Nonuniqueness of the Bose-Einstein statistical processes for the black body radiation

A.N. Pechenkov
Institute of Metal Physics
Ural Division of the Rus.Acad.Sci.
18,S. Kovalevskaya str., GSP – 170
620041, Ekaterinburg, Russia
pechenkov@imp.uran.ru

An infinite number of stationary random processes with continuous energy distribution and upper energy limit of photons are possible as an explanation for black body radiation.

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As it is well known, M. Planck's initial aim was to find a continuous distribution of radiating oscillator energy at a given frequency ν, to explain his famous formula for black body radiation. But he (and many other great physicists, like: A.Einstein, P.Debye, H.Lorentz, etc.) used the Boltzmann energy equilibrium distribution for particles with infinite possible energies, which requires a discrete equidistant series of possible oscillator energies (0, hν, 2hν, 3hν, …), or a discrete number of photons (0, 1, 2, 3, …) with the energy hν, to explain the Planck formula. It was shown by S.Bose, that there is a maximum probable distribution (Bose – Einstein distribution) for the number of such discrete photons, which give us the Planck formula. This picture of discrete energies is the basis of the contemporary quantum theory of light.

But it is interesting that there are early uninvestigated possibilities of continuous distributions of photon energies which can explain some experiments as well as the discrete energy picture. In this paper I will begin to consider such continuous distributions with the natural assumption that oscillator or photon energies are restricted by some upper limit on the given frequency ν.

To better understand the investigation, let us first to do a few numerical estimations.

Let us consider the Planck formula for the spectral energy density in the volume V [1]:

$$dE = \frac{8\pi\nu^2 V}{c^3}\frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}d\nu \qquad (1)$$

Let ν ~ 5*1014 c-1 (λ ~ 5000 Ao , visible diapason), T ~ 3*103 Ko . Quantum of energy is hν ~ 6.6 *10-27 * 5*1014 erg = 3.3*10-12 erg. Thermal energy is kT ~ 1.4*10-16 *3*103 erg = 4.2*10-13 erg. Then exp(hν/ kT)~ exp(8.3) ~ 103.

In the classical wave theory there are the following number of possible standing waves in the volume V in the frequency diapason dν at given frequency ν:

$$dn = \frac{8\pi\nu^2 V}{c^3}d\nu \sim \frac{8\pi 25*10^{28}}{27*10^{30}}Vd\nu \sim 0.23Vd\nu \qquad (2)$$

Every standing wave can have amplitude and energy as stationary random functions. The average energy of the wave in (1) is:

$$\bar{\varepsilon} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \sim 10^{-3} h\nu \sim 3.3 * 10^{-15} erg \tag{3}$$

As we can see, the average wave energy on the given frequency is much less than quantum of energy in the example.

How can there be such small values of average energy? It is clear, that it can be done by statistical process with very small probability of huge energy levels. If we could to do very fast measurements, then in Bose-Einstein processes we would not be able to see any wave in most measurements, and only very rare waves with energy Nhν (N is the random number). But there are an infinite number of other stationary processes with continuous energy distribution and finite upper energy limit of photons, which can yield the same small mean energy. In such processes we will measure some wave in every measurement.

In [2] the process with follow continuous energy distribution was considered:

$$w(x) = \frac{u}{kT} e^{\frac{h\nu}{kT} x} \quad , \quad x = \frac{\varepsilon}{\varepsilon_m} \in [0,1] \tag{4}$$

u – mean wave energy on the frequency ν;

k – Boltzmann constant;

T – absolute temperature;

ε - wave energy;

ε_m – upper limit of wave energy on given frequency ν.

The distribution (4) derives Planck formula simply by normalization:

$$\int_0^1 w(x)dx = 1 \tag{5}$$

or:

$$\int_0^1 w(x)dx = \frac{u}{kT}\frac{kT}{h\nu}(e^{\frac{h\nu}{kT}} - 1) = 1$$

and:

$$u = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \tag{6}$$

Using the condition $u = \overline{\varepsilon}$, we find upper limit εm of wave energy on given frequency ν:

$$\overline{x} = \frac{\overline{\varepsilon}}{\varepsilon_m} = \frac{u}{\varepsilon_m} = \int_0^1 xw(x)dx = \frac{u}{h\nu}\left\{e^{\frac{h\nu}{kT}}\left[1 - \frac{kT}{h\nu}\right] + \frac{kT}{h\nu}\right\}$$

$$\varepsilon_m = \frac{h\nu}{\left\{e^{\frac{h\nu}{kT}}\left[1 - \frac{kT}{h\nu}\right] + \frac{kT}{h\nu}\right\}} \tag{7}$$

The dispersion of the wave energy in (4) is:

$$\overline{(x-\bar{x})^2} = \overline{x^2} - \bar{x}^2 = \left(\frac{u}{h\nu}\right)\left[\begin{matrix} e^{\frac{h\nu}{kT}}\left[1-2\frac{kT}{h\nu}+2\left(\frac{kT}{h\nu}\right)^2\right]- \\ -2\left(\frac{kT}{h\nu}\right)^2 \end{matrix}\right]- \tag{8}$$

$$-\left(\frac{u}{h\nu}\right)^2\left[e^{\frac{h\nu}{kT}}\left[1-\frac{kT}{h\nu}\right]+\frac{kT}{h\nu}\right]^2$$

Of course, it is not the same dispersion as in Bose – Einstein statistics.

As we can see from (7), the upper limit εm of wave energy in (4) is also much less than the quantum of energy.

The question arises: can we coordinate the small possible wave energies in (4) with the empirical photoeffect equation:

$$E_m = h\nu - A \tag{9}$$

Em – maximum kinetic energy of photoelectron;

A – work of exit of photoelectron.

I think there are two possible explanations of (9). The first explanation is as follows. In spite of the small wave energy, we have significant energy flow from 1cm^2 of black body surface, at a given frequency:

$$r = \frac{1}{4}c\frac{dE}{d\nu} \sim 1.7*10^6 \text{ h}\nu \;\; erg/cm^2 \tag{10}$$

Let all this flow fall on the 1cm^2 of metal surface. The usual point of view is that the photoelectron needs a very ;pmg time (many years) to collect sufficient energy from the flow, because it is much smaller than 1cm^2. But this point of view is doubtful if the electrons in the metal can exchange energies. Then some electrons will quickly receive the energy hν [3] or less, sufficient to exit from the metal.

The second interesting explanation of (9) consists in rejecting the distribution (4). We can build an infinite number of distributions w(x) with mean energy (6). Also we can make the additional requirement εm = hν, as follows from (9). If the distribution w(x,a,b) has two undetermined parameters (a,b), then we have two equations for determination of the parameters:

$$\int_0^1 w(x,a,b)dx = 1$$

$$\bar{x} = \frac{\overline{\varepsilon}}{\varepsilon_m} = \frac{u}{\varepsilon_m} = \int_0^1 xw(x,a,b)dx \qquad (11)$$

For example, let us take the distribution (4) in the common view:

$$w(x,a,b) = ae^{bx}$$

$$\int_0^1 w(x,a,b)dx = \frac{a}{b}(e^b - 1) = 1 \quad \Rightarrow \frac{a}{b} = \frac{1}{(e^b - 1)} \qquad (12)$$

Then we have a nonlinear equation for the determination of parameter b, under given (ν,T):

$$\bar{x} = \frac{\overline{\varepsilon}}{\varepsilon_m} = \frac{u}{h\nu} = \frac{1}{e^{\frac{h\nu}{kT}} - 1} = \int_0^1 xw(x,a,b)dx = \frac{a}{b}[e^b - \frac{1}{b}(e^b - 1)] =$$

$$= [\frac{e^b}{(e^b - 1)} - \frac{1}{b}] = 1 - \frac{1}{b} + \frac{1}{(e^b - 1)} \qquad (13)$$

Note again that in that case I explain the existence of maximum kinetic energy of photoelectron in (9) by the existence of maximum wave energy εm = hν of photons. From this point of view, from the basic formulas in quantum theory of atoms E2 – E1 = hν can be considered as energy conservation law for the photons with maximum possible energies.

Note also, that in the same way we can use any other empirical moments of energy distribution (the variance, for example), if the distribution contains additional undetermined parameters.

In conclusion I would like to note that the distributions (4) or (12) are the simple canonical Gibbs equilibrium distributions for the noninteracting particles (or waves), but with an upper limit on the energy of the particles. In fact, for such distributions we have generally[1]:

$$w(\varepsilon) = Ae^{\pm a\varepsilon} \tag{14}$$

Unfortunately, statistical physics books usually consider only distributions without an upper limit on the energy of the particles, i.e. with the "–" sign. Therefore, the "+"sign in the exponent power is usually removed from consideration (except for statistical ensembles with negative temperatures), because we cannot do the normalization of the distribution. But with upper limit on the energy of the particles the sign "+" is valid.

Conclusions:

- an infinite number of continuous energy distributions w(x) are possible to derive the Planck formula. This means that the photon can have energy less than hν;
- statistical properties of such photon ensembles must be investigated in more detail;
- the contemporary quantum theory of light can be changed to include such photons;
- there have been no direct experiments on photon energy distribution measurements in monochromatic black body

radiation. This could be done with photoeffect like experiments. Usually, the energy distribution of photoelectrons in such experiments is determined by the energy distribution of electrons in metal [4], not by the energy distribution of photons. To obtain the energy distribution of photons in such experiments, we must prepare a specific electron ensemble, with near equal energies of electrons (in the ideal case, with equal energies);

- we must find as new way to measure single photon energy;
- we must use the postulate E = hν only for photons with maximal energy.

References

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