

Detailing Minimum Parameters for the Red Shift, Frequency, Strain, and Wavelength of Gravitational Waves / Gravitons, and Impact Upon GW astronomy for Experimentally Falsifiable Measurements to Evaluate Current models of Cosmology

Andrew Walcott Beckwith
Chongqing University
Department of Physics
P.R. China, 400014
abeckwith@uh.edu

This document briefly outlines some of the issues pertinent to early inflation and how they affect strain readings for a GW detector, GW wavelengths, and the number of gravitons which may be collected per phase space, among other issues. Different inflation models will also be briefly explored to explain in part rates of wavelength alterations of GW's

from their pre-inflation genesis to inflationary generation. We also describe a standard for GW measurement and how the ‘metric’ of measurement varies between the different cosmological models, thereby allowing them to be distinguished experimentally. The best chances for relic GW measurements are within the $0.1\text{Hz} < f < 10\text{GHz}$ range according to the pre big bang models and the QIM model. Conventional red shift calculations indicate that past the big bang massive red shifting continues and would eliminate HFGW as presently measurable from the big bang. If there are still in the present day HFGW detectable from the big bang, then we have either an absence of extreme post big bang red shifting, inflation as configured is not quite what we think in the initial inflationary era, or there are even higher frequency GW generated in the onset of creation.

Keywords: strain, wavelength, gravitons, entropy

Introduction

Historical remarks: the fundamental issue of relic gravitational waves was introduced in the literature by the paper L.P. Grishchuk,[1] and as an inflationary scenario in A. A. Starobinsky,[2] The value of the gravitational l wave spectrum also first appeared in Starobinsky (1979).[2]. More in general, stimulated creation of quanta in an expanding universe first appeared in L. Parker, [3] What is attempted is to do a refinement as to how to study the initial introduction of gravitational waves while giving full credit to the initial pioneers of this subject.

The linkage to $SO(4)$ gauge theory and gravitons was brought up by [4] Kuchiev, M. Yu, which we believe leads to a kink-anti kink pair tie in for attendant gravitons. Note that Kuchiev [4] writes that “Conventional non-Abelian $SO(4)$ gauge theory is able to describe gravity provided the gauge field possesses a polarized vacuum state.

In this vacuum the instantons and anti-instantons have a preferred direction of orientation.”, and furthermore “Gravitons appear as the mode describing propagation of the gauge field which strongly interacts with the oriented instantons” Furthermore, as given by Andrić, Jonke and Jurman,[5] what is called an n-soliton solution is shown to have an equivalence “semi classical solutions” corresponding to

1. Modeling of entropy, generally, as kink-anti-kinks pairs with \tilde{N} the number of the kink-anti-kink pairs. This number, \tilde{N} is, initially in tandem with entropy production, brought up by Beckwith. [6]
2. The tie in with entropy and gravitons is that the two structures are related to each other in terms of kinks and anti-kinks. It is asserted that how they form and break up is due to the same phenomenon: a large insertion of vacuum energy leads to a breakup of both entropy levels and gravitons. When a second-order phase transition occurs, there is a burst of relic gravitons. Similarly, there is an initial breakup of net entropy levels, and after a second-order phase transition, another rapid increase in entropy.

The supposition we are making here is that the value of N is proportional to a numerical graviton density we refer to as $\langle n \rangle$ [7],[8], provided that there is a bias toward HFGW, which would mandate a very small value for $V \sim volume \sim \lambda^3$. Furthermore, structure formation arguments, given by Perkins [6] give ample evidence that if we use an energy scale, m , over a Planck mass value M_{Planck} , as well as contributions from field amplitude φ , and using

the contribution of scale factor behavior $\frac{\dot{a}}{a} \equiv H \approx -m \cdot \frac{\varphi}{3 \cdot \dot{\varphi}}$, where

we assume $\ddot{\varphi} \cong 0$ due to inflation, then

$$\frac{\Delta\rho}{\rho} \sim H\Delta t \sim \frac{H^2}{\dot{\phi}} \sim \left(\frac{m}{M_{Planck}}\right) \times \left(\frac{\phi}{M_{Planck}}\right) \sim 10^{-5} \quad (1)$$

At the very onset of inflation, $\phi \ll M_{Planck}$, and if m (assuming $\hbar = c = 1$) is due to inputs from a prior universe, we have a wide range of parameter space as to ascertain where $\Delta S \approx \Delta N_{gravitons} \neq 10^{88}$ comes from and how it plays a role in the development of entropy in cosmological evolution ‘information’. If $S_{initial} \sim 10^5$ is transferred from a prior universe to our own universe at the onset of inflation, then at times less than Planck time $t_p \sim 10^{-44}$ seconds enough information **MAY** exit for the preservation of the prior universe’s cosmological constants, i.e. \hbar, G, α (fine structure constant) and the like. We do not have a reference for this. Lee Simolin [9] suggested this supposition first, but the mechanism for this is being described here for the first time. Times after time $t \approx t_{Planck} \sim 10^{-44}$ seconds are not important because by then the ‘constant’s memory’ is already imprinted in the universe. Confirmation of this hypothesis depends upon models of how much ‘information’ \hbar, G, α actually require to be set in place, at the onset of our universe’s inflation, a topic which we currently have no experimental way of testing at this current time.

Furthermore, finding out if or not it is either a drop in viscosity [12], [13] when $\left|\frac{\eta}{s} \approx \varepsilon^+\right| \ll \frac{1}{4\pi}$, or a major increase in entropy density may tell us how much information is, indeed, transferred from a prior universe to our present. If it is $s \rightarrow \infty$, the moment after the pre big bang configuration, likely then there will be a high degree of ‘information’ from a prior universe exchanged to our present universe. If on the other hand, $\eta \rightarrow 0^+$ due to restriction of

‘information from four dimensional ‘geometry’ to a variable fifth dimension then it is likely that significant data compression has occurred. As indicated by Hawking’s theorem, infinite density is the usual *modus operandi* for a singularity, but this assumption may have to be revisited. Natário (2006) [14] has more details on the different types of singularities involved. The supposition is that the value of N is proportional to a numerical DM density referred to as $\langle n \rangle$ $\Big|_{\text{Dark-matter}}$. HFGW would play a role if $V \approx R_H^3 \approx \lambda^3$ has each λ of the order of being within an order of magnitude of the Planck length value, as implied by Beckwith (2009) [15] examined, and linked to modeling gravity as an effective theory, as well as giving credence to how to avoid $dS/dt = \infty$ at $S=0$. If so, then one can look at the research results of Mathur [16] (2007). This is part of what has been developed in the case of massless radiation, where for D space-time dimensions, and E , the general energy is

$$S \sim E^{(D-1/D)} \quad (2)$$

This suggests that entropy scaling is proportional to a power of the vacuum energy, i.e., entropy \sim vacuum energy, if $E \sim E_{total}$ is interpreted as a total net energy proportional to vacuum energy, as given below. Conventional brane theory actually enables this instanton structure analysis, as can be seen in the following. This is adapted from a lecture given at the ICGC-07 conference by Beckwith [17]

$$\frac{\Lambda_{Max} V_4}{8 \cdot \pi \cdot G} \sim T^{00} V_4 \equiv \rho \cdot V_4 = E_{total} \quad (3)$$

The approximation in this treatment initially is that $E_{total} \propto V(\varphi)$ equating to the potential energy term.[18] For an exponential potential (effective potential energy)

$$V(\varphi) = g \cdot \varphi^\alpha \quad (4)$$

In the limits of pre and post Planckian space time regimes, when looking at consistency of the emergent structure gives the following [19]

$$V(\phi) \propto \varphi^{|\alpha|} \text{ for } t < t_{PLanck} \quad (5)$$

And

$$V(\phi) \propto 1/\varphi^{|\alpha|} \text{ for } t \gg t_{PLanck} \quad (6)$$

The switch between Eq. (5) and Eq. (6) is not justified analytically. I.e. it breaks down. Beckwith et al (2011) designated this as the boundary of a causal discontinuity. Now according to Weinberg [18],

if $\epsilon = \frac{\lambda^2}{16\pi G}$, $H = 1/\epsilon t$ so that one has a scale factor behaving as

$$a(t) \propto t^{1/\epsilon} \quad (7)$$

Then, if

$$|V(\varphi)| \ll (4\pi G)^{-2} \quad (8)$$

There are no quantum gravity effects worth speaking of. I.e., if one uses an exponential potential a scalar field, when there is a drop in a field from φ_1 to φ_2 for flat space geometry and times t_1 to t_2 [18,19]

$$\varphi(t) = \frac{1}{\lambda} \ln \left[\frac{8\pi G g \epsilon^2 t^2}{3} \right] \quad (9)$$

Then the scale factors, from Planckian time scale as [18,19]

$$\frac{a(t_2)}{a(t_1)} = \left(\frac{t_2}{t_1} \right)^{1/\epsilon} = \exp \left[\frac{(\varphi_2 - \varphi_1) \lambda}{2 \epsilon} \right] \quad (10)$$

The more $\frac{a(t_2)}{a(t_1)} \gg 1$, then the less likely there is a tie in with quantum gravity. Note those that the way this potential is defined is for a flat, Robertson-Walker geometry, and that if and when $t_1 < t_{Planck}$ then what is done in Eq. (10) no longer applies, and that one is no longer having any connection with even an octonionic Gravity regime. Note to tame the incommensurate metrics, use for all models, the approximation given below is used as a start

$$h_0^2 \Omega_{GW} \sim 10^{-6} \quad (11)$$

Next, after we tabulate results with this measurement standard, we will commence to note the difference and the variances from using $h_0^2 \Omega_{GW} \sim 10^{-6}$ as a unified measurement which will be in the different models discussed right afterwards

Wavelength, sensitivity and other such constructions from Maggiore, with our adaptations and comments

We will next give several of our basic considerations as to early universe geometry which we think are appropriate as to Maggiore's [20] treatment of both wavelength, strain, and Ω_{GW} among other things. As far as early universe geometry and what we may be able to observe, such considerations are critical to the role of early universe geometry and the generation of GW at the start of the universe. To begin with, we will look at Maggiore's [20] Ω_{GW} formulation, his ideas of strain, and what we did with observations from L. Crowell [21] which may tie in with the ten to the tenth power increase for wavelengths from pre Planckian physics to 1-10 GHz early inflationary GW frequencies. The idea will be to look at how the ten to the tenth stretch out of generated wavelength may tie in with early universe models. We will from there proceed to look at, and speculate

how the presented conclusions factor in with information exchange between different universes.

We begin with the following tables, Table 1 and Table 2. The idea is if one has $h_0 = .51 \pm .14$, as a degree of measurement uncertainty yjrm we can begin to understand what may be affecting an expansion of the wavelengths of pre Planckian GW / gravitons. What we have stated below in Table 1 and Table 2 will have major consequences as far as not only information flow from a prior to present universe, but also fine tuning the degree of GW variance.

The table before uses, among other things, Maggiore's [20] $h_0^2 \Omega_{GW}$ analytical expression. The range of frequencies is consistent with ultra high frequencies to the low point given for very long GW frequencies red shifted dramatically by inflation and presumably the aftermath of inflation. The longest possible GW wave lengths could be of the order of almost a light year in length.

Table 1: Managing GW generation from Pre Planckian physics

$h_C \leq 2.82 \times 10^{-33}$	$f_{GW} \sim 10^{12} \text{ Hertz}$	$\lambda_{GW} \sim 10^{-4} \text{ meters}$
$h_C \leq 2.82 \times 10^{-31}$	$f_{GW} \sim 10^{10} \text{ Hertz}$	$\lambda_{GW} \sim 10^{-2} \text{ meters}$
$h_C \leq 2.82 \times 10^{-29}$	$f_{GW} \sim 10^8 \text{ Hertz}$	$\lambda_{GW} \sim 10^0 \text{ meters}$
$h_C \leq 2.82 \times 10^{-27}$	$f_{GW} \sim 10^6 \text{ Hertz}$	$\lambda_{GW} \sim 10^2 \text{ meters}$
$h_C \leq 2.82 \times 10^{-25}$	$f_{GW} \sim 10^4 \text{ Hertz}$	$\lambda_{GW} \sim 10^1 \text{ kilometer}$
$h_C \leq 2.82 \times 10^{-23}$	$f_{GW} \sim 10^2 \text{ Hertz}$	$\lambda_{GW} \sim 10^3 \text{ kilometer}$
$h_C \leq 2.82 \times 10^{-21}$	$f_{GW} \sim 10^0 \text{ Hertz}$	$\lambda_{GW} \sim 10^5 \text{ kilometer}$
$h_C \leq 2.82 \times 10^{-19}$	$f_{GW} \sim 10^{-2} \text{ Hertz}$	$\lambda_{GW} \sim 10^7 \text{ kilometer}$
$h_C \leq 2.82 \times 10^{-17}$	$f_{GW} \sim 10^{-4} \text{ Hertz}$	$\lambda_{GW} \sim 10^9 \text{ kilometer}$

$h_C \leq 2.82 \times 10^{-15}$	$f_{GW} \sim 10^{-6} \text{ Hertz}$	$\lambda_{GW} \sim 10^{11} \text{ kilometer}$
$h_C \leq 2.82 \times 10^{-13}$	$f_{GW} \sim 10^{-8} \text{ Hertz}$	$\lambda_{GW} \sim 10^{13} \text{ kilometer}$
$h_C \leq 2.82 \times 10^{-11}$	$f_{GW} \sim 10^{-10} \text{ Hertz}$	$\lambda_{GW} \sim 10^{15} \text{ kilometer}$

What we are expecting, as given to us by L. Crowell, is that initial waves, synthesized in the initial part of the Planckian regime would have about $\lambda_{GW} \sim 10^{-14} \text{ meters}$ for $f_{GW} \sim 10^{22} \text{ Hertz}$ which would turn into $\lambda_{GW} \sim 10^{-1} \text{ meters}$, for $f_{GW} \sim 10^9 \text{ Hertz}$, and sensitivity of $h_C \leq 2.82 \times 10^{-30}$. This is assuming that $h_0^2 \Omega_{GW} \sim 10^{-6}$, using Maggiore's [20] $h_0^2 \Omega_{GW}$ analytical expression. It is important to note in all of this, that when we discuss the different models that the $h_0^2 \Omega_{GW} \sim 10^{-6}$ is the first measurement metric which is drastically altered. h_C which is mentioned in Eqn. (13) below should be also noted to be an upper bound. In reality, only the 2nd and 3rd columns in table 1 above escape being seriously off and very different. , since the interactions of gravitational waves / gravitons with quark – gluon plasmas and even neutrinos would serve to deform by at least an order of magnitude h_C . So for table 1, the first column is meant to be an upper bound which, even if using Eqn. (13) may be off by an order of magnitude. More seriously, the number of gravitons per unit volume of phase space as estimated, and are heavily dependent upon $h_0^2 \Omega_{GW} \sim 10^{-6}$. If that is changed, which shows up in the models discussed right afterwards, the degree of fidelity with Eqn. (12) drops.

The question to now raise is as follows. If ultra high frequency GW from the big bang are detectable, then does this mean that not all GW waves are red shifted after the big bang. If not, then why not?

Table 2: Managing GW count from Planckian physics/unit-phase-space[20]

$\lambda_{GW} \sim 10^{-4} \text{ meters} \Rightarrow n_f \propto 10^{-6} \text{ graviton / unit - phase - space ;}$
$\lambda_{GW} \sim 10^{-2} \text{ meters} \Rightarrow n_f \propto 10^2 \text{ graviton / unit - phase - space}$
$\lambda_{GW} \sim 10^0 \text{ meters} \Rightarrow n_f \propto 10^{10} \text{ graviton / unit - phase - space}$
$\lambda_{GW} \sim 10^2 \text{ meters} \Rightarrow n_f \propto 10^{18} \text{ graviton / unit - phase - space}$
$\lambda_{GW} \sim 10^1 \text{ kilometer} \Rightarrow n_f \propto 10^{26} \text{ graviton / unit - phase - space}$
$\lambda_{GW} \sim 10^3 \text{ kilometer} \Rightarrow n_f \propto 10^{34} \text{ graviton / unit - phase - space}$
$\lambda_{GW} \sim 10^5 \text{ kilometer} \Rightarrow n_f \propto 10^{42} \text{ graviton / unit - phase - space}$
$\lambda_{GW} \sim 10^7 \text{ kilometer} \Rightarrow n_f \propto 10^{50} \text{ graviton / unit - phase - space}$
$\lambda_{GW} \sim 10^9 \text{ kilometer} \Rightarrow n_f \propto 10^{58} \text{ graviton / unit - phase - space}$
$\lambda_{GW} \sim 10^{11} \text{ kilometer} \Rightarrow n_f \propto 10^{66} \text{ graviton / unit - phase - space}$
$\lambda_{GW} \sim 10^{13} \text{ kilometer} \Rightarrow n_f \propto 10^{74} \text{ graviton / unit - phase - space}$
$\lambda_{GW} \sim 10^{13} \text{ kilometer} \Rightarrow n_f \propto 10^{74} \text{ graviton / unit - phase - space}$

The particle per phase state count will be given as, if $h_0^2 \Omega_{GW} \sim 10^{-6}$ [20]

$$n_f \sim h_0^2 \Omega_{GW} \cdot \frac{10^{37}}{3.6} \cdot \left[\frac{1000 \text{ Hz}}{f} \right]^4 \quad (12)$$

Secondly we have that a detector strain for device physics is given by [20]

$$h_c \leq \left(2.82 \times 10^{-21} \right) \cdot \left(\frac{1 \text{ Hz}}{f} \right) \quad (13)$$

These values of strain, the numerical count, and also of n_f give a bit count and entropy which will lead to possible limits as to how much information is transferred. Note that per unit space, if we have an entropy count of , after the start of inflation with having the following , namely at the beginning of relic inflation $\lambda_{GW} \sim 10^{-1} \text{ meters} \Rightarrow n_f \propto 10^6 \text{ graviton / unit - phase - space}$ for $f_{GW} \sim 10^9 \text{ Hertz}$. This is to have a starting point in pre inflationary physics of $f_{GW} \sim 10^{22} \text{ Hertz}$ when $\lambda_{GW} \sim 10^{-14} \text{ meters}$, i.e. a change of $\sim 10^{13}$ orders of magnitude in about 10^{-25} seconds, or less. However, these gravity waves propagate on a space which continues to expand, and has expanded enormously to the dimensions of the universe we currently observe. Those gravity waves would then have been stretched out, or red shifted, by a factor of about $10^{\{26\}}$. This means the gravity waves are stretched out to around $10^{\{25\}}$ to $10^{\{28\}}$ cm, or 10^7 to 10^{10} light years. The search for B-modes in the CMB from Planck spacecraft data is one possible test for the production of early graviton-gravity waves by the early universe. These B-modes are non-Gaussian features in the CMB which are fingerprints for these early gravity waves. Usual standard lore is given in [20].

The question to ask is, do all gravity waves get red shifted after the big bang to such an extreme degree? I submit that this is to be answered only by measurements. Then, as indicated in the beginning abstract, either

- 1) The red shifting post big bang is not nearly as great as people think. How or why this would be possible would be a serious source of model building expertise to consider
- 2) Even higher frequency initial GW would happen before the onset of the big bang

3) The big bang is then over stated as far as its initial expansion.

For those who deny that there are questions as to the degree of initial expansion, the author refers the readers as to Steinhardt's recent work critiquing the big bang [41].

GW detectors will be the only way to resolve this issue.

Establishing GW astronomy in terms of a choice between models

A change of $\sim 10^{13}$ orders of magnitude is expected in about 10^{-25} seconds, or less in terms of one of the variants of inflation. As has been stated elsewhere [22], [23] in a publication under development, there are several models which may be affecting this change of magnitude. The following is a summary of what may be involved:

A) The relic GWs in the pre-big-bang model.

Here, the relic GWs have a broad peak bandwidth from 1 Hz to 10 GHz [24] We can refer to other such publications for equivalent information [25] In this spectral region the upper limit of energy density of relic GWs is almost a constant $\Omega_{gW} \sim 6.9 \times 10^{-6}$, but it will rapidly decline in the region from 1 Hz to 10^{-3} Hz. Thus direct detection of the relic GWs should be focused in intermediate and high-frequency bands. Amplitude upper limits of relic GWs range from $h \sim 10^{-23}$ at frequencies around 100 Hz to $h \sim 10^{-30}$ at frequencies around 2.9 GHz. This means that frequencies around 100 Hz and frequencies around 2.9 GHz would be two key detection windows. If the relic GWs in the pre-big-bang model (or other similar models such as the cyclic model of the universe [26] can be

detectable, then its contribution to contemporary cosmological perspectives would be substantial

B) The relic GWs in the quintessential inflationary model (QIM).

The peak and maximal signal of relic GWs in the QIM are localized in the GHz band [26, 27], and the strength of relic GWs in both the QIM and the pre-big-bang model in the GHz band have almost the same magnitude (e.g., $h \sim 10^{-30}$ at 2.9GHz). But the peak bandwidth of the QIM (from 1GHz to 10GHz) (21) is less than that of the pre-big-bang model (from 1Hz to 10GHz) [24]

C) The relic GWs in the cosmic string model.

Unlike relic GWs in the pre-big-bang model and in the QIM, the peak energy density Ω_{gw} of relic GWs in the cosmic string model is in the low-frequency region of $\sim 10^{-7}$ Hz to 10^{-1} Hz, and the upper limit of Ω_{gw} may be $\sim 4 \times 10^{-6}$ at frequencies around 10^{-6} Hz. When $\nu < 10^{-7}$ Hz, the energy density decays quickly. Therefore, LISA and ASTROD will have sufficient sensitivity to detect low-frequency relic GWs in the region of $\sim 10^{-7}$ Hz $< \nu < 10^{-3}$ Hz predicted by the model [24], [27], [28]. Moreover, the energy density of relic GWs is an almost constant $\Omega_{gw} \sim 10^{-8}$ from 10^{-1} Hz to 10^{10} Hz, and the relic GWs at frequencies around 100 Hz should be detectable by advanced LIGO, but the amplitude upper limit of relic GWs in the GHz band may be only $h \sim 10^{-31}$ to 10^{-32} , which cannot be directly detected by current technologies.

D) The relic GWs in the ekpyrotic scenario

Relic GWs in the ekpyrotic scenario [26] and in the pre-big-bang [27], [28] model have some common and similar features. The initial

state of universe described by both is a large, cold, nearly empty universe, and there is no beginning of time in both, and they are faced with the difficult problem of making the transition between the pre- and post-big bang phase. However, the difference of physical behavior of relic GWs in both is obvious. First, the peak energy density of relic GWs in the ekpyrotic scenario is $\Omega_{gw} \sim 10^{-15}$, and it is localized in frequencies around 10^7 Hz to 10^8 Hz. Therefore the peak of Ω_{gw} in the former is less than corresponding value in the latter.

E) The relic GWs in the ordinary inflationary model

Also, for ordinary inflation [29] the energy density of relic GWs holds constant ($\Omega_{gw} \sim 10^{-14}$) in a broad bandwidth from 10^{-16} Hz to 10^{10} Hz, but the upper limit of the energy density is less than that in the pre-big-bang model from 10^{-3} Hz to 10^{10} Hz, in the cosmic string model from 10^{-7} Hz to 10^{10} Hz, and in the QIM from 10^{-1} Hz to 10^{10} Hz. For example, this model predicts $h_{max} \sim 10^{-27}$ at 100 Hz, $h_{max} \sim 10^{-33}$ at 100 MHz and $h_{max} \sim 10^{-35}$ at 2.9 GHz. To summarize, what we expect is that appropriate strain sensitivity values plus predictions as to frequencies may confirm or falsify each of these five inflationary candidates, and perhaps lead to completely new model insights. We hope that we can turn GW research into an actual experimental science.

TABLE 3: Variance of the Ω_{GW} parameters as given by the above mentioned cosmology models.

Relic pre big bang	QIM	Cosmic String model
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$\Omega_{GW} \sim 6.9 \times 10^{-6}$ <i>when</i> $f \geq 10^{-1} \text{ Hz}$ $\Omega_{GW} \ll 10^{-6}$ <i>when</i> $f < 10^{-1} \text{ Hz}$	$\Omega_{GW} \sim 4 \times 10^{-6}$ $1 \text{ GH} < f < 10 \text{ GH}$	$\Omega_{GW} \sim 4 \times 10^{-6}$ $f \propto 10^{-6} \text{ Hz}$ $\Omega_{GW} \sim 0$ <i>otherwise</i>
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In addition, the following should be compared with the relic Pre big bang, QIM and Cosmic String model. Consider this to be an addition to the Table 3.

Ekpyrotic
$\Omega_{GW} \sim 10^{-15}$ $10^7 \text{ Hz} < f < 10^8 \text{ Hz}$ $\Omega_{GW} \sim 0$ <i>otherwise</i>

The best targets of opportunity, for viewing $\Omega_{GW} \sim 10^{-6}$ are in the $.01 \text{ Hz} < f < 10 \text{ GHz}$ range, with another possible target of opportunity in the $f \propto 10^{-6} \text{ Hz}$ range. Other than that, it may be next to impossible to obtain relic GW signatures. Now that we have said it, it is time to consider the next issue. Having said that, it is now time to consider what is also vital. I.e. finding if information from a prior universe may be transmitted to our own universe.

Minimum amount of information needed to initiate placing values of fundamental cosmological parameters.

A.K. Avessian's [30] article (2009) about alleged time variation of Planck's constant from the early universe depends heavily upon initial starting points for $\hbar(t)$, as given below, where we pick :

$$\hbar(t) \equiv \hbar_{initial} [t_{initial} \leq t_{Planck}] \cdot \exp[-H_{macro} \cdot (\Delta t \sim t_{Planck})] \quad (14)$$

The idea is that we are assuming a granular, discrete nature of space time. Furthermore, after a time we will state as $t \sim t_{Planck}$ there is a transition to a present value of space time, It is easy to, in this situation, to get an inter relationship of what $\hbar(t)$ is with respect to the other physical parameters, i.e. having the values of α written as $\alpha(t) = e^2 / \hbar(t) \cdot c$, as well as note how little the fine structure constant actually varies. Note that if we assume an unchanging Planck's mass $m_{Planck} = \sqrt{\hbar(t)c/G(t)} \sim 1.2 \times 10^{19} GeV$, this means that G has a time variance, too. This leads to us asking what can be done to get a starting value of $\hbar_{initial} [t_{initial} \leq t_{Planck}]$ recycled from a prior universe, to our present universe value. What is the initial value, and how does one insure its existence? We obtain a minimum value as far as 'information' via appealing to Hogan's [26] (2002) argument with entropy stated as

$$S_{max} = \pi / H^2 \quad (15)$$

and this can be compared with A.K. Avessian's article [30] (2009) value of, where we pick $\Lambda \sim 1$

$$H_{macro} \equiv \Lambda \cdot [H_{Hubble} = H] \quad (16)$$

I.e. a choice as to how $\hbar(t)$ has an initial value, and entropy as scale valued by $S_{max} = \pi / H^2$ gives us a ball park estimate as to compressed

values of $\hbar_{initial} [t_{initial} \leq t_{Planck}]$ which would be transferred from a prior universe, to today's universe. If $S_{max} = \pi/H^2 \sim 10^5$, this would mean an incredibly small value for the INITIAL H parameter, i.e. in pre inflation, we would have practically NO increase in expansion, just before the introduction vacuum energy, or emergent field energy from a prior universe, to our present universe.

Unanswered questions and what this suggests for future research endeavors

As far back as 1982, Linde, [32] when analyzing a potential of the form

$$V(\varphi) = \frac{m^2 \varphi^2}{2} + \lambda \varphi^4 + V(0) \quad (17)$$

This is when the 'mass' has the form, (here M is the bare mass term of the field φ in de Sitter space, which does not take into account quantum fluctuations)

$$m^2(t) = M^2 + \frac{3\lambda H^3}{4\pi} \cdot (t - t_0) \quad (18)$$

Specified non linearity of $\langle \varphi^2 \rangle$ at a time from the big bang, of the form

$$\Delta t_1 \approx \frac{3H}{2M} \quad (19)$$

The question raised repeatedly in whether or not i) if higher dimensions are necessary, and whether or not ii) mass gravitons are playing a role as far as the introduction of DE speed up of cosmological expansion may lead to an improvement over what was specified for density fluctuations and structure formation

(The galaxy hierarchy problem) of density fluctuations given as

$$\frac{\delta\rho}{\rho} \sim 10^{-4} \Leftrightarrow \lambda \leq 10^{-10} \quad (20)$$

Eq. (18) is for four space, a defining moment as to what sort of model would lead to density fluctuations. It totally fails as to give useful information as to the galaxy hierarchy problem, above. Furthermore is considering the spectral index problem, where the spectral index is [32]

$$n_s - 1 \cong -\frac{3}{8\pi} \cdot \left(\frac{V_\varphi}{V}\right)^2 + \frac{1}{4\pi} \cdot \left(\frac{V_{\varphi\varphi}}{V}\right)^2 \quad (21)$$

Usual experimental values of density fluctuations experimentally are $\frac{\delta\rho}{\rho} \sim 10^{-5}$, instead of $\frac{\delta\rho}{\rho} \sim 10^{-4}$, and this is assuming that λ is small.

In addition, Linde [32] (1982) had $\frac{d}{d\varphi^2} V = m^2 \leq \frac{H}{40} = \frac{1}{40} \cdot \frac{\dot{a}}{a}$ inside a false vacuum bubble. If something other than the Klein Gordon relationship $\frac{\dot{a}}{a} \Rightarrow \ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi = 0$ occurs, then different models of how density fluctuation may have to be devised. A popular model of density fluctuations with regards to the horizon is [32]

$$\left(\frac{\delta\rho}{\rho}\right)_{Horizon} \cong \frac{k^{3/2} |\delta_k|}{\sqrt{2\pi}} \propto \frac{k^{(3/2)+3\alpha-3/2}}{\sqrt{2\pi}} \approx \left(1/\sqrt{2\pi}\right) \cdot k^{3\alpha} \quad (22)$$

Where $-0.1 < \alpha < 0.2$, and $\alpha \equiv 0 \Leftrightarrow n_s \equiv 1$ and to first order, $k \cong Ha$. The values, typically of [33] $n_s \neq 1$ If working

with $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left[\left(\frac{\rho}{3M_4^2} + \frac{\rho^2}{36M_{Planck}^2} \right) + \frac{C}{a^4} \right]$, and with a density value [28], [29] $\rho \equiv \rho_0 \cdot \left(\frac{a_0}{a}\right)^3 - \left[\frac{m_g c^6}{8\pi G \hbar^2} \right] \cdot \left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2} \right)$ where $m_g \approx 10^{-65}$ grams, and $\alpha < 0.2$ is picked to avoid over production of black holes, a complex picture emerges. Furthermore, if $\alpha < 0.2$ and $\alpha \neq 0$

$$\left(\frac{\delta\rho}{\rho} \right)_{Horizon} \cong \left(1 / \sqrt{2}\pi \right) \cdot k^{3\alpha} \sim \frac{H^2}{\dot{\phi}} \propto 10^{-4} - 10^{-5} \quad (23)$$

The above equation gives inter relationships between the time evolution of a pop up inflaton field ϕ , and a Hubble expansion parameter H, and a wave length parameter $\lambda = (2\pi/k) \cdot a(t)$ for a mode given as δ_k . What should be considered is the inter relationship of the constituent components of Eqn. (21) and $\lambda \leq H^{-1}$. What the author thinks is of import is to look at whether equation below also holds. [32]

$$\left(\frac{\delta\rho}{\rho} \right) \cong Ak^{\left(\frac{n_s-1}{2}\right)} \propto 10^{-4} - 10^{-5} \quad (24)$$

To first order, variations of $\alpha < 0.2$ and $\alpha \neq 0$, should be compared with admissible values of $\left([n_s - 1]/2\right)$ which would closely correspond to $\alpha \neq 0$ and $0 < \alpha \ll 0.2$. What we hope is that if we can determine what are the appropriate conditions for plotting sensitivities for strain, and frequency, for GW astronomy, that in due time we will be able to give inputs into Eqn. (24) above to understand

structure formation in the early universe. What is brought up in Eqn.(24) is important also if we wish to understand massive Gravitons [34]. In addition, a proper understanding of Eq. (24) is also important if we wish to understand how GW and neutrinos may interact with each other, which could be part of what is happening in, as an example, low Lithium stars, as brought up by Beckwith in Erice, nuclear physics 2009 [35]

Serious technical problems which need to be addressed in order to improve the quality of research for relic signals.

An important, direct connection between the strain of relic gravitational waves and the inflaton field has been released by Dr. Corda [36] as far as the formula he derived for an inflaton and inputs of strain upon the inflaton field. This was given by Dr. Corda as[36]

$$\varphi = \frac{H^2}{2A_h^2} \quad (25)$$

Here, H is given as the evolving Hubble parameter, and A_h represents the averaged amplitude of the perturbations of the RSBGWs, where RSBGWs is an abbreviation for relic stochastic background of gravitational waves (RSBGWs) which is proposed by the Pre-Big-Bang Theory. Below we work with an amplitude $A_h \sim A_{hc} \propto 10^{-44}$, as compared to $A_h \sim A_{hc} \propto 10^{-44} - 10^{-51}$ for a frequency range Corda gave as for when one has $H \sim 10^{22} Hz$ for the Hubble parameter when setting for a narrower frequency band width given $10Hz < f < 10KHz$. The upshot as claimed by Corda is for that range of GW that $\varphi \geq 10^{-5} grams$ as a lower bound for the inflaton field. If so, then the inflaton field may have a different lower bound if, as an

example one looks at $1\text{Hz} < f < 10\text{GHz}$, even if one looks at $H \sim 10^{22}\text{Hz}$. The lower bound of the inflaton field becomes especially significant, if as an example inflaton fields are connected with initial entropy conditions which Beckwith picked as $n \sim \text{particle-count} \sim 10^5$. Being able to connect, say $\text{particle-count} \sim 10^5$, or falsify it, via experiment may settle the question of if the inflaton, and its initial values generate entropy, in relic conditions.

Furthermore we also should mention that Relic gravitational waves represent a stochastic background, therefore, for a potential detection, a cross correlation between, at least, two detectors is needed. An important discussion on this issue which is important here has been released in B. Allen,[37].

I.e. the role of how cross correlation works in refining accurate measurements of initial entropy conditions which Beckwith picked as $n \sim \text{particle-count} \sim 10^5$. Being able to connect, say $\text{particle-count} \sim 10^5$ with inflaton models should not be underestimated, as a way to confirm or falsify if inflations are connected with initial entropy, The relevant work by Lee Samuel Finn, Shane L. Larson, Joseph D. Romano, [38] specify that the separation and relative orientation of the two detectors plays a crucial role in determining the frequency dependent sensitivity of each detector pair to the stochastic background. I.e. how the detector geometry is set up will either allow confirmation or refutation of the role of inflatons in entropy generation, which has the potential of allowing falsifiable criteria being developed to either link inflatons to measurable GW generation, or kill the inflaton, model wise.

Note that as given by [37,38] the overlap reduction function of a pair of gravitational-wave detectors cited to be “the collection of

geometric factors, associated with the relative position and orientation of the detector pair, that appear in the cross-correlation of the detector pair's response". Key to the fidelity of the overlap reduction pair's contribution will also be in how accurate the planar wave approximation to GW actually is, and remains, even in relic conditions to the present..

In all, as noticed especially by Corda [36] that the framework of the relic gravitational waves in the ordinary inflationary model has been recently generalized to extended theories of gravity as mentioned by C. Corda,[39]. In this context, in C. Corda,[40] it has been shown that detectors for gravitational waves will be important to confirm or to rule out, in an ultimate way, the physical consistency of General Relativity, eventually becoming an observable endorsing of Extended Theories of Gravity. Getting the role of the overlap reduction function right, and also determining to what degree the planar approximation of GW is correct for high fidelity measurements involving two or more detectors will be crucial to the development of GW cosmology as a falsifiable scientific endeavor.

It is useful to also note that Stephenson [41] noted, that for optimal detector strategies that Collins work [42] allowed Stephenson to write "In the case of discrete events, multiple detectors, even detectors in completely different locations, can be combined via the mechanism of delay histograms. Referring to Figure 1, the signal from detector #1 can be compared with the signal in detector #2 by checking a range of possible delays between the signal of #1 and #2."

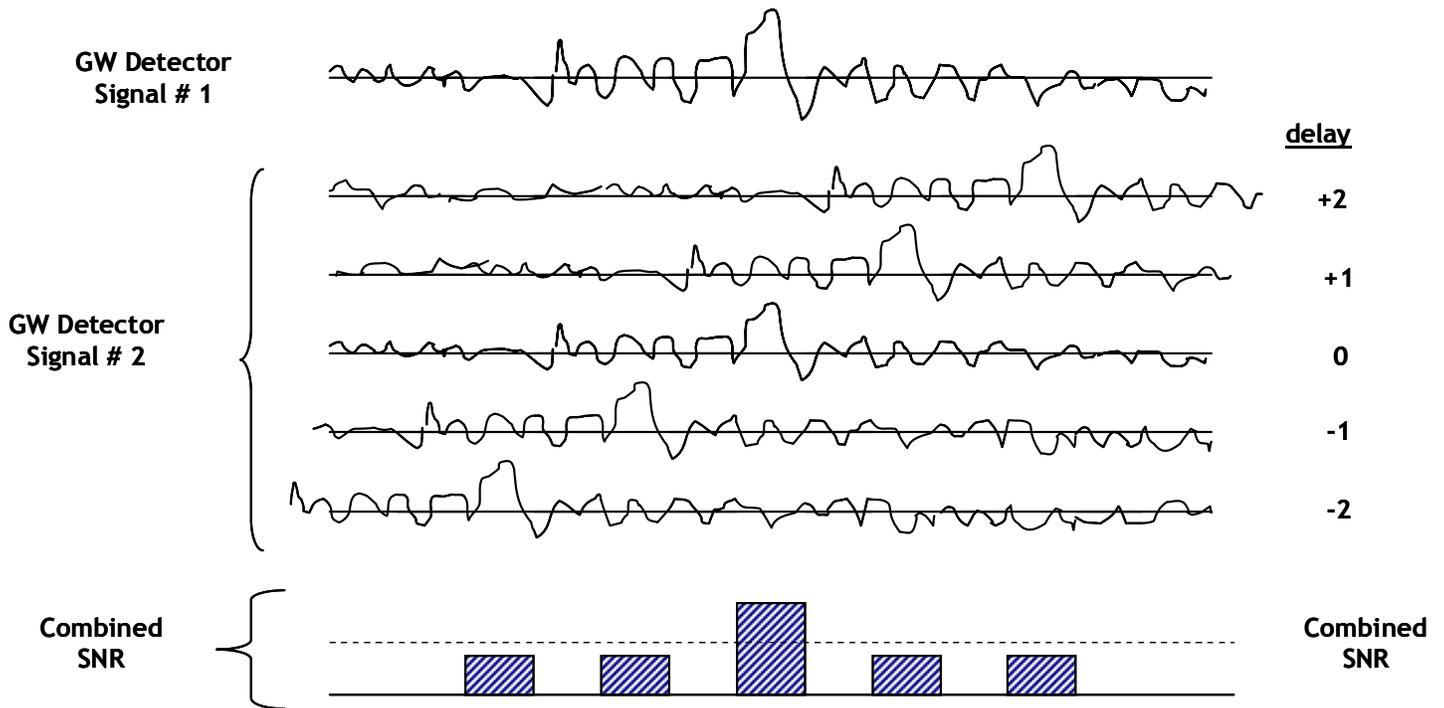


Figure 1, as given by Stephenson [41], ‘An example of a delay histogram when used to align detection events,’ first given by (Collins, 2004) [423].’

We hope for a similar refinement as to work done in refining the work needed to ascertain if GW measurements made initially from the onset of creation of our universe’s cosmology can become an experimental science.

Conclusions, as to how to look at early universe topology and later flat space

Resolution of which add more detail to a wave function of the universe we can approximate in early pre inflationary conditions as

$\Psi \sim \left[R / R_{eq} \right]^{3/2}$ [43]. I.e. spatial variation due to inflation is not in itself sufficient to understand how space time geometry evolved in the early universe. Our discussion has, in fact outlined $\Omega_{GW} \sim 10^6$ as in the $1Hz < f < 10 GHz$ range for either the QIM and / or the pre big bang models as the best chance of obtaining signatures of GW physics in relic GW conditions.

It is clear that gravitational wave density is faint, even if we make the approximation that $H \equiv \frac{\dot{a}}{a} \cong \frac{m\dot{\phi}}{\sqrt{6}}$ as stated by Linde (2008) [44], where we are following $\dot{\phi} = -m\sqrt{2/3}$ in evolution, so we have to use different procedures to come up with relic gravitational wave detection schemes to get quantifiable experimental measurements so we can start predicting relic gravitational waves. This is especially true if we make use of the following formula for gravitational radiation, as given by L.Kofman [45] (2008), with $M = V^{1/4}$ as the energy scale, with a stated initial inflationary potential V . This leads to an initial approximation of the emission frequency, using present-day gravitational wave detectors.

$$f \cong \frac{(M = V^{1/4})}{10^7 \text{ GeV}} \text{ Hz} \quad (26)$$

What we would like to do for future development of entropy would be to consider a way to ascertain if or not the following is really true, and to quantify it by an improvement of a supposition advanced by Kiefer, Polarski, and Starobinsky [46] as of (2000). I.e. the author, Beckwith, has in this document presented a general question of how to avoid having $dS/dt = \infty$ at $S=0$,

1. Removes any chance that early universe nucleation is a quantum based emergent field phenomena
2. Goldstone gravitons would arise in the beginning due to a violation of Lorentz invariance. I.e. we have a causal break, and merely having the above condition does not qualify for a Lorentz invariance breakdown. Kiefer, Polarski, and Starobinsky as of (2000) [46] presented the idea of presenting the evolution of relic entropy via the evolution of phase spaces, with Γ/Γ_0 being

the ratio of ‘final (future)’ / ‘initial’ phase space volume, for k modes of secondary GW background.

$$S(k) = \ln \frac{\Gamma}{\Gamma_0} \quad (27)$$

If the phase spaces can be quantified, as a starting point of say $l_{\text{min-length-string}} \equiv 10^\alpha \cdot l_{\text{Planck}}$, with l_{Planck} being part of how to form the ‘dimensions’ of Γ_0 , and $l_{\text{min-length-string}}$ part of how to form the dimensions of Γ , and 10^α being, for a given $\alpha > 0$, and in certain cases $\alpha \gg 0$, then avoiding having $dS/dt = \infty$ at $S=0$ will be straight forward Determining the run up as to avoiding infinite change of entropy/ early universe GW production and an infinite, unphysical spurt of gravitons at the onset of inflation is part and parcel of turning GW astronomy into an empirical science. What we intend to do, is to use Eqn. (13) as part of making sense of the two tables, and also the point of Eqn.(13)’s break down as an aid to distinguishing between the five models brought up in this document, plus the possibility that there is a multi verse to be investigated. The entropy so outlined in Eqn. (25) with a graviton count, along the lines of what was brought up by Beckwith [47] for a relationship of entropy with particle count may be a way to obtain relic GW traces, provided we obtain conditions for turning GW physics into GW astro physics.

We wish to, once again, to close with the following summary of possibilities as to if there are relic HFGW measurable today, namely either

1. The red shifting post big bang is not nearly as great as people think. How or why this would be possible would be a serious source of model building expertise to consider
2. Even higher frequency initial GW would happen before the onset of the big bang

3. The big bang is then over stated as far as its initial expansion.

Experiments carefully done are the only way to get to this fundamental issue on an experimentally verified stand point. We look forward to the day when experimental platforms measuring GW can be built of sufficient sensitivity which may bring closure to a resolution of this most important experimental measurement issue in contemporary cosmology

It is important to note that recently Corda [50] has on his own considerably refined the relative characteristics of the inflaton, as he wrote recently, namely in that

“By releasing a formula that directly connects the average amplitude of the relic stochastic gravitational-wave background (SGWB) with the Inflaton field and the equation for the characteristic amplitude h_c for the relic SGWB, in this paper the upper bounds on the relic SGWB from the Wilkinson Microwave Anisotropy Probe (WMAP) and the Laser Interferometer Gravitational-wave Observatory (LIGO) Scientific Collaboration (LSC) data are translated in lower bounds on the Inflaton field”

Corda’s important contribution is to parameterize detection of the SGWB so as to permit a direct measure of the value of the Inflaton field. We thank Dr. Corda for his insight as to this important development in GW physics. What we wish to do, while building upon Corda’s research is to also determine yet another issue, i.e. is the inflaton, as constituted in his Eqn.(27) [50] actually driving initial entropy. If there is a connection between inflaton physics, as given in Corda’s Eqn(27) [50] which is represented in Eqn. (25) of our document

A careful analysis of this point and looking at the relative role of inflaton physics as given in the evolution of entropy may allow determining if conditions for initial entropy , as given by Eqn. (27)

above are accurate may help making a determination if HFGW can be detected experimentally, i.e. answering the 3 possibilities given in page 12 above.

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Bibliography

- [1] L.P. Grishchuk, Zh. Eksp. Teor. Fiz. 67, 825 (1974)
- [2] A. A. Starobinsky, ETP Lett. 30, 682 (1979).
- [3] L. Parker, Phys. Rev. Lett. 21 562 (1968).
- [4] M. Maggiore, "Gravitational Wave Experiments and Early Universe Cosmology", Physics Reports 331 (2000) 283}367
- [5] Kuchiev, M. Yu, "Can gravity appear due to polarization of instantons in SO(4) gauge theory?" Class. Quantum Grav. 15 1895-1913 doi: 10.1088/0264-9381/15/7/008
- [6] Andrić, I, Jonke , L. and Jurman , D."Solitons and giants in matrix models", JHEP12(2006)006
- [7] A.W. Beckwith Entropy growth in the early universe, and the search for determining if gravity is classical or quantum, part I (Confirmation of initial big bang conditions?), <http://vixra.org/pdf/0909.0016v1.pdf>

- [8] A. W. Beckwith, “Energy Content of Gravitation as a Way to Quantify Both Entropy and Information Generation in the Early Universe”, *Journal of Modern Physics*, 2011, 2, 58-61
- [9] Lee Simolin “Life of the Cosmos” (1997),
- [10] Y. J. Ng, *Entropy* 10(4), pp. 441-461 (2008);
 Y. J. Ng and H. van Dam, *Found. Phys.* 30, pp. 795–805 (2000);
 Y. J. Ng and H. van Dam, *Phys. Lett. B*477, pp. 429–435 (2000);
 Jack Ng, “Quantum Foam and Dark Energy”, International work shop on the Dark Side of the Universe,
<http://ctp.bue.edu.eg/workshops/Talks/Monday/QuantumFoamAndDarkEnergy.pdf>
- [11] D. Perkins, *Particle Astrophysics*, Oxford University press, Oxford, UK (2003)
- [12] Asakawa M., Hatsuda, T. and Nakahara, Y., *Prog. Part. Nucl. Phys.* 46, 459(2001)
 Asakawa M, Bass SA, and Müller B.,” Anomalous viscosity of an expanding quark-gluon plasma”, *Physical review letters* 96(25):252301 2006 Jun 30
 Asakawa M. , Hatsuda, T. and Nakahara, Y., *Prog. Part. Nucl. Phys.* 46, 459(2001)
- [13] G.Torrieri and I. Mushuntin, *Phys.Rev.C*78:021901,2008,
<http://arxiv.org/abs/0805.0442>
- [14] J. Natário, (2006). *Relativity and Singularities - A Short Introduction for Mathematicians*. March 8. <http://arxiv.org/abs/math.DG/0603190>.
- [15] A.W. Beckwith, “Relic High Frequency Gravitational waves from the Big Bang, and How to Detect Them”, <http://arxiv.org/abs/0809.1454> (2009), *AIP Conf.Proc.*1103:571-581,2009 (SPESIF)
- [16] S. Mathur, and B. Chowdhury, “ Fractional Brane States in the Early Universe”, *Classical and Quantum Gravity*, 24 (2007), pp. 2689-2720
- [17] A.W. Beckwith, “Instanton formation of vacuum energy via the Reissner-Nordstrom geometry of a wormhole bridge between a prior to our present universe.”, arXiv:0710.3788, October 2007
- [18] S. Weinberg,” *Cosmology*”, Oxford University Press, 2008
- [19] A.W. Beckwith, “Instanton formation of vacuum energy via the Reissner-Nordstrom geometry of a wormhole bridge between a prior to our present universe.”, arXiv:0710.3788, October 2007

- [20] M. Maggiore, “Gravitational Wave Experiments and Early Universe Cosmology”, *Physics Reports* 331 (2000) 283}367
- [21] L. Crowell, e mail to author, February 28, 2011
- [22] . R. Clive Woods , Robert M L Baker, Jr., Fangyu Li, Gary V. Stephenson , Eric W. Davis and Andrew W. Beckwith (2011), “A new theoretical technique for the measurement of high-frequency relic gravitational waves,” submitted for possible publication. <http://vixra.org/abs/1010.0062>
- [23] Fangyu Li, Zhenyun Fang, Robert M. L. Baker, Jr., Andrew W. Beckwith , Gary V. Stephenson, R. Clive Woods, Eric W. Davis, Kai Lin, Nan Yang, Jin Li, Hao Wen,” Extending the High-Frequency Gravitational Wave Spectrum: Cosmology Predictions and Measurement Methods”, article in preparation which is for submission to a journal reflecting HFGW astronomy intersecting with inflation models to be delineated
- [24] B. P. Abbott, et al., “An upper limit on the stochastic gravitational-wave background of cosmological origin”, *Nature* 460, 990 (2009)
- [25] <http://www.phys.ufl.edu/~tanner/ligo-papers.html>
- [26] P. J. Steinhardt and N. Turok, “A Cyclic Model of the Universe”, *Science* 296, 1436 (2002).
- [27] A. Ruediger, 2002, “ Detecting gravitationalwaves with ground and space interferometers—with special attention to the space project ASTROD”, *Int. J. Mod. Phys. D* 11 963–94
- [28] W. T. Ni, “Gravitational Waves, Dark Energy And Inflation” *Modern Phys. Lett. A*, 922 (2010) , <http://arxiv.org/pdf/1003.3899>; W. T. Ni, et al.,2002 *Int. J. Mod. Phys. D* 11 1035-1048
- [29] W. T. Ni, *Modern Phys. Lett. A*, 922 (2010)
- [30] A.K. Avessian,” Plancks constant evolution as a cosmological evolution test for the early universe”, *Gravitation and Cosmology*, 2009, Vol 15, number 1, pp 10-12
- [31] C. Hogan, “Holographic discreteness of Inflationary Perturbations”, arXIV astro-ph/0201020 v 2, May27, 2002
- [32] J. Camp and N. Cornish., “Gravitational wave Astronomy”, pp 525-577, as part of “ Annual Review of Nuclear and Particle Science”, Volume 54, with Kayser, B., Holstein, B., and Jawahery, A. as editors. Menlo Park, California, 2004

- [33] A.W. Beckwith, “Applications of Euclidian Snyder Geometry to the Foundations of Space-Time Physics”, EJTP 7, No. 24 (2010) 241–266, <http://www.ejtp.com/articles/ejtpv7i24p241.pdf>
- [34] E. Alves, O. Miranda. and J. de Araujo, “ Can Massive Gravitions be an Alternative to Dark Energy?”, arXiv: 0907.5190 (July 2009)
- [35] A. W. Beckwith, “Entropy, neutrino physics, and the lithium problem: Why are there stars with essentially no lithium due to serious lithium deficiency in certain spatial regions in the early universe?”, in particles and nuclear physics journal , as recorded in <http://vixra.org/pdf/0910.0030v3.pdf>; See version 3
- [36] C. Corda, Gen. Rel Grav. 42, 1323-1333 (2010).
- [37] B. Allen, Proceedings of the Les Houches School on Astrophysical Sources of Gravitational Waves, eds. Jean-Alain Marck and Jean-Pierre Lasota (Cambridge University Press, Cambridge, England 1998)
- [38] Lee Samuel Finn, Shane L. Larson, Joseph D. Romano,” Detecting a Stochastic Gravitational-Wave Background: The Overlap Reduction Function”, Phys.Rev.D79:062003,2009 ; <http://arxiv.org/abs/0811.3582>
- [39] C. Corda. Eur. Phys. J. C 65 1-2, 257-267 (2010).
- [40] C. Corda, Int. J. Mod. Phys. D18, 2275-2282 (2009)
- [41] G.V. Stephenson, “Lessons for Energy Resonance HFGW Detector Designs Learned from Mass Resonance and Interferometric LFGW Detection Schemes,” in the Proceedings of the Space, Propulsion and Energy Sciences International Forum (SPESIF), edited by Glen Robertson, American Institute of Physics Conference Proceedings 1103, Melville, New York, (2009a), pp. 532-541. <http://www.gravwave.com/docs/Detector%20Development.pdf>
- [42] H. Collins, Gravity’s Shadow, University of Chicago Press, Chicago, (2004).
- [43] S. Capozziello, A. Feoli, G. Lambiase, G. Papini, G., ‘ Thin shell quantization in Weyl Space – time’, Physics Letters A, 273 (2000), pp 25-30
- [44] L. Koffman, “Pre heating After Inflation “, pp 55-79, in Lecture notes in Physics 738, Inflationary cosmology , editors M. Lemoine, J. Martin, P. Peter, as Editors, Springer Verlag, 2008 Berlin, Federal Republic of Germany [45] A.Linde, “Inflationary cosmology”, pp 1-54, , in Lecture notes in Physics, 738, Inflationary cosmology , editors M. Lemoine, J. Martin, P. Peter, as Editors, Springer Verlag, 2008 Berlin, Federal Republic of Germany

- [46] L. Koffman, “Pre heating After Inflation “, pp 55-79, in Lecture notes in Physics 738, Inflationary cosmology , editors M. Lemoine, J. Martin, P. Peter, as Editors, Springer Verlag, 2008 Berlin, Federal Republic of Germany
- [47] C. Kiefer, D. Polarski, A.A. Starobinsky,” Entropy of gravitons produced in the early Universe”, *Phys. Rev. D* 62, 043518 (2000) [6 pages]; arxiv gr-qc/9910065
- [48] L. Krauss, S. Dodelson, S. Meyer,” Primordial Gravitational Waves and Cosmology”, http://arxiv.org/PS_cache/arxiv/pdf/1004/1004.2504v1.pdf
- [49] P. Steinhardt, “Is the theory at the heart of modern cosmology deeply flawed?”, 36 *Scientific American*, April 2011, <http://www.physics>
- [50] C. Corda, “Primordial Gravity’s breath”, <http://arxiv.org/pdf/1110.1772>