Derivation of $\Delta E=\Delta mc^2$ Revisited

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Einstein's September 1905 paper in which $\Delta L=\Delta mc^2$ (light energy-mass equation) is derived, is not completely studied; and is only valid under special conditions of involved parameters. The origin of $\Delta E=\Delta mc^2$ from $\Delta L=\Delta mc^2$ is completely speculative in nature. The factor c^2 has been arbitrarily brought into the picture by Einstein. To obtain $L = \Delta mc^2$ Einstein retained the term $v^2/2c^2$ (compared to unity) without giving numerical values to v. If the value of v is considered in a typical classical region, 1cm/s say ($v^2/2c^2 = 5.55\times10^{-22}$ is negligible) then the result is $M_b = M_a$. Thus the conversion factor c^2 is arbitrarily brought into the picture as both the results i.e. $\Delta L=\Delta mc^2$ and $M_b = M_a$ are equally probable. Einstein derived $\Delta L=\Delta mc^2$ under special conditions and speculated from it that $\Delta E=\Delta mc^2$ without mathematical derivation. These are the limitations of the derivation. The same derivation also gives $L \propto \Delta mc^2$ or $L = A\,\Delta mc^2$, where A is a coefficient of proportionality. There are numerous values of coefficients of proportionality in the existing physics. This is a mathematical critical analysis, have no implications on the experimentally established status of $\Delta L=\Delta mc^2$ ($E=\Delta mc^2$).

1.0 Description and critical analysis of Einstein's Thought Experiment

In Einstein's derivation the basic equation is

$$\ell^* = \ell \frac{\left[1 - \frac{v}{c}\cos\phi\right]}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

where ℓ is light energy emitted by a body in (x,y,z) frame, ℓ^* is the light energy measured in (ξ, η, ζ) system, and v is the velocity with which the frame or (ξ, η, ζ) system is moving. This is the equation for the Doppler principle for light for any velocity whatsoever [2].

Einstein's perception [1]: Let a system of plane waves of light, referred to the (x, y, z) system of coordinates, possesses the energy *l*. Let the direction of the ray (the wave-normal) make an angle ϕ with the x-axis of the system [1]. Energy is a scalar quantity, having magnitude only, but according to eq. (1) it also depends upon angle.

Introduce a new system of co-ordinates (ξ, η, ζ) moving in uniform parallel translation with respect to the (x, y, z) system, and having its origin of coordinates in motion along the x-axis with the velocity *v*.

Thus *v* is the relative velocity between system (x, y, z) and system (ξ, η, ζ) . The body which emits light energy is considered stationary in the system (x,y,z) and also remains stationary after emission of light energy in the system (x,y,z).

Let E_0 and H_0 be the energies in the (x, y, z) coordinate system and the (ξ, η, ζ) coordinate system respectively, before emission of light energy. Furthermore, E_1 and H_1 are the energies of the body in both systems after it emits light energy. E_i and H_i include all the energies possessed by the body in the two systems respectively. The various meanings of the E_i's and H_i's are shown in Table I.

Table I. Energies emitted before and after emission by a body in Einstein's September 1905 derivation.

Sr No	System (x,y,z) at rest	System(ξ, η, ζ) moving with velocity *v*
1	Before Emission E_0	Before Emission H_0
2	After Emission E_1	After Emission H_1

Then Einstein assumed that the body emits two light waves, each of energy $0.5L$, in the (x,y,z) system where the energy is E_0. Thus,

Energy before Emission = Energy after emission $+0.5L + 0.5L$ (1)

$$E_0 = E_1 + 0.5L + 0.5L = E_1 + L \tag{2}$$

Energy of the body in system (ξ, η, ζ) is

$$H_0 + H_1 + 0.5\beta L\left\{\left(1 - \frac{v}{c}\cos\phi\right) + \left(1 + \frac{v}{c}\cos\phi\right)\right\} \tag{3}$$

where

$$\beta = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

$$H_0 = H_1 + \beta L \tag{5}$$

or

$$\left(H_0 - E_0\right) - \left(H_1 - E_1\right) = L\left(\beta - 1\right) \tag{6}$$

Einstein maintained that

$$\left(H_0 - E_0\right) = K_0 + C = \frac{M_b v^2}{2} + C$$

$$\left(H_1 - E_1\right) = K_1 + C = \frac{M_a v^2}{2} + C$$

Einstein defined C as an additive constant which depends on the choice of the arbitrary additive constants of the energies *H* and *E*. The arbitrary additive constant C is regarded as equal in both cases. Kinetic energy of the body before emission of light energy, K_0, $\left(\frac{M_b v^2}{2}\right)$ and the kinetic energy of the body after emission of light energy, K_1, $\left(\frac{M_a v^2}{2}\right)$ gives

$$K_0 - K_1 = L\left\{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right\} \tag{7}$$

Einstein considered the velocity in the classical region, applying the binomial theorem thus,

$$K_0 - K_1 = L\left(1 + \frac{v^2}{2c^2} + 3\frac{v^4}{8c^4} + \ldots - 1\right) \tag{8}$$

Further Einstein [1] neglected magnitudes of the fourth (v^4/c^4) and higher (v^6/c^6, v^8/c^8 …….) orders so that,

$$K_0 - K_1 = L\frac{v^2}{2c^2} \tag{9}$$

$$\frac{M_b v^2}{2} - \frac{M_a v^2}{2} = L\frac{v^2}{2c^2} \tag{10}$$

or

$$L = (M_b - M_a)\, c^2 = \Delta mc^2 \tag{11}$$

or

Mass of body after emission (M_a) = Mass of body before emission

$$(M_b) - \frac{L}{c^2}\,. \tag{12}$$

Then Einstein generalized the result for every energy and asserted that the mass of a body is a measure of the energy content (every energy that is included in a collection). Fadner [11] has mentioned that in the paper Einstein neither wrote $E = \Delta mc^2$ nor E in the paper. It is concluded that Einstein's statement means $E = \Delta mc^2$. It can be obtained by replacing L (light energy) by E (energy-content or every energy). Einstein wrote,

$$E = (M_b - M_a)c^2 = \Delta mc^2 \tag{13}$$

or

Mass of body after emission (M_a) = Mass of body before emission $(M_b) - \dfrac{E}{c^2}$ (14)

When energy is emitted the mass decreases. Thus Einstein did not differentiate between Light Energy and other energies in the derivation.

1.1 Typical comments regarding the classical region of velocity (not given by Einstein).

Einstein's derivation also offers the most mysterious situation in physics, explained below with the help of this equation,

$$M_{motion} = \frac{M_{rest}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{15}$$

Let the velocity be in classical region e.g. 10m/s (36 km/hr, i.e. ordinary speed of a motor vehicle), then there is no increase in the mass of the object when it moves with this velocity. The speed of an aeroplane is over 400km/hr, and no increase in mass is observed.

$$M_{motion} = M_{rest}\left(1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \ldots\right) \tag{16}$$

(i) If the velocity of the (x,y,z) system is $v = 0$, then $M_{\text{motion}} = M_{\text{rest}}$.

(ii) If v = 1cm/s (0.036 km/hr) then

$$M_{\text{motion}} = M_{\text{rest}}\,[\,1 + 5.55\times10^{-22} + 4.166\times10^{-42} + \ldots\ldots\ldots\ldots\ldots\ldots]\tag{17}$$

$$M_{\text{motion}} = M_{\text{rest}} + M_{\text{rest}}\,5.55\times10^{-22} + M_{\text{rest}}\,4.166\times10^{-42} + \ldots$$

Here even the term $M_{\text{rest}}\ 5.55\times10^{-22}$ is regarded as negligible compared to M_{rest}, and $M_{\text{rest}}\ 4.166\times10^{-42}$ is further negligible; thus

$$M_{motion} = M_{rest} \tag{18}$$

Thus the term 5.55×10^{-22} can be neglected only when both masses are equal.

$$M_{\text{motion}} = M_{\text{rest}}\,[1 + \mathbf{v^2/2c^2} + 3v^4/8c^4 + \ldots\ldots\ldots\ldots\ldots]$$

$$= M_{\text{rest}}\,[\,1 + \mathbf{5\times10^{-9}} + 3.75\times10^{-17} + \ldots\ldots\ldots\ldots\ldots\ldots]$$

$$M_{\text{motion}} = M_{\text{rest}} + M_{\text{rest}}\,5\times10^{-9} + M_{\text{rest}}\,3.75\times10^{-17} \tag{19}$$

The mass of Earth remains the same i.e. 5.98×10^{24} kg always. Thus here also the term $v^2/2c^2$ (5×10^{-9}) is neglected compared to unity; or it implies that the term M_{rest} 5×10^{-9} has to be neglected compared to $M_{rest.}$ It implies increase in mass of the Earth of about 5×10^{-7} %. If the term $v^2/2c^2$ **(5×10^{-9})** is neglected then

(iii) Similarly the orbital velocity of the Earth is 30km/s or 3,0000m/s i.e. $v/c = 10^{-4}$ thus

M_{motion} [mass of earth in motion] = M_{rest} [mass of earth at rest] (20) The vari

Table II: Terms neglected in calculations and their effects.

Sr. No.	velocity	$M_{rel} = M_{rest}[\ 1+v^2/2c^2+3v^4/8c^4 + \ldots.\ ]$	Neglected term	Result
1	0	$M_{rel} = M_{rest}$	none	$M_{rel} = M_{rest}$
2	Earth's orbital velocity 30km/s or 3×10^4m/s	$M_{rel} = M_{rest}$ [1+ **5×10^{-9}** + 3.75×10^{-17} + ………]	**5×10^{-9}**	$M_{rel} = M_{rest}$
3	v=1cm/s or 0.036km/s	$K_b - K_a$ = L [1+ **5.55×10^{-22}** +4.166×10^{-42} + ………-1] or $M_b = M_a$	**5.55×10^{-22}**	$M_b = M_a$ Mass before emission = Mass after Emission

1.2 Appearance of c^2 in L= Δmc^2 is apparently arbitrary.

$$K_0 - K_1 = L\left(1 + \frac{v^2}{2c^2} + 3\frac{v^4}{8c^4} + ... - 1\right) \quad (8)$$

Now consider the same case when velocity is 1cm/s or 0.036km/hr. Under this conditions eq. (8) becomes

$$\frac{M_b v^2}{2} - \frac{M_a v^2}{2} = L\left[1 + 5.55\times10^{-22} + 4.166\times10^{-42} + ... - 1\right] \quad (21)$$

(i) Einstein has neglected the term $3v^4/8c^4$ retained the term as $v^2/2c^2$, and obtained equation L = Δmc^2.
(ii) If the velocity is very small then $v^2/2c^2$ can be neglected compared to unity. If the velocity is 1cm/s (classical region), then $v^2/2c^2$ is 5.55×10^{-22}. Depending upon the orbital velocity of the Earth (30km/s or 3,0000m/s i.e. $v/c = 10^{-4}$) the term $v^2/2c^2$ (5×10^{-9}) can be neglected compared to unity; only then the equation i.e

M_{motion} [mass of earth in motion] = M_{rest} [mass of earth at rest] is justified.

In the typical classical region (v =1cm/s), $\boldsymbol{v^2/2c^2}$ = 5.55×10^{-22} is neglected compared to unity (as $\mathbf{5\times10^{-9}}$ is neglected) then

M_b (mass before emission) = M_a (mass after emission) (21)

Thus both L= Δmc^2 and $M_b = M_a$ are equally probable but have an entirely different nature. This discussion also validates the necessity of categorisation of sub ranges of velocity in the classical region or up to which magnitude of the term to be neglected comparatively.

2.0 Einstein took only super special values of variables and its effects.

The following arguments can be given that Einstein's derivation is true under special conditions [11-37].

1. Einstein [1] has imposed a condition on state of the body: Let there be a **stationary body** in the system (*x, y, z*), and let its energy referred to the system (*x, y, z*) be E_0. Let the energy of the body relative to the system (ξ, η, ζ) moving as above with the velocity *v*, be H_0. The body also remains stationary in system (x, y, z) after emission of energy. Einstein also assumed that the body remains stationary before and after emission of light energy, which is a super special condition. But practically this condition (light emitting body is stationary) is not obeyed in many other cases.

(i) Nuclear fission is caused by the thermal neutrons which have velocity 2,185m/s. The uranium atom also moves as it is split up into barium and krypton, and emits energy.

(ii) When a gamma ray photon of energy at least 1.02MeV, moves near the field of a nucleus it is split up into an electron and positron pair [2]. The gamma ray photon is in motion and similar is the state of the electron and positron pair.

(iii) Similarly particle and antiparticle move towards each other for annihilation. The particle and antiparticle collide then annihilation takes place. In nuclear fusion the atoms are set in motion. Fission is only caused by thermal neutrons (0.025eV or having velocity 2,185m/s). Thus there are characteristic or inherent conditions on the process in inter-conversion of mass and energy. These phenomena were not discovered in Einstein's time.

(iv) When a piece of paper burns it also sets in motion, and energy is emitted various forms.

(v) When deuterium and tritium fuse, but only after these are set in motion under conditions of high temperature. In the nuclear fusion of deuterium –tritium the energy of emitted neutrons is 14.1MeV (moving at 52,000km/s); their mass must increase by about 15.36%. It may increase the mass considerably. The velocity of the reactants is not necessarily uniform and gradually they overcome the force of electrostatic repulsion. Chemical reactions were discovered in Einstein's time. Einstein never discussed this phenomenon in his works. Thus derivation under the condition that body remains stationary in the emission process, is not conceptually useful or applicable in other cases.

Other conditions on Einstein's derivation.

Einstein's September 1905 derivation [1] of $\Delta L = \Delta mc^2$ is true under super special conditions or handpicked conditions only. It is justified below. In the derivation of $\Delta L = \Delta mc^2$ there are FOUR variables, i.e.

(a) Number of waves emitted,

(b) magnitude of light energy l,

(c) Angle ϕ at which light energy is emitted, and

(d) Uniform velocity (relative velocity), v. The fast neutrons are slowed down and are called thermal neutrons; thus their velocities are not necessarily uniform and can be variable while they cause fission of other nuclei.

Nature of *v*

According to Einstein: v is the relative velocity between system (x, y, z) and system (ξ, η, ζ). If system (x,y,z) is at rest and system (ξ, η, ζ) moves with velocity v, then v is relative velocity. If the system (x,y,z) and system (ξ, η, ζ) both move with same velocity then relative velocity *v* is zero. Further Einstein strictly took the value of velocity as uniform. The law of inter-conversion of mass and energy holds good if

(i) Velocity *v* is in the classical region
(ii) Velocity *v* is in the relativistic region
(iii) Velocity *v* is zero i.e. if both systems move with same velocity or system (ξ, η, ζ) is at rest.
(iv) Velocity *v* is variable or uniform.

These variables have numerous values. The law of inter-conversion of mass and energy holds good under all conditions, but Einstein has considered just one i.e. velocity in the classical region. It does not hold good under relativistic conditions. Such a significant derivation must be independent of velocity.

2.1 Genuine cases neglected in Einstein's derivation

Einstein took super special or handpicked values of parameters in the derivation. Thus for complete analysis the derivation can be repeated with all possible values of parameters. In all cases the law of conservation of momentum is obeyed (which is discussed in next sub-section).

(i) The body can emit a large number of light waves but Einstein has taken only **TWO light waves emitted by a luminous body.** Why were one or n light energy waves neglected?

(ii) The energy of two emitted light waves may have different magnitudes but **Einstein has taken two light waves of EQUAL magnitudes (0.5*L* each).** Why were other magnitudes (0.500001*L* and 0.499999*L*) neglected by Einstein?

(iii) A body may emit a large number of light waves of different magnitudes of energy making different angles **(other than 0º and 180º as assumed by Einstein).** Why were other angles (such as 0º and 180.001º**,** 0.9999 º and 180º etc.) neglected by Einstein? Thus a body needs to be specially fabricated; other forms of energy such as invisible energy are not taken in account. Furthermore Einstein has considered only light energy, not other forms of energy which can be emitted by body.

(iv) Einstein has taken velocity in the classical region ($v<<c$ and applied the binomial theorem at the end) has not at all used velocity in the relativistic region. If velocity is regarded as in the relativistic region (v is comparable with c), then the equation for relativistic variation of mass with velocity i.e.

$$M_{motion} = \frac{M_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{15}$$

is taken in account. It must be noted that before Einstein's work this equation was given by Lorentz [3-4] and firstly confirmed by Kaufman [5] and afterwards more convincingly by Bucherer [6]. Einstein, on June 19, 1948 wrote a letter to Lincoln Barnett [7] and advocated abandoning relativistic mass and suggested that it is better to use the expression for the momentum and energy of a body in motion, instead of relativistic mass.

It is a strange suggestion as Einstein has used relativistic mass in his work, including it in the expression of relativistic kinetic energy [8] from which rest mass energy is derived [9-10].

So Einstein's equation for inter-conversion of mass to energy highly depends upon velocity theoretically whereas practically the mass energy inter-conversion phenomena are applicable in all cases.

(v) Einstein has considered body emits light energy only, but simultaneously a body may also emit heat energy which is not taken into account in Einstein's derivation. A burning body emits heat, sound, light energies and energy in the form of invisible radiations simultaneously, along with visible radiations. For a proper description of heat energy-mass inter-conversion we need an equation equivalent to eq.(1). Similar is the case for other energies. In a nuclear explosion energies exist in many forms e.g. light energy, sound energy, heat energy, energy in form of various invisible radiations. Einstein has only considered light energy and other energies are neglected to derive $L = \Delta mc^2$.

Furthermore Einstein has considered that a body emits energy in the visible region. But energy can also be emitted in the invisible region and Einstein did not mention at all heat and sound energies (emitted along with the light energy). Thus energies other than light energy are also emitted but not taken in account. So energies are not taken in account completely. The various values of parameters neglected and taken into account in the derivation are shown in Table III.

Table III The values of various parameters considered by Einstein and neglected by Einstein in the derivation of Light Energy Mass equation $L = \Delta mc^2$.

Sr No	Parameters	Einstein considered	Einstein neglected (No reason was given by Einstein why parameters are neglected).
1	No. of light waves	Two Light Waves	One, three, four or n waves
2	Energy of light wave	Equal 0.5*L* and 0.5*L* each	Energies of the order of 0.500001*L* and 0.499999*L* are also possible. There are numerous such possibilities, which need to be probed. Bodies can emit more than two waves. The invisible waves of energy are not taken in account.
3	Angle	0° and 180°	The angles can be 0° and 180.001° or 0.9999° and 180° are also possible. There are numerous such possibilities which need to be probed.
4	Velocity	Classical region	The velocity may tend to zero. The velocity v can also be zero i.e. $v = 0$. The velocity can be in the relativistic region; $v \sim c$ and mass increases.
5	Velocity	Velocity is uniform in the classical region	The law of inter-conversion of mass to energy also holds good, when velocity is variable.

Deductions: Einstein has taken only super-special values of parameters, and neglected many realistic values.

3.0 Effects of general values of variables

If a body recoils when waves of different light energy are emitted: eq.(1) is the main equation in Einstein's derivation.

(i) If a light emitting body is at rest then the relative velocity of the measuring system (ξ, η, ζ) is only v.

(ii) If a light emitting body recoils away from the body with velocity V_R, then relative velocity will be $v + V_R$.

(iii) If a light emitting body drifts towards the measuring system (ξ, η, ζ) with velocity V_R then relative velocity will be $v - V_R$. But in this case the recoil velocity V_R is of the order of 5×10^{-33} m/s and with this velocity a body can travel a distance of 1.57×10^{-24} m in 10 years (which is undetectable; hence by definition the body is at rest). Thus in this case the body is regarded as at rest. The body may move towards or away from the observer, after emitting light energy; then the velocity becomes ($v \pm V_R$) or say ($v + V_R$). Thus

$$\ell^* = \ell \frac{\left[1 - \frac{v + V_R}{c}\cos\phi\right]}{\sqrt{1 - \frac{[v + V_R]^2}{c^2}}} \tag{23}$$

Now this equation can be applied to study variations in mass when light energy is emitted as in case of eq. (1).

3.1 Conservation of momentum in general cases helps in calculations of recoil velocity.

The law of conservation of energy and momentum are two significant laws.

(i) In the derivation of $L = \Delta mc^2$, the law of conservation of energy is taken into account as in eqs. (2) and (3). Similar is the status in the derivation under general conditions.

(ii) Furthermore, the law of conservation of momentum holds good in such cases and can be used to calculate the recoil velocity, V_R. The value of the recoil velocity in this case turns out to be much less, i.e. 5×10^{-33} m/s; thus the body can be regarded as at rest. This case is similar to the recoil of the gun when a bullet is fired, but here two waves are emitted. The momentum is conserved irrespective of the fact that the body remains at rest or recoils after emission of light energy [38].

In case of Einstein's derivation momentum is confirmed in special and general cases.

(i) When a light emitting body remains at rest after emission. In this case the recoil velocity is zero i.e. V_R =0; it is calculated by applying the law conservation of momentum.

(ii) When a light emitting body recoils due to emission of two waves in different directions. In this case the velocity of recoil is non-zero and is calculated by applying the law of conservation of momentum. Einstein did not consider that case.

Calculations of recoil velocity in system (x,y,z)

The recoil velocity V_R can be calculated by applying the law of conservation of momentum. In this case V_R will be non-zero. Now in various calculations eq.(23) will be used as eq. (1) is used in Einstein's derivation.

The law of conservation of momentum can be used to calculate the velocity of recoil in this case too. Let the body of mass 1 kg emit two waves in the visible region of wavelength 5000ºA; it corresponds to energy $2hc/\lambda$ or 7.9512×10^{-19} J. Let this energy be divided in two waves. Let body emits light energy (towards the observer, φ= 0º) 0.50001L

$$E_1 = 3.975607\times10^{-19}\ \text{J} \tag{24}$$

and momentum

$$p_1 = E_1/c = 1.325202\times10^{-27}\ \text{kg m/s} \tag{25}$$

Secondly, the body emits a light wave of energy (away from the observer, φ= 180º) 0.49999L i.e.

$$E_2 = 3.975592\times10^{-19}\ \text{J} \tag{26}$$

$$\text{Momentum}\ p_2 = E_2/c = 1.325197\times10^{-27}\ \text{kg m/s.} \tag{27}$$

Let us assume that when the body emits light waves of energy in the system (x,y,z) and recoils (if it actually does) with velocity V_R (say). The body will recoil opposite in direction to that of the wave having energy E_1 (more energetic wave). Mathematically,

$$\text{Initial momentum of waves + initial momentum of luminous body} = 0 + 0 \tag{28}$$

$$\text{Final momentum of waves + final momentum of body due to recoil} = -p_1 + p_2 - MV_b \tag{29}$$

One wave having momentum p_2 moves towards the direction in which the body recoils and the other wave moves in the opposite direction. Here the energy of wave 1 is E_1 ($E_1 = 3.975607\times10^{-19}$ J, $p_1 = E_1/c = 1.325202\times10^{-27}$ kg m/s), which is more than the energy of wave E_2 ($E_2 = 3.975592\times10^{-19}$J, $p_2 = E_2/c = 1.325197\times10^{-27}$ kg m/s). The wave 1 is emitted towards the observer's system (ξ, η, ζ).

Thus the body recoils in the direction opposite to which wave 1 is emitted. It is like a bullet fired from a gun; then the system recoils in the backward direction. Hence here momentum p_1 (greater in magnitude) having a backward direction to the measuring system is taken as negative.

The direction of V_R is also regarded as the same as that of p_1.The recoil velocity is calculated as 5×10^{-33} m/s; hence relative velocity of the system (ξ, η, ζ) becomes $(v+V_r)$. Then according to law of conservation of momentum we get

$$0 = -p_1 + p_2 - M_b V_r \tag{30}$$

$V_R = (-p_1 + p_2)/M_b = (-1.3252202\times10^{-27} + 1.325197\times10^{-27})\times10^{-27} = -0.000005\times10^{-27}$

$$= -5\times10^{-33} \text{ m/s} \tag{31}$$

The velocity 5×10^{-33} m/s means the body remains at rest. A body is said to at rest if it does not change its position w.r.t. its surroundings. The velocity is too small to be detected. It is analogous to the observation that a car cannot move when head and rear lights are switched on. Thus conservation of momentum requires that a body should move with velocity 5×10^{-33} m/s away from the observer. So the law of conservation of energy is obeyed. The distances travelled by the body in 100 or 10 years can be calculated as,

$$S(100 \text{ years}) = 5\times10^{-33} \text{ m/s} \times 3.14\times10^{7} \times 100 = 1.57\times10^{-23} \text{ m} \tag{32}$$

$$S(10 \text{ years}) = 1.57\times10^{-24} \text{ m}$$

which is undetectable by any means and hence the body can be regarded as at rest. Thus the body will tend to move with velocity 5×10^{-33} m/s (away from the observer) which is immeasurably small or undetectable by any means; hence the body remains at rest by the definition of rest.

This recoil velocity (V_R) i.e. 5×10^{-33} m/s is negligible compared to the velocity of the measuring system i.e. $v+V_R = v+5\times10^{-33}$ m/s = v:

$$\text{Size of nucleus} = 10^{-14}\text{ m} \tag{33}$$

$$S(100\text{ years}) = 1.57\times10^{-9}\text{ size of nucleus} \tag{34}$$

$$S(10\text{ years}) = 1.57\times10^{-10}\text{ size of nucleus} \tag{35}$$

Thus the body moves a distance of 1.57×10^{-24} m which is immeasurable. Hence the body can be regarded as at rest, as in the case of Einstein's derivation when two waves of energy are emitted. Even if this velocity is taken into account for the sake of completeness then the results are the same as in the previous case.

Even greater numerical values are neglected in calculations, e.g. in the relativistic variation of mass

$$M_{motion} = \frac{M_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{15}$$

$$M_{motion} = M_{rest}\left(1+\frac{v^2}{2c^2}+\frac{3v^4}{8c^4}+...\right)$$

$$= M_{rest}\left(1+5.55\times10^{-18}+4.629\times10^{-31}...\right)$$

Here velocity is regarded as 1m/s (3.6km/hr) in the classical region and even the term 5.55×10^{-18} is regarded as negligible, thus $M_{\text{motion}} = M_{\text{rest}}$. Hence recoil velocity 5×10^{-33} m/s is also negligible. Thus equations for recoil momentum and recoil kinetic energy KE recoil will be

$$P_{\text{recoil}} = 5\times10^{-33}\text{ kgm/s} \tag{36}$$

$$KE_{\text{recoil}} = 1.25\times10^{-65}\text{ kgm}^2/\text{s}^2 \tag{37}$$

Due to this uniform relative velocity *v*, the system (ξ, η, ζ) will not change within measurable limits, however the effect of V_R can be considered for completeness. If a body does not change its position so it can be regarded at rest by definition, as $v+5\times10^{-33}$ m/s $=v$, x+ 1.57×10^{-24} m = x, in 10 years. Thus the law of conservation of momentum helps us in calculations of recoil velocity, which changes the magnitude of *v* in eq.(1). Hence the equation of relativistic variation of light energy i.e. Doppler principle for light for any velocities whatsoever, becomes

$$\ell^* = \ell \frac{\left[1 - \frac{v+V_R}{c}\cos\phi\right]}{\sqrt{1-\frac{[v+V_R]^2}{c^2}}} \tag{23}$$

Now if the body recoils then eq.(23) has to be used instead of eq.(1) in the calculation of energy. However the magnitude of eq. (23) and eq.(1) is the practically the same as $v+V_R = v+5\times10^{-33}$ m/s $= v$.

4.0 $L \propto \Delta mc^2$ or $L=A\Delta mc^2$ is equally feasible

In Einstein's derivation [1] all types of energies of a body in *E* and *H* are taken into account. Hence the equation for every energy source or energy content is speculated as $E = \Delta mc^2$. While a body recoils a small amount of heat energy, sound energy etc. may also be produced and energy is dissipated against friction, depending upon velocity of recoil. In Einstein's case recoil velocity is zero. Thus when a body recoils then energies after emission are denoted by E_1 and H_1 or different notations may be given to them. These energies (E_i's and H_i's) include all types of energies and Einstein has applied the law of conservation of energy. Einstein has considered a body emitting two light waves of energy 0.5*L* each just in opposite directions. Let in this case the luminous body emit two light waves of energy 0.500001*L* and 0.499999*L* in system (x,y,z) emitted in opposite directions. As discussed in eq. (31), the body remains at rest in this case also. Then the amount of light energy measured in both systems are related as (equivalent to case of Einstein)

$$E_0 = E_1 + L \tag{2}$$

$$H_0 = H_1 + 0.500001L\beta\left(1 - \frac{v}{c}\cos 0\right) + 0.499999L\left(1 - \frac{v}{c}\cos 180\right) \tag{38}$$

where

$$\beta = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{39}$$

The eqs.(37) and (38) correspond to the law of conservation of energy, like eqs.(2) and (3).

$$H_0 = H_1 + L\beta\left(1 - 0.000002\frac{v}{c}\right) \tag{40}$$

$$H_0 - E_0 = H_1 - E_1 + l\beta\left(1 - 0.000002\frac{v}{c}\right) - L \tag{41}$$

$$\left(H_0 - E_0\right) - \left(H_1 - E_1\right) = L\left[\beta\left(1 - 0.000002\frac{v}{c}\right) - 1\right] \tag{42}$$

$$K_0 - K_1 = L\left[\beta\left(1 - 0.000002\frac{v}{c}\right) - 1\right]$$

$$= L\left(1+\frac{v^2}{2c^2}\right)\left[\left(1-0.000002\frac{v}{c}\right)-1\right]$$

$$= L\left(1-\frac{0.000002v}{c}+\frac{v^2}{2c^2}+\ldots-1\right)$$

$$K_0 - K_1 = L\left(-\frac{0.000002v}{c}+\frac{v^2}{2c^2}\right) \quad (43)$$

$$M_b\frac{v^2}{2} - M_a\frac{v^2}{2} = L\left(-0.000002\frac{v}{c}+\frac{v^2}{2c^2}\right) \quad (44)$$

$$M_b - M_a = \frac{-0.000004L}{cv}+\frac{L}{c^2}$$

$$M_b - M_a = L\left(\frac{-0.000004}{cv}+\frac{1}{c^2}\right) \quad (45)$$

$$\Delta mc^2 = L\left(\frac{-0.000004c}{v}+1\right) \quad (46)$$

$$L = \frac{\Delta mc^2}{\left(-0.000004\frac{c}{v}+1\right)} \quad (47)$$

$$-0.000004\frac{c}{v} = -0.000004\times 3\times 10^8/10 = -120 \quad (48)$$

$$L = \frac{\Delta mc^2}{\left(-120+1\right)} = \frac{\Delta mc^2}{-119} \quad (49)$$

$$L \propto \Delta mc^2 \quad or \quad L = A\Delta mc^2$$

Even if the recoil velocity V_R is included i.e. eq. (23) is taken into account, the results remain the same.

In the derivation Einstein used eq. (1) for relativistic variation of light energy, which was speculated in the previous paper [8]. But this equation is only meant for light energy, not at all for other energies; hence any deduction from it must be applicable for light energy only. The equation

$$\ell^* = \ell \frac{\left[1 - \frac{v}{c}\cos\phi\right]}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

is not meant for

(i) Sound energy. In the Doppler effect change in frequency of sound is estimated, not variation in mass. The speed of sound is 332m/s. Eq. (1) is not associated with any other energy.

(ii) Heat energy. There is no equation like eq. (1) which relates variation of heat energy. Similar is the case for other types of energies.

(iii) Chemical energy.

(iv) Nuclear energy.

(v) magnetic energy.

(vi) Electrical energy.

(vii) Energy emitted in form of invisible radiations.

(viii)Attractive binding energy of nucleus.

(ix) Energy emitted in cosmological and astrophysical phenomena.

(x) Energy emitted in volcanic reactions.

(xi) Energies co-existing in various forms etc.

Then why are results based upon eq. (1) applied to the above energies (i –xi)? The reason is that all energies have a different type of nature, and the energies are not confirmed to obey the same equation i.e. eq.(1).

Einstein initially derived a 'light energy'–mass inter-conversion equation $L = \Delta mc^2$, then speculated the 'every energy' –mass inter conversion equation $E = \Delta mc^2$ from $L = \Delta mc^2$. Eq. (1) is only meant for light energy, not for other energies. Hence speculative transition to $E = \Delta mc^2$ from $L = \Delta mc^2$ is absolutely without any mathematical basis.

4.2 If the measuring system is at rest (*v*=0) and a body emits two light waves as in Einstein's derivation then the derivation is not applicable; *v* can also be zero if system (x,y,z) and system (ξ, η, ζ) move with the same velocity.

However in this case experimentally when light energy is emitted mass decreases. It is a serious limitation of Einstein's derivation. When the measuring system (ξ, η, ζ) is at rest $v = 0$ then

$$l^* = l \tag{50}$$

$$H_0 = H_1 + \frac{L}{2} + \frac{L}{2} \tag{51}$$

$$E_0 = E_1 + L \tag{2}$$

$$\left(H_0 - E_0\right) - \left(H_1 - E_1\right) = 0 \tag{52}$$

As a body is at rest and the measuring system (ξ, η, ζ) is also at rest, then $(H_o - E_o)$ or $(H_1 - E_1)$ cannot be interpreted as kinetic energy. Hence further derivation is not applicable.

Thus a critical analysis of Einstein's derivation leads to many interesting facts. Einstein derived L =Δmc^2 under certain conditions.

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