$\text{Mass of body after emission } (M_a) = \text{Mass of body before emission } (M_b) - \frac{E}{c^2} \quad (26)$$

If the whole mass is annihilated ($M_a = 0$) i.e. no mass is left after emission, then

$$M_b c^2 = E \quad (27)$$

In Einstein's derivation M_b is the mass before emission and the body is regarded at rest [14]; thus $E = M_b c^2$ may be regarded as rest mass energy i.e. $E_{\text{rest}} = M_{\text{rest}} c^2$. It is energy equivalent to rest mass energy. It is a logical deduction from Einstein's derivation. In Einstein's this derivation $E_{\text{rest}} = M_{\text{rest}} c^2$ is not obtained from a non-existent equation as in the previous derivation, hence this derivation is correct.

Acknowledgements

The author is highly indebted to Stephen J. Crothers and Anjana Sharma for critical comments. Also the assistance of Secretary, DST, Dr. T. Ramasami in various stages of the work is gratefully acknowledged.

References

- [1] Sharma, A *Progress in Physics*, Vol. 3 pp. 76-83 (2008)
- [2] Sharma, A. *Apeiron* Vol. 18, No. 3 , (2011) 270-296.
- [3] Sharma, A *International Journal of Nuclear Science and Technology* Vol. 3 No 4. pp. 370-377 (2007)
- [4] Sharma, A. *Galilean Electrodynamics*, Vol. 18, No. 5, pp 99-100 (2007).
- [5] Sharma, A. Kurt Godel Society Collegium Logicum Volume IX pp. 67-71 (2007)
- [6] Sharma, A. *Concepts of Physics*, Vol. III, No. 4, pp.345–373 (2006), http://merlin.fic.uni.lodz.pl/concepts/2006_4/2006_4_351.pdf
- [7] Sharma, A. *Physics Essays*, Vol. 17, pp.195–222 (2004)
- [8] Sharma, A. *Proc. Int. Conf. on Number, Time, Relativity*, United Physical Society of Russian Federation, Moscow, p.81-82 (2004)
- [9] Einstein, A *Ann. der Phys.* **17**, 891-921 (1905).
- [10] Einstein, A *Jahr. Rad. Elektr.* **4**, 411 (1907).
- [11] Robert Resnick, *Introduction to Special Theory of Relativity* (John Wiley and Sons, New York, 1968), p. 120. (1968)
- [12] Arthur Beiser, *Concepts of Modern Physics*, 4th edition (McGraw-Hill International Edition, New York, 1987), pp. 25, 27, 31 (1987),

- [13] A. Pais, *Subtle Is the Lord* (Oxford University Press, Oxford, 1983), p. 149 (1983)
- [14] Ives, H. *J. Opt. Soc. Am.* 42, 540 (1952)
- [15] Einstein, A. *Annalen der Physik* **18** (1905) 639-641.