Particle mass generation from physical vacuum

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We present an approach for particle mass generation in which the physical vacuum is assumed as a medium at zero temperature and where the dynamics of the vacuum is described by the Standard Model without the Higgs sector. In this approach fermions acquire masses from interactions with vacuum and gauge bosons from charge fluctuations of vacuum. The obtained results are consistent with the physical mass spectrum, in such a manner that left-handed neutrinos are massive. Masses of electroweak gauge bosons are properly predicted in terms of experimental fermion masses and running coupling constants of strong, electromagnetic and weak interactions. An existing empirical relation between the top quark mass and the electroweak gauge boson masses is explained by means of this approach.

Keywords: Particle mass generation, physical vacuum, Standard Model without Higgs sector, self-energy, polarization tensor.

Introduction

The Higgs mechanism is the current accepted procedure to generate masses of electrically charged fermions and electroweak bosons in particle physics [1]. The implementation of this mechanism requires the existence of a sector of scalar fields which includes a Higgs potential and Yukawa terms in the Lagrangian density of the model. In the Minimal Standard Model (MSM), the Higgs field is a doublet in the $SU(2)_L$ space carrying a non-zero hypercharge, and a singlet in the $SU(3)_C$ space of color. The Higgs mechanism is based on the fact that the neutral component of the Higgs field doublet spontaneously acquires a non-vanishing vacuum expectation value. Since the vacuum expectation value of Higgs field is different from zero, the Higgs field vacuum can be interpreted as a medium with a net weak charge. In this way the $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken into the $SU(3)_C \times U(1)_{em}$ symmetry [2].

As a consequence of the MSM Higgs mechanism, the electroweak gauge bosons acquire their masses so that the masses depend on the vacuum expectation value of Higgs field, which is a free parameter in the MSM. This parameter can be fixed by means of calculating the muon decay at tree level using the Fermi effective coupling constant. Simultaneously, Yukawa couplings between the Higgs field and fermion fields lead to the generation of masses for electrically charged fermions that depend on Yukawa coupling constants, which also are free parameters in the MSM. These constants can be fixed by means of experimental values of fermionic masses. The above mechanism implies the existence of a neutral Higgs boson in the physical spectrum being its mass a free parameter in the MSM. Because it just exists left-handed neutrinos in the Lagrangian density of the MSM, then neutrinos remain massless after the spontaneously electroweak symmetry breaking.

In the current picture of Higgs mechanism [1], masses of the MSM particles spectrum are generated through interactions between electroweak gauge bosons and electrically charged fermions with weakly charged Higgs field vacuum. However there are some physical aspects in this picture of mass generation that are not completely satisfactory summarized in the following questions: What is the possible description of interactions between fermions and electroweak bosons with Higgs field vacuum? How possible is it to show fundamentally that particle masses are generated by these interactions? Why is it that the origin of particle masses is just related to the weak interaction? Why is it that the most-intense interactions (strong and electromagnetic) are not related to the mass generation mechanism? Why are there no interactions between weakly charged left-handed neutrinos and the weakly charged Higgs field vacuum? Why left-handed neutrinos are massless if they have a weak charge?

All the above questions might have a trivial answer if we only look at things through the current picture of Higgs mechanism. However we are interested in exploring a possible physics behind the Higgs mechanism. On this manner we propose an approach for particle mass generation in which fermions acquire their masses from theirs interactions with physical vacuum and gauge bosons from charge fluctuations of vacuum.

Physical vacuum is the state of lowest energy of all gauge bosons and fermion fields [3]. As it is well known from the covariant formulation of Quantum Field Theory [4], the state of lowest energy of gauge bosons and matter fields has an infinite energy. This physical vacuum is a rich medium where there are processes involving particles and antiparticles with unlimited energy. The physical vacuum is then assumed as a medium at zero temperature which is formed by fermions and antifermions interacting among themselves by exchanging gauge bosons. From a cosmological point of view the physical vacuum can be assumed as an almost equilibrated medium which corresponds to an infinitely evolving vacuum [5, 6].

The fundamental model describing the dynamics of physical vacuum is the Standard Model without the Higgs Sector (SMWHS), which is based on the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry group. We assume that each fermion in the physical vacuum has associated a chemical potential which describes the excess of antifermions over fermions. Then there are twelve fermionic chemical potentials μ_f associated with the six leptons and the six quarks implying an antimatter-matter asymmetry in the physical vacuum. Hence the physical vacuum is considered as a virtual medium having antimatter finite density. This antimatter-matter asymmetry of physical vacuum is related to CP violation by electroweak interactions. Naturally the mentioned asymmetry has an inverse sign respect to the one of the matter-antimatter asymmetry of Universe. The existence of fermionic chemical potentials in physical vacuum does not imply that this vacuum itself carries net charges. This can be understood in a similar way as the existence of the maximal matter-antimater asymmetry of Universe which does not mean that the baryonic matter carries net charges. This idea to have fermionic vacuum densities responsible for distinct fermion masses has also been suggested by [7].

The masses of fermions are obtained starting from their self-energies which give account of the fundamental interactions of massless fermions with physical vacuum [3]. While quark masses are generated by strong, electromagnetic and weak interactions, the electrically charged lepton masses are only generated by electromagnetic and weak interactions and the neutrino masses are generated from the weak interaction. On the other hand, gauge boson masses are obtained from charge fluctuations of the physical vacuum which are described by the vacuum polarization tensors [3].

We use the following general procedure to calculate particle masses: Initially we write one-loop self-energies and one-loop polarization tensors at finite temperature and density, next we calculate dispersion relations by obtaining the poles of fermion and gauge boson propagators, from these dispersion relations we find fermion and gauge boson effective masses at finite temperature and density, finally we identify these effective masses at zero temperature with physical masses. This identification can be performed because the virtual medium at zero temperature is representing the physical vacuum.

From a different perspective other works have intended to show that the inertial reaction force appearing when a macroscopic body is accelerated by an external agent [9]. This reaction force is originated as a reaction by the physical vacuum that opposes the accelerating action [9]. All these works involve just the electromagnetic quantum vacuum and have been able to yield the expression $F = ma$ as well as its relativistic generalization. An expression for the contribution by the electromagnetic quantum vacuum to the inertial mass of a macroscopic object has been found and this has been extended to the gravitational case. Originally these works have used a semiclassical approach

[9] which has been easily extended to a quantum version [10].

We find that the fermion and gauge boson masses are functions of the vacuum fermionic chemical potentials μ_f which are fixed using experimental fermion masses. From the values of all fermionic chemical potentials obtained we calculate the masses of electroweak gauge bosons obtaining an agreement with their experimental values. In this approach for particle mass generation is obtained that left-handed neutrinos are massive because they have a weak charge. The weak interaction among massless neutrinos and the physical vacuum is the source of neutrino masses. Additionally this approach can explain an existing empirical relation between the top quark mass and the electroweak gauge boson masses.

Before considering the case of physical vacuum described by the SMWHS, in section 2 we first show how to obtain gauge invariant masses of fermions and gauge bosons for the case in which the dynamics of the vacuum is described by a non-abelian gauge theory. In section 3 we consider the SMWHS as the model which describes the dynamics of physical vacuum and we obtain fermion (quark and lepton) and electroweak gauge boson ($W^{\pm}$ and Z^0) masses. We have consistently the masses of electroweak gauge bosons in terms of the masses of fermions and running coupling constants of the three fundamental interactions. In section 4 we focus our interest in finding a restriction about the possible number of families. Additionally we predict the mass of the quark top and a highest value for the sum of the square of neutrino masses. Our conclusions are summarized in section 5.

Non-abelian gauge theory case

In this section we first consider a more simple case in which the dynamics of vacuum is described by a non-abelian gauge theory, and in this context we calculate fermion and gauge boson masses. The vacuum is assumed to be a quantum medium at zero temperature constituted by fermions and antifermions interacting among themselves through non-abelian gauge bosons. We also assume that there exist an excess of antifermions over fermions in vacuum. This antimatter-matter asymmetry of vacuum is described by non-vanishing fermionic chemical potentials μ_{f_i}, where f_i represents different fermion species. In this section, for simplicity we will take $\mu_{f_1} = \mu_{f_2} = \ldots = \mu_f$.

The non-abelian gauge theory describing the dynamics of vacuum is given by the following Lagrangian density [11]

$$\mathcal{L} = -\frac{1}{4} F_A^{\mu\nu} F_{\mu\nu}^A + \bar{\psi}_m \gamma^\mu \left(\delta_{mn} i \partial_\mu + g L_{mn}^A A_\mu^A \right) \psi_n, \qquad (1)$$

where A runs over the generators of the gauge group and m, n run over the states of the fermion representation. The covariant derivative is $\mathrm{D}_\mu = \delta_\mu + igT_A A_\mu^A$, being T_A the generators of the $SU(N)$ gauge group and g the gauge coupling constant. The representation matrices L_{mn}^A are normalized by $Tr(L^A L^B) = T(R)\delta^{AB}$ where $T(R)$ is the index of the representation. In the calculation of fermionic self-energy appears $(L^A L^A)_{mn} = C(R)\delta_{mn}$, where $C(R)$ is the quadratic Casimir invariant of the representation [11].

At finite temperature and density, Feynman rules for vertices are the same as those at $T = 0$ and $\mu_f = 0$, while propagators in the Feynman gauge for massless gauge bosons $D_{\mu\nu}(p)$, massless scalars $D(p)$ and massless fermions $S(p)$ are [12]

$$D_{\mu\nu}(p) = -g_{\mu\nu}\left[\frac{1}{p^2+i\epsilon} - i\Gamma_b(p)\right], \tag{2}$$

$$D(p) = \frac{1}{p^2+i\epsilon} - i\Gamma_b(p), \tag{3}$$

$$S(p) = \frac{\not p}{p^2+i\epsilon} + i\not p\Gamma_f(p), \tag{4}$$

where p is the particle four-momentum and the medium temperature T is introduced through the functions $\Gamma_b(p)$ and $\Gamma_f(p)$ which are given by

$$\Gamma_b(p) = 2\pi\delta(p^2)n_b(p), \tag{5}$$

$$\Gamma_f(p) = 2\pi\delta(p^2)n_f(p), \tag{6}$$

with

$$n_b(p) = \frac{1}{e^{|p\cdot u|/T} - 1}, \tag{7}$$

$$n_f(p) = \theta(p\cdot u)n_f^-(p) + \theta(-p\cdot u)n_f^+(p), \tag{8}$$

being $n_b(p)$ the Bose-Einstein distribution function. Fermi-Dirac distribution functions for fermions $n_f^-(p)$ and for anti-fermions $n_f^+(p)$ are

$$n_f^{\mp}(p) = \frac{1}{e^{(p\cdot u\mp\mu_f)/T} + 1}. \tag{9}$$

In the distribution functions (7) and (8), u^α is the four-velocity of the center-of-mass frame of the medium, with $u^\alpha u_\alpha = 1$.

Self-energy and fermion mass

We first consider the propagation of a massless fermion in a medium at finite temperature and density. The finite density of the medium is associated with the fact that medium has more antifermions than fermions. The fermion mass is calculated by following the general procedure that we have described in the introduction.

For a non-abelian gauge theory with parity and chirality conservation, the real part of the self-energy for a massless fermion is written as

$$\mathrm{Re}\,\Sigma'(K) = -a\not{K} - b\not{u}, \tag{10}$$

a and b are Lorentz-invariant functions and K^α the fermion momentum. These functions depend on Lorentz scalars ω and k defined as $\omega \equiv (K \cdot u)$ and $k \equiv [(K \cdot u)^2 - K^2]^{1/2}$. For convenience $u^\alpha = (1, 0, 0, 0)$ and then we have $K^2 = \omega^2 - k^2$, where ω and k can be interpreted as the energy and three-momentum respectively. From (10) we can write

$$a(\omega, k) = \frac{1}{4k^2}\left[Tr(\not{K}\mathrm{Re}\,\Sigma') - \omega Tr(\not{u}\,\mathrm{Re}\,\Sigma')\right], \tag{11}$$

$$b(\omega, k) = \frac{1}{4k^2}\left[(\omega^2 - k^2)Tr(\not{u}\,\mathrm{Re}\,\Sigma') - \omega Tr(\not{K}\mathrm{Re}\,\Sigma')\right] \tag{12}$$

The fermion propagator including only mass corrections is given by [14]

$$S(p) = \frac{1}{\not{K} - \mathrm{Re}\,\Sigma'(K)} = \frac{1}{r}\frac{\gamma^0\omega n - \gamma_i k^i}{n^2\omega^2 - k^2}, \tag{13}$$

where $n = 1 + b(\omega, k)/r\omega$ and $r = 1 + a(\omega, k)$. Propagator poles can be found when

$$\left[1+\frac{b(w,k)}{w(1+a(w,k))}\right]^2 w^2-k^2=0. \tag{14}$$

We observe in (14) that n plays a role similar to that of the index of refraction in optics. To solve the equation (14), $a(\omega,k)$ and $b(\omega,k)$ are first calculated from the relations (11) and(12) in terms of the real part of the fermionic self-energy. The contribution to the fermionic self-energy from the one-loop diagram which can be constructed in this theory is given by

$$\Sigma(K)=ig^2C(R)\int\frac{d^4p}{(2\pi)^4}D_{\mu\nu}(p)\gamma^\mu S(p+K)\gamma^\nu, \tag{15}$$

where g is the interaction coupling constant and $C(R)$ is the quadratic Casimir invariant of the representation. For the fundamental representation of SU(N), $C(R)=(N^2-1)/2N$ [13]. We have that $C(R)=1$ for the U(1) gauge symmetry group, $C(R)=1/4$ for SU(2) and $C(R)=4/3$ for SU(3).

Substituting (2) and (4) into (15), the fermionic self-energy can be written as $\Sigma(K)=\Sigma(0)+\Sigma'(K)$, where $\Sigma(0)$ is the zero-density and zero-temperature contribution and $\Sigma'(K)$ is the contribution at finite temperature and density. Then we have that

$$\Sigma(0)=-ig^2C(R)\int\frac{d^4p}{(2\pi)^4}\frac{g_{\mu\nu}}{p^2}\gamma^\mu\frac{\not{p}+\not{K}}{(p+K)^2}\gamma^\nu \tag{16}$$

and

$$\begin{aligned}\Sigma'(K) &= 2g^2C(R)\int\frac{d^4p}{(2\pi)^4}(\not{p}+\not{K})\\ &\times\left[\frac{\Gamma_b(p)}{(p+K)^2}-\frac{\Gamma_f(p+K)}{p^2}+i\Gamma_b(p)\Gamma_f(p)\right].\end{aligned} \tag{17}$$

If we take only the real part ($\mathrm{Re}\,\Sigma'(K)$) of the contribution at finite temperature and density we obtain

$$\begin{aligned}\mathrm{Re}\,\Sigma'(K) &= 2g^2C(R)\int\frac{d^4p}{(2\pi)^4} \\ &\times\left[(\not{p}+\not{K})\Gamma_b(p)+\not{p}\Gamma_f(p)\right]\frac{1}{(p+K)^2}. \qquad (18)\end{aligned}$$

Now we multiply (18) by either $\not{K}$ or $\not{u}$, then we take the trace and perform the integrations over p_0 and the two angular variables, and finally we find that functions (11) and (12) can be written as

$$a(\omega,k)=g^2C(R)A(w,k,\mu_f), \qquad (19)$$

$$b(\omega,k)=g^2C(R)B(w,k,\mu_f), \qquad (20)$$

where we have used the notation given in [15]. In the last expression, $A(\omega,k,\mu_f)$ and $B(\omega,k,\mu_f)$ are integrals over the modulus of the three-momentum $p=|\vec{p}|$ and they are defined as

$$\begin{aligned}A(\omega,k,\mu_f) &= \frac{1}{k^2}\int_0^\infty\frac{dp}{8\pi^2}\left[2p-\frac{\omega p}{k}\log\left(\frac{\omega+k}{\omega-k}\right)\right] \\ &\times\left[2n_b(p)+n_f^-(p)+n_f^+(p)\right], \qquad (21)\end{aligned}$$

$$\begin{aligned}B(\omega,k,\mu_f) &= \frac{1}{k^2}\int_0^\infty\frac{dp}{8\pi^2}\left[\frac{p(\omega^2-k^2)}{k}\log\left(\frac{\omega+k}{\omega-k}\right)-2\omega p\right] \\ &\times\left[2n_b(p)+n_f^-(p)+n_f^+(p)\right]. \qquad (22)\end{aligned}$$

The integrals (21) and (22) have been obtained using the high density approximation, *i.e.* $\mu_f \gg k$ and $\mu_f \gg \omega$, and keeping the leading terms in temperature and chemical potential [16]. Evaluating these integrals we obtain that $a(\omega, k)$ and $b(\omega, k)$ are given by

$$a(\omega, k) = \frac{M_F^2}{k^2}\left[1 - \frac{\omega}{2k}\log\frac{\omega+k}{\omega-k}\right], \quad (23)$$

$$b(\omega, k) = \frac{M_F^2}{k^2}\left[\frac{\omega^2 - k^2}{2k}\log\frac{\omega+k}{\omega-k} - \omega\right], \quad (24)$$

where fermion effective mass M_F is

$$M_F^2(T, \mu_f) = \frac{g^2 C(R)}{8}\left(T^2 + \frac{\mu_f^2}{\pi^2}\right). \quad (25)$$

The value of M_F given by (25) is in agreement with [17]-[20]. We are interested in the effective mass at $T = 0$, which corresponds precisely to the case in which the vacuum is described by a medium at zero temperature. For this case

$$M_F^2(0, \mu_F) = M_F^2 = \frac{g^2 C(R)}{8}\frac{\mu_f^2}{\pi^2}. \quad (26)$$

Substituting (23) and (24) into (14), we obtain for the limit $k \ll M_F$ that

$$\omega^2(k) = M_F^2\left[1 + \frac{2}{3}\frac{k}{M_F} + \frac{5}{9}\frac{k^2}{M_F^2} + \ldots\right]. \quad (27)$$

This dispersion relation is gauge invariant due to that the calculation has been done at leading order in temperature and chemical potential [16].

It is well known that the relativistic energy in the vacuum for a massive fermion at rest is $\omega^2(0) = m_f^2$. From (27) we have that for $k = 0$ then $\omega^2(0) = M_F^2$ and thereby we can identify the fermion effective mass at zero temperature as the rest mass of the fermion, i. e. $m_f^2 = M_F^2$. So the gauge invariant fermion mass which is generated by the SU(N) gauge interaction of the massless fermion with the vacuum is

$$m_f^2 = \frac{g^2 C(R)}{8} \frac{\mu_f^2}{\pi^2}, \tag{28}$$

where μ_f is a free parameter.

Polarization tensor and gauge boson mass

The gauge boson mass is due to the charge fluctuations of vacuum. This mass is calculated following the general procedure presented in the introduction. The most general form of the polarization tensor which preserves invariance under rotations, translations and gauge transformations is [21]

$$\Pi_{\mu\nu}(K) = P_{\mu\nu}\Pi_T(K) + Q_{\mu\nu}\Pi_L(K), \tag{29}$$

where Lorentz-invariant functions Π_L and Π_T, which characterize the longitudinal and transverse modes respectively, are obtained by contraction

$$\Pi_L(K) = -\frac{K^2}{k^2} u^\mu u^\nu \Pi_{\mu\nu}, \tag{30}$$

$$\Pi_T(K) = -\frac{1}{2}\Pi_L + \frac{1}{2} g^{\mu\nu}\Pi_{\mu\nu}. \tag{31}$$

Bosonic dispersion relations are obtained by looking at the poles of the full propagator which results from adding all vacuum polarization insertions. The full bosonic propagator is [21]

$$D_{\mu\nu}(K) = \frac{Q_{\mu\nu}}{K^2 - \Pi_L(K)} + \frac{P_{\mu\nu}}{K^2 - \Pi_T(K)} - (\xi - 1)\frac{K_\mu K_\nu}{K^4}, \quad (32)$$

where ξ is a gauge parameter. The gauge invariant dispersion relations describing the two propagation modes are found from

$$K^2 - \Pi_L(K) = 0, \quad (33)$$

$$K^2 - \Pi_T(K) = 0. \quad (34)$$

The one-loop contribution to vacuum polarization from the diagram which can be constructed in this theory is given by

$$\Pi_{\mu\nu}(K) = ig^2 C(R) \int \frac{d^4p}{(2\pi)^4} Tr\left[\gamma_\mu S(p) \gamma_\nu S(p+K)\right], \quad (35)$$

where S is the fermion propagator (4). Substituting (4) into (35) the polarization tensor can be written as $\Pi_{\mu\nu}(K) = \Pi_{\mu\nu}(0) + \Pi'_{\mu\nu}(K)$, where $\Pi_{\mu\nu}(0)$ is the contribution at zero density and temperature and $\Pi'_{\mu\nu}(K)$ is the contribution at finite temperature and density.

The real part of the contribution at finite temperature and density to the polarization tensor $\mathrm{Re}\,\Pi'_{\mu\nu}(K)$ is given by

$$\mathrm{Re}\,\Pi'_{\mu\nu}(K) = \frac{g^2 C(R)}{2} \int \frac{d^4p}{\pi^4} \times \frac{(p^2 + p\cdot K)g^{\mu\nu} - 2p^\mu p^\nu - p^\mu K^\nu - p^\nu K^\mu}{(p+K)^2} \Gamma_f(p). \quad (36)$$

Substituting (36) in (30) and (31) and keeping the leading terms in temperature and chemical potential we obtain that for the high density approximation ($\mu_f \gg k$ and $\mu_f \gg \omega$)

$$\mathrm{Re}\,\Pi'_L(K) \quad = 3M_B^2\left[1-\frac{\omega}{2k}\log\frac{\omega+k}{\omega-k}\right], \tag{37}$$

$$\mathrm{Re}\,\Pi'_T(K) \quad = \frac{3}{2}M_B^2\left[\frac{\omega^2}{k^2}+\left(1-\frac{\omega^2}{k^2}\right)\frac{\omega}{2k}\log\frac{\omega+k}{\omega-k}\right], \tag{38}$$

where the gauge boson effective mass M_B is

$$M_B^2(T,\mu_f) = \frac{1}{6}Ng^2T^2+\frac{1}{2}g^2C(R)\left[\frac{T^2}{6}+\frac{\mu_f^2}{2\pi^2}\right], \tag{39}$$

being N the gauge group dimension. The non-abelian effective mass (39) is in agreement with [20]. The abelian gauge boson associated with a U(1) gauge invariant theory acquires an effective mass $M_{B(a)}$ defined by

$$M_{B(a)}^2(T,\mu_f) = e^2\left[\frac{T^2}{6}+\frac{\mu_f^2}{2\pi^2}\right], \tag{40}$$

where e is the interaction coupling constant associated with the U(1) abelian gauge group. The abelian effective mass (40) is in agreement with [22]. Because the vacuum is described by a virtual medium at $T = 0$, then the non-abelian gauge boson effective mass generated by quantum fluctuations of vacuum is

$$M_{B(na)}^2(0,\mu_f) = M_{B(na)}^2 = g^2C(R)\frac{\mu_f^2}{4\pi^2}, \tag{41}$$

and the abelian gauge boson effective mass generated by quantum fluctuations of vacuum is

$$M^2_{B(a)}(0,\mu_f) = M^2_{B(a)} = e^2 \frac{\mu_f^2}{2\pi^2}, \tag{42}$$

in agreement with the result obtained at finite density and zero temperature [23]. For the limit $k \ll M_{B_\mu}$, we can obtain the dispersion relations for the transverse and longitudinal propagation modes [21]

$$\omega_L^2 = M_B^2 + \frac{3}{5}k_L^2 + \dots \tag{43}$$

$$\omega_T^2 = M_B^2 + \frac{6}{5}k_T^2 + \dots \tag{44}$$

We note that (43) and (44) have the same value when the three-momentum goes to zero. We van observe from (43) and (44) that for $k = 0$ then $\omega^2(0) = M_B^2$ and we recognize the gauge boson effective mass as a real gauge boson mass. The non-abelian gauge boson mass is

$$m^2_{b(na)} = M^2_{B(na)} = g^2 C(R) \frac{\mu_f^2}{4\pi^2}, \tag{45}$$

and the abelian gauge boson mass is

$$m^2_{b(a)} = M^2_{B(a)} = e^2 \frac{\mu_f^2}{2\pi^2}. \tag{46}$$

We observe that the gauge boson mass is a function on the chemical potential that is a free parameter on this approach. We note that if the fermionic chemical potential has an imaginary value, then the gauge boson effective mass is given by (45) or (46), and it would be negative [24].

SMWHS case

In this section we follow the same procedure as the previous one. We calculate fermion and electroweak gauge boson masses for the case in which the dynamics of physical vacuum is described by the SMWHS. The physical vacuum is assumed to be a medium at zero temperature constituted by quarks, antiquarks, leptons and antileptons interacting among themselves through gluons G (for the case of quarks and antiquarks), electroweak gauge bosons $W^{\pm}$, non-abelian gauge bosons W^3 and abelian gauge bosons B. In this quantum medium there exist an excess of virtual antifermions over virtual fermions. This fact is described by non-vanishing chemical potentials associated with different fermion flavors. The chemical potentials for the six quarks are represented by $\mu_u, \mu_d, \mu_c, \mu_s, \mu_t, \mu_b$. For the chemical potentials of charged leptons we use μ_e, μ_μ, μ_τ and for neutrinos $\mu_{\nu_e}, \mu_{\nu_\mu}, \mu_{\nu_\tau}$. These non-vanishing chemical potentials are input parameters in the approach of particle mass generation.

The dynamics of the vacuum associated with the strong interaction is described by Quantum Chromodynamics (QCD), while the electroweak dynamics of physical vacuum is described by the $SU(2)_L \times U(1)_Y$ electroweak standard model without the Higgs sector. This last model is defined by the following Lagrangian density

$$\mathcal{L}_{ew} = \mathcal{L}_{YM} + \mathcal{L}_{FB} + \mathcal{L}_{GF} + \mathcal{L}_{FP}, \tag{47}$$

where $\mathcal{L}_{YM}$ is the Yang-Mills Lagrangian density, $\mathcal{L}_{FB}$ is the fermionic-bosonic Lagrangian density, $\mathcal{L}_{GF}$ is the gauge fixing Lagrangian density and $\mathcal{L}_{FP}$ is the Fadeev-Popov Lagrangian density. The $\mathcal{L}_{YM}$ is given by

$$\mathcal{L}_{YM} = -\frac{1}{4} W_A^{\mu\nu} W_{\mu\nu}^A - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \tag{48}$$

where $W_{\mu\nu}^A = \partial_\mu W_\nu^A - \partial_\mu W_\mu^A + g_w F^{ABC} W_\mu^B W_\nu^C$ is the energy-momentum tensor associated with the group $SU(2)_L$ and $F_{\mu\nu} = \partial_\mu B_\nu - \partial_\mu B_\mu$ is the one associated with the group $U(1)_Y$. The $\mathcal{L}_{FB}$ is written as

$$\mathcal{L}_{FB} = i\bar{\mathrm{L}}\gamma^\mu \mathrm{D}_\mu \mathrm{L} + i\psi_R^i \gamma^\mu \mathrm{D}_\mu \psi_R^i + i\psi_R^I \gamma^\mu \mathrm{D}_\mu \psi_R^I, \tag{49}$$

where $\mathrm{D}_\mu \mathrm{L} = (\partial_\mu + i g_e Y_L B_\mu/2 + i g_w T_i W_\mu^i)\mathrm{L}$ and $\mathrm{D}_\mu \mathrm{R} = (\partial_\mu + i g_e Y_R B_\mu/2)\mathrm{R}$, being g_w the gauge coupling constant associated with the group $SU(2)_L$, g_e the one associated with the group $U(1)_Y$, $Y_L = -1$, $Y_R = -2$ and $T_i = \sigma_i/2$. The $SU(2)_L$ left-handed doublet (L) is given by

$$\mathrm{L} = \begin{pmatrix} \psi^i \\ \psi^I \end{pmatrix}_L . \tag{50}$$

Masses of fermions

Initially we consider the propagation of massless fermions in a medium at finite temperature and density. The fermion masses are calculated following the same procedure as on previous section. For a non-abelian gauge theory with parity violation and chirality conservation like the SMWHS, the real part of the self-energy for a massless fermion is [15]

$$\mathrm{Re}\,\Sigma'(K) = -\not{K}(a_L P_L + a_R P_R) - \not{u}(b_L P_L + b_R P_R), \tag{51}$$

where $P_L \equiv \frac{1}{2}(1-\gamma_5)$ and $P_R \equiv \frac{1}{2}(1+\gamma_5)$ are the left- and right-handed chiral projectors respectively. The functions a_L, a_R, b_L and b_R are the chiral projections of Lorentz-invariant functions a, b and they are defined as follows

$$a = a_L P_L + a_R P_R, \tag{52}$$

$$b = b_L P_L + b_R P_R. \tag{53}$$

The inverse fermion propagator is given by

$$S^{-1}(K) = \not{\mathcal{L}} P_L + \not{\Re} P_R, \tag{54}$$

where

$$\mathcal{L}^\mu = (1+a_L)K^\mu + b_L u^\mu, \tag{55}$$

$$\Re^\mu = (1+a_R)K^\mu + b_R u^\mu. \tag{56}$$

The fermion propagator follows from the inversion of (54)

$$S = \frac{1}{D}\left[\left(\mathcal{L}^2 \not{\Re}\right) P_L + \left(\Re^2 \not{\mathcal{L}}\right) P_R\right], \tag{57}$$

where $D(\omega, k) = \mathcal{L}^2 \Re^2$. The poles of the propagator correspond to values ω and k for which the determinant D in (57) vanishes

$$\mathcal{L}^2 \Re^2 = 0. \tag{58}$$

In the rest frame of the dense plasma $u = (1, \vec{0})$, Eq.(58) leads to fermion dispersion relations for a chirally invariant gauge theory with parity violation, as from the case of the SMWHS. Thus fermion dispersion relations are given by [15]

$$\left[\omega(1+a_L) + b_L\right]^2 - k^2\left[1+a_L\right]^2 = 0, \tag{59}$$

$$\left[\omega(1+a_R) + b_R\right]^2 - k^2\left[1+a_R\right]^2 = 0. \tag{60}$$

Left- and right-handed components of the fermion dispersion relations are decoupled relations. The Lorentz invariant functions $a(\omega, k)$ and $b(\omega, k)$ are calculated from expressions (11) and (12) through the real part of fermion self-energy. This self-energy is obtained adding all posible gauge boson contributions admitted by the Feynman rules of the SMWHS.

We work on the basis of gauge bosons given by B_μ, W^3_μ, $W^\pm_\mu$, where the charged electroweak gauge bosons are $W^\pm_\mu = (W^1_\mu \mp iW^2_\mu)/\sqrt{2}$. The diagrams with an exchange of a $W^\pm$ gauge boson induce a flavor change in the incoming fermion i to a different outgoing fermion j.

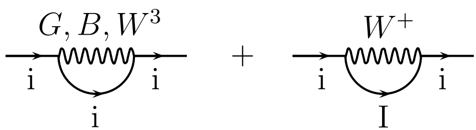

FIGURE 1: Feynmann diagrams contributing to the self-energy of the left-handed quark i.

Quark masses

The quark masses are obtained from flavor change contributions previously mentioned. For the quark sector, the flavor i (I) of the internal quark (inside the loop) runs over the up (i) or down (I) quark flavors according to the type of the external quark (outside the loop). Feynman diagrams at one-loop order which contribute to the self-energy of the left-handed quark i ($i = u_L, c_L, t_L$) are shown in Figure 1. As for each contribution to the quark self-energy is proportional to (21)-(22), the functions a_L, a_R, b_L and b_R are given by

$$a_L(\omega,k)_{ij} = [f_S + f_{W^3} + f_B]A(\omega,k,\mu_i) + \sum_I f_{W^\pm}A(\omega,k,\mu_I), \tag{61}$$

$$b_L(\omega,k)_{ij} = [f_S + f_{W^3} + f_B]B(\omega,k,\mu_i) + \sum_I f_{W^\pm}B(\omega,k,\mu_I), \tag{62}$$

$$a_R(\omega,k)_{ij} = [f_S + f_B]A(\omega,k,\mu_i), \tag{63}$$

$$b_R(\omega,k)_{ij} = [f_S + f_B]B(\omega,k,\mu_i). \tag{64}$$

In the last expressions the coefficients f are

$$f_S = \frac{4}{3}g_s^2\delta_{ij}, \tag{65}$$

$$f_{W^3} = \frac{1}{4}g_w^2\delta_{ij}, \tag{66}$$

$$f_B = \frac{1}{4}g_e^2\delta_{ij}, \tag{67}$$

$$f_{W^\pm} = \frac{1}{2}g_w^2 K_{il}^+ K_{lj}, \tag{68}$$

where K represents the CKM matrix and g_s is the strong running coupling constant associated with the group $SU(3)_C$. The integrals $A(\omega,k,\mu_f)$ and $B(\omega,k,\mu_f)$ are obtained in a high density approximation ($\mu_f \gg k$ and $\mu_f \gg \omega$). These integrals keeping the leading terms in temperature and chemical potential are given by

$$A(\omega, k, \mu_f) = \frac{1}{8k^2}\left(T^2 + \frac{\mu_f^2}{\pi^2}\right)\left[1 - \frac{\omega}{2k}\log\frac{\omega + k}{\omega - k}\right], \quad (69)$$

$$B(\omega, k, \mu_f) = \frac{1}{8k^2}\left(T^2 + \frac{\mu_f^2}{\pi^2}\right)\left[\frac{\omega^2 - k^2}{2k}\log\frac{\omega + k}{\omega - k} - \omega\right] \quad (70)$$

The chiral projections of the Lorentz-invariant functions are

$$\begin{aligned} a_L(\omega, k)_{ij} &= \frac{1}{8k^2}\left[1 - F(\frac{\omega}{k})\right] \\ &\times \left[l_{ij}(T^2 + \frac{\mu_i^2}{\pi^2}) + h_{ij}(T^2 + \frac{\mu_i^2}{\pi^2})\right], \end{aligned} \quad (71)$$

$$\begin{aligned} b_L(\omega, k)_{ij} &= -\frac{1}{8k^2}\left[\frac{\omega}{k} + (\frac{k}{\omega} - \frac{\omega}{k})F(\frac{\omega}{k})\right] \\ &\times \left[l_{ij}(T^2 + \frac{\mu_i^2}{\pi^2}) + h_{ij}(T^2 + \frac{\mu_i^2}{\pi^2})\right], \end{aligned} \quad (72)$$

$$a_R(\omega, k)_{ij} = \frac{1}{8k^2}\left[1 - F(\frac{\omega}{k})\right]\left[r_{ij}(T^2 + \frac{\mu_i^2}{\pi^2})\right], \quad (73)$$

$$\begin{aligned} b_R(\omega, k)_{ij} &= -\frac{1}{8k^2}\left[\frac{\omega}{k} + (\frac{k}{\omega} - \frac{\omega}{k})F(\frac{\omega}{k})\right] \\ &\times \left[r_{ij}(T^2 + \frac{\mu_i^2}{\pi^2})\right], \end{aligned} \quad (74)$$

where $F(x)$ is

$$F(x) = \frac{x}{2}\log\left(\frac{x+1}{x-1},\right) \quad (75)$$

and the coefficients l_{ij}, h_{ij} and r_{ij} are given by

$$l_{ij} = \left(\frac{4}{3}g_s^2 + \frac{1}{4}g_w^2 + \frac{1}{4}g_e^2\right)\delta_{ij}, \tag{76}$$

$$h_{ij} = \sum_l \left(\frac{g_w^2}{2}\right) K_{il}^+ K_{lj}, \tag{77}$$

$$r_{ij} = \left(\frac{4}{3}g_s^2 + \frac{1}{4}g_e^2\right)\delta_{ij}. \tag{78}$$

Substituting (71)-(72) into (59), and (73)-(74) into (60), for the limit $k \ll M_{(i,I)_{L,R}}$ we obtain

$$\omega^2(k) = M^2_{(i,I)_{L,R}}\left[1 + \frac{2}{3}\frac{k}{M_{(i,I)_{L,R}}} + \frac{5}{9}\frac{k^2}{M^2_{(i,I)_{L,R}}} + \dots\right], \tag{79}$$

where

$$M^2_{(i,I)_L}(T,\mu_f) = (l_{ij}+h_{ij})\frac{T^2}{8} + l_{ij}\frac{\mu^2_{(i,I)_L}}{8\pi^2} + h_{ij}\frac{\mu^2_{(I,i)_L}}{8\pi^2}, \tag{80}$$

$$M^2_{(i,I)_R}(T,\mu_f) = r_{ij}\frac{T^2}{8} + r_{ij}\frac{\mu^2_{(i,I)_R}}{8\pi^2}. \tag{81}$$

As it was explained on previous section, we are interested in effective masses at $T = 0$. For this case

$$M^2_{(i,I)_L}(0,\mu_f) = l_{ij}\frac{\mu^2_{(i,I)_L}}{8\pi^2} + h_{ij}\frac{\mu^2_{(I,i)_L}}{8\pi^2}, \tag{82}$$

$$M^2_{(i,I)_R}(0,\mu_f) = r_{ij}\frac{\mu^2_{(i,I)_R}}{8\pi^2}. \tag{83}$$

Keeping the same argument as in section 2, we can identify quark effective masses at zero temperature with the rest masses of quarks. Coming from the left-handed and right-handed representations, we find that masses of the left-handed quarks are

$$m_i^2 = \left[\frac{4}{3}g_s^2 + \frac{1}{4}g_w^2 + \frac{1}{4}g_e^2\right]\frac{\mu_{i_L}^2}{8\pi^2} + \left[\frac{1}{2}g_w^2\right]\frac{\mu_{I_L}^2}{8\pi^2}, \quad (84)$$

$$m_I^2 = \left[\frac{4}{3}g_s^2 + \frac{1}{4}g_w^2 + \frac{1}{4}g_e^2\right]\frac{\mu_{I_L}^2}{8\pi^2} + \left[\frac{1}{2}g_w^2\right]\frac{\mu_{i_L}^2}{8\pi^2}, \quad (85)$$

and the masses of the right-handed quarks are

$$m_i^2 = \left[\frac{4}{3}g_s^2 + \frac{1}{4}g_e^2\right]\frac{\mu_{i_R}^2}{8\pi^2}, \quad (86)$$

$$m_I^2 = \left[\frac{4}{3}g_s^2 + \frac{1}{4}g_e^2\right]\frac{\mu_{I_R}^2}{8\pi^2}, \quad (87)$$

where the couple of indexes (i, I) run over quarks (u, d), (c, s) and (t, b). It is known that the masses of left-handed quarks are the same as the masses of right-handed quarks. This means that left-handed quark chemical potentials μ_{q_L} are different from right-handed quark chemical potentials μ_{f_R}.

If we call

$$a_q = \frac{1}{8\pi^2}\left[\frac{4}{3}g_s^2 + \frac{1}{4}g_w^2 + \frac{1}{4}g_e^2\right], \quad (88)$$

$$b_q = \frac{1}{8\pi^2}\left[\frac{1}{2}g_w^2\right], \quad (89)$$

the expressions (84) and (85) lead us to

$$\mu_{u_L}^2 = \frac{a_q m_u^2 - b_q m_d^2}{a_q^2 - b_q^2}, \quad (90)$$

$$\mu_{d_L}^2 = \frac{-b_q m_u^2 + a_q m_d^2}{a_q^2 - b_q^2}. \quad (91)$$

Naming

$$c_q = \frac{1}{8\pi^2}\left[\frac{4}{3}g_s^2 + \frac{1}{4}g_e^2\right], \quad (92)$$

the expressions (86) and (87) can be written as

$$\mu^2_{u_R} = \frac{m_u^2}{c_q}, \quad (93)$$

$$\mu^2_{d_R} = \frac{m_u^2}{c_q}, \quad (94)$$

and similar expressions for the other two quark doublets (c, s) and (t, b).

If we take the experimental central values for the strong constant as $\alpha_s = 0.1184$, the fine-structure constant as $\alpha_e = 7.2973525376 \times 10^{-3}$ and the cosine of the electroweak mixing angle as $\cos\theta_w = M_W/M_Z = 80.399/91.1876 = 0.88168786$ [8], then $g_s = 1.21978$, $g_w = 0.641799$ and $g_e = 0.343457$. Fixing the central values for quark masses as [8] $m_u = 0.0025$ GeV, $m_d = 0.00495$ GeV, $m_c = 1.27$ GeV, $m_s = 0.101$ GeV, $m_t = 172.0$ GeV, $m_b = 4.19$ GeV, into the expressions (90), (91), (93) and (94), we obtain that the squares of left-handed quark chemical potentials are given by

$$\mu^2_{u_L} = 1.4559 \times 10^{-4}, \quad (95)$$

$$\mu^2_{d_L} = 9.0 \times 10^{-4}, \quad (96)$$

$$\mu^2_{c_L} = 60.7141, \quad (97)$$

$$\mu^2_{s_L} = -5.5280, \quad (98)$$

$$\mu^2_{t_L} = 1.1143 \times 10^{6}, \quad (99)$$

$$\mu^2_{b_L} = -1.0778 \times 10^{5}, \quad (100)$$

and the squares of right-handed quark chemical potentials are

$$\mu^2_{u_R} = 2.4511 \times 10^{-4}, \tag{101}$$
$$\mu^2_{d_R} = 9.6092 \times 10^{-4}, \tag{102}$$
$$\mu^2_{c_R} = 63.254, \tag{103}$$
$$\mu^2_{s_R} = 0.4, \tag{104}$$
$$\mu^2_{t_R} = 1.1602 \times 10^{6}, \tag{105}$$
$$\mu^2_{b_R} = 688.508, \tag{106}$$

where the left- and right-handed chemical potentials are given in GeV^2 units.

Lepton masses

For the lepton sector, the contributions to the fermion self-energy are proportional to (21)-(22) and functions a_L, a_R, b_L and b_R are given by

$$a_L(\omega,k)_{ij} = [f_{W^3} + f_B]A(\omega,k,\mu_i) + \sum_I f_{W^\pm} A(\omega,k,\mu_I), \tag{107}$$
$$b_L(\omega,k)_{ij} = [f_{W^3} + f_B]B(\omega,k,\mu_i) + \sum_I f_{W^\pm} B(\omega,k,\mu_I), \tag{108}$$
$$a_R(\omega,k)_{ij} = [f_B]A(\omega,k,\mu_i), \tag{109}$$
$$b_R(\omega,k)_{ij} = [f_B]B(\omega,k,\mu_i), \tag{110}$$

where $f_{W^\pm} = g_w^2/2$ and the other coefficients f_{W^3} and f_B are given by (66) and (67) respectively.

The dispersion relation for leptons are similar to the relations (79), but in this case the effective masses (80) and (81) are given by

$$M^2_{(i,I)_L}(T,\mu_f) = (l+h)\frac{T^2}{8} + l\frac{\mu^2_{(i,I)_L}}{8\pi^2} + h\frac{\mu^2_{(I,i)_L}}{8\pi^2}, \quad (111)$$

$$M^2_{(i,I)_R}(T,\mu_f) = r\frac{T^2}{8} + r\frac{\mu^2_{(i,I)_R}}{8\pi^2}, \quad (112)$$

where the coefficients l, h and r for the charged leptons are given by

$$l = \left(\frac{1}{4}g_w^2 + \frac{1}{4}g_e^2\right), \quad (113)$$

$$h = \left(\frac{1}{2}g_w^2\right), \quad (114)$$

$$r = \left(\frac{1}{4}g_e^2\right), \quad (115)$$

and for the neutrinos these coefficients are

$$l = \left(\frac{1}{4}g_w^2\right), \quad (116)$$

$$h = \left(\frac{1}{2}g_w^2\right), \quad (117)$$

$$r = 0. \quad (118)$$

The leptonic effective masses (111) and (112) at zero temperature can be interpreted as lepton masses. Coming from left-handed and right-handed representations, we find that masses of left-handed leptons are given by

$$m_i^2 = \left[\frac{1}{4}g_w^2\right]\frac{\mu_{i_L}^2}{8\pi^2} + \left[\frac{1}{2}g_w^2\right]\frac{\mu_{I_L}^2}{8\pi^2}, \tag{119}$$

$$m_I^2 = \left[\frac{1}{4}g_w^2 + \frac{1}{4}g_e^2\right]\frac{\mu_{I_L}^2}{8\pi^2} + \left[\frac{1}{2}g_w^2\right]\frac{\mu_{i_L}^2}{8\pi^2}, \tag{120}$$

and masses of the right-handed charged leptons are

$$m_I^2 = \left[\frac{1}{4}g_e^2\right]\frac{\mu_{I_R}^2}{8\pi^2}, \tag{121}$$

where couple of indexes (i, I) run over leptons (ν_e, e), (ν_μ, μ) and (ν_τ, τ). We observe that the approach predicts that neutrinos are massive. The W^3 and $W^\pm$ interactions among massless neutrinos with physical vacuum are the source for left-handed neutrinos masses, as we can observe from (119).

If we call

$$a_l = \frac{1}{8\pi^2}\left[\frac{1}{4}g_w^2\right], \tag{122}$$

$$b_l = \frac{1}{8\pi^2}\left[\frac{1}{2}g_w^2\right], \tag{123}$$

$$c_l = \frac{1}{8\pi^2}\left[\frac{1}{4}g_w^2 + \frac{1}{4}g_e^2\right], \tag{124}$$

then the expressions (119) and (120) lead us to

$$\mu_{\nu_L}^2 = \frac{c_l m_\nu^2 - b_l m_e^2}{a_l c_l - b_l^2}, \tag{125}$$

$$\mu_{e_L}^2 = \frac{-b_l m_\nu^2 + a_l m_e^2}{a_l c_l - b_l^2}. \tag{126}$$

Calling

$$d_l = \frac{1}{8\pi^2}\left[\frac{1}{4}g_e^2\right], \tag{127}$$

the expression (121) can be written as

$$\mu_{e_R}^2 = \frac{m_e^2}{d_l}, \tag{128}$$

and similar expressions for the other two lepton doublets (ν_μ, μ) and (ν_τ, τ). Assuming that neutrinos are massless $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$, and fixing the experimental central values for charged lepton masses as [8] $m_e = 0.510998910 \times 10^{-3}$ GeV, $m_\mu = 0.105658367$ GeV, $m_\tau = 1.77682$ GeV, into the expressions (125), (126) and (128), we obtain that the squares of left-handed lepton chemical potentials are

$$\mu_{\nu_{e_L}}^2 = 1.4699 \times 10^{-4}, \tag{129}$$

$$\mu_{e_L}^2 = -7.3492 \times 10^{-5}, \tag{130}$$

$$\mu_{\nu_{\mu_L}}^2 = 6.30867, \tag{131}$$

$$\mu_{\mu_L}^2 = -3.1543, \tag{132}$$

$$\mu_{\nu_{\tau_L}}^2 = 1.7841 \times 10^3 \tag{133}$$

$$\mu_{\tau_L}^2 = -8.9205 \times 10^3, \tag{134}$$

and the squares of right-handed charged lepton chemical potentials are

$$\mu_{e_R}^2 = 6.9637 \times 10^{-4}, \tag{135}$$

$$\mu_{\mu_R}^2 = 29.8888, \tag{136}$$

$$\mu_{\tau_R}^2 = 8.4526 \times 10^3, \tag{137}$$

where the left- and right-handed chemical potentials are given in GeV^2 units. Experimental neutrino masses are unknown but experimental results show that neutrino masses are of order 1 eV [8], and cosmological interpretations from five-year WMAP observations find a limit over the total mass of massive neutrinos of $\Sigma m_\nu < 0.6$ eV (95% CL) [25]. These results assure that values of left-handed lepton chemical potentials obtained by taking neutrinos to be massless will change a little if we have the real small neutrinos masses.

We observe that for five from the six fermion doublets the square of the chemical potential associated to the down fermion of the doublet has a negative value. This behavior is observed if there is a large difference between the masses of the two fermions of the doublet. This means that for the quark doublet which is formed by the up and down quarks this behavior is not observed due to that the masses for these two quarks are quite near. In this case, the chemical potentials associated to these two quarks are positive.

From expressions (84), (85), (119) and (120) it can be seen that our approach does not predict fermion mass values owing to the fermionic chemicals potentials μ_{f_i} are free parameters. However, we have fixed the values of these μ_{f_i} starting from the known experimental values for fermion masses. This limitation of our approach is similar to what happens in the MSM with Higgs mechanism in the sense that fermion masses depend on the Yukawa coupling constants which are free parameters. Similarly to occur here when we find the values of vacuum chemical potentials, the Yukawa coupling constants can be fixed by means of the experimental values of fermion masses.

Masses of electroweak gauge bosons

The masses of electroweak gauge bosons are originated from charge fluctuations of vacuum. These masses are calculated following a sequence of steps that we present now: On the outset we write the one-loop polarization tensor at finite temperature and density, then we calculate the one-loop bosonic dispersion relations in the high density approximation by obtaining the poles of gauge boson propagators, next from these dispersion relations we obtain the electroweak gauge boson effective masses at finite temperature and density, finally we identify these effective masses at zero temperature with masses of the electroweak gauge bosons.

To evaluate the bosonic polarization tensor associated with the $W_\mu^\pm$, W_μ^3, B_μ gauge boson propagators, we follow the same procedure as in section 2.2. Applying the expressions (45) and (46) in the SMWHS we obtain that the masses of the gauge bosons are

$$M_{W^\pm}^2 = \frac{g_w^2}{2}\,\frac{S(\mu_{q_L}^2) + \sum_{i=1}^3(\mu_{\nu_{i_L}}^2 - \mu_{e_{i_L}}^2)}{4\pi^2}, \qquad (138)$$

$$M_{W^3}^2 = \frac{g_w^2}{4}\,\frac{S(\mu_{q_L}^2) + \sum_{i=1}^3(\mu_{\nu_{i_L}}^2 - \mu_{e_{i_L}}^2)}{2\pi^2}, \qquad (139)$$

$$M_B^2 = \frac{g_e^2}{4}\,\frac{S(\mu_{q_L}^2) + \sum_{i=1}^3(\mu_{\nu_{i_L}}^2 - \mu_{e_{i_L}}^2)}{2\pi^2}, \qquad (140)$$

where $S(\mu_{q_L}^2) = \mu_{u_L}^2 + \mu_{d_L}^2 + \mu_{c_L}^2 - \mu_{s_L}^2 + \mu_{t_L}^2 - \mu_{b_L}^2$ and the sum runs over the three lepton families. We remind you that if the left-handed fermionic chemical potential has an imaginary value, then its contribution to the gauge boson effective mass, as in the case (45) or (46), would be negative. This fact means that finally the contribution from each fermionic chemical potential to gauge boson masses is always positive.

Substituting the obtained left-handed fermionic chemical potential values (101)-(106) and (129)-(137) into the expressions (138)-(140), we obtain

$$M_{W^{\pm}} = M_{W^3} = 79.9344\,\mathrm{Ge}V, \tag{141}$$

$$M_B = 42.7767\,\mathrm{Ge}V, \tag{142}$$

We observe that the value of M_W is smaller respect to its experimental value given by $M_W^{exp} = 80.399 \pm 0.023$ GeV [8].

Due to well known physical reasons W_μ^3 and B_μ gauge bosons are mixed. After diagonalization of the mass matrix, we obtain that the physical fields A_μ and Z_μ corresponding to massless photon and neutral Z^0 boson of mass M_Z respectively are related by means of [26, 27]

$$M_Z^2 = M_W^2 + M_B^2, \tag{143}$$

$$\cos\theta_w = \frac{M_W}{M_Z} \quad , \quad \sin\theta_w = \frac{M_B}{M_Z}, \tag{144}$$

where θ_w is the weak mixing angle

$$Z_\mu^0 = B_\mu \sin\theta_w - W_\mu^3 \cos\theta_w, \tag{145}$$

$$A_\mu = B_\mu \cos\theta_w + W_\mu^3 \sin\theta_w. \tag{146}$$

Substituting (141) and (142) into (143) we obtain

$$M_Z = 90.6606\,\mathrm{Ge}V, \tag{147}$$

which is also smaller respect to its experimental value given by $M_Z^{exp} = 91.1876 \pm 0.0021$ GeV [8].

Substituting the expressions for the fermionic chemical potentials given by (90), (91), (125), (126) into the expressions (138), (139), (140) we obtain that the masses of the electroweak gauge bosons W and Z are given by

$$
\begin{aligned}
M_W^2 &= g_w^2(A_1 + A_2 + A_3 - A_4), && (148)\\
M_Z^2 &= (g_e^2 + g_w^2)(A_1 + A_2 + A_3 - A_4), && (149)
\end{aligned}
$$

where the parameters A_1, A_2, A_3 and A_4 are

$$
\begin{aligned}
A_1 &= \frac{m_u^2 + m_d^2}{B_1}, && (150)\\
A_2 &= \frac{m_c^2 - m_s^2 + m_t^2 - m_b^2}{B_2}, && (151)\\
A_3 &= \frac{3(m_e^2 + m_\mu^2 + m_\tau^2)}{B_3}, && (152)\\
A_4 &= \frac{(3 + g_e^2/g_w^2)(m_{\nu_e}^2 + m_{\nu_\mu}^2 + m_{\nu_\tau}^2)}{B_3}, && (153)
\end{aligned}
$$

being

$$
\begin{aligned}
B_1 &= \frac{4}{3}g_s^2 + \frac{3}{4}g_w^2 + \frac{1}{4}g_e^2, && (154)\\
B_2 &= \frac{4}{3}g_s^2 - \frac{1}{4}g_w^2 + \frac{1}{4}g_e^2, && (155)\\
B_3 &= \frac{3}{4}g_w^2 - \frac{1}{4}g_e^2. && (156)
\end{aligned}
$$

We observe that M_W and M_Z are written in terms of the masses of fermions and running coupling constants of the strong, weak and electromagnetic interactions.

If we take the experimental central values for the strong running coupling constant as $\alpha_s = 0.1184$, the fine-structure constant as $\alpha_e = 7.2973525376 \times 10^{-3}$ and the cosine of the electroweak mixing angle as $\cos\theta_w = M_W/M_Z = 80.399/91.1876 = 0.88168786$ [8], then $g_s = 1.21978$, $g_w = 0.641799$ and $g_e = 0.343457$. Substituting the values of g_s, g_w and g_e and the values for the experimental masses of the electrically charged fermions, given by [8] $m_u = 0.0025$ GeV, $m_d = 0.00495$ GeV, $m_c = 1.27$ GeV, $m_s = 0.101$ GeV, $m_t = 173.0015$ GeV, $m_b = 4.19$ GeV, $m_e = 0.510998910 \times 10^{-3}$ GeV, $m_\mu = 0.105658367$ GeV, $m_\tau = 1.77682$ GeV, into the expressions (148) and (149), and assuming neutrinos as massless particles, $m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$, we obtain that theoretical masses of the W and Z electroweak gauge bosons are given by

$$M^{th}_{W^\pm} = 79.9344 \pm 1.0208\, GeV, \tag{157}$$

$$M^{th}_{Z} = 90.6606 \pm 1.1587\, GeV. \tag{158}$$

These theoretical masses are in agreement with theirs experimental values given by $M^{exp}_W = 80.399 \pm 0.023$ GeV and $M^{exp}_Z = 91.1876 \pm 0.0021$ GeV [8]. Central values for parameters A_1, A_2, A_3 and A_4 in expressions (148) and (149) are $A_1 = 1.32427 \times 10^{-5}$, $A_2 = 15478$, $A_3 = 34.0137$ and $A_4 = 0$. We observe that A_2 is very large respect to A_3 and A_1. Taking into account the definition of parameter A_2 given by (151) we can conclude that masses of electroweak gauge bosons coming specially from top quark mass m_t and strong running coupling constant g_s.

We obtain also a prediction for top quark mass starting from the expression (149). Using central experimental values $M_W^{exp} = 80.399$ GeV, $M_Z^{exp} = 91.1876$ GeV and considering the uncertainties for running coupling constants and for fermion masses, and assuming neutrinos as massless particles, we predict from (149) that top quark mass is $m_t^{th} = 173.0015 \pm 0.6760$ GeV. This theoretical value is in agreement with the experimental value for top quark mass given by [8] $m_t^{exp} = 172.0 \pm 2.2$ GeV.

Some relations between m_t and M_W, M_Z

The square of the electroweak gauge boson masses M_W and M_Z were written in terms of the fermion masses and the running coupling constants of strong, weak and electromagnetic interactions, such as shown in (148) and (149). If we sum up (148) and (149) we can write that

$$M_W^2 + M_Z^2 = (g_e^2 + 2g_w^2)(A_1 + A_2 + A_3 - A_4). \tag{159}$$

Since the top quark mass m_t is very large in comparison to other fermion masses, it is very easy to prove that

$$A_1 + A_2 + A_3 - A_4 \approx \frac{m_t^2}{B_2}. \tag{160}$$

Substituting (160) into (159) we can obtain that the square of the top quark mass and the squares of the electroweak gauge boson masses are related as

$$m_t^2 = C_1(M_W^2 + M_Z^2), \tag{161}$$

where

$$C_1 = \frac{B_2}{g_e^2 + 2g_w^2}. \tag{162}$$

On the other hand, if we take the square root of expressions (148) and (149) we can prove that

$$(M_W + M_Z)^2 = (g_w + \sqrt{g_w^2 + g_e^2})^2 (A_1 + A_2 + A_3 - A_4). \tag{163}$$

Substituting (160) into (163) and after taking the square root, we can obtain that the top quark mass and the electroweak gauge boson masses satisfy the following relation

$$m_t = C_2(M_W + M_Z) \approx M_W + M_Z, \tag{164}$$

where

$$C_2 = \frac{\sqrt{B_2}}{g_w + \sqrt{g_w^2 + g_e^2}}. \tag{165}$$

Substituting the values $g_s = 1.21978$, $g_w = 0.641799$ and $g_e = 0.343457$ into (162) and (165) we obtain that $C_1 = 2.02843$ and $C_2 = 1.00907$. Using the central values for the electroweak gauge boson masses $M_W^{exp} = 80.399$ GeV and $M_Z^{exp} = 91.1876$ GeV, from (161) and (164) we obtain that $m_t = 173.143$ GeV, which is in agreement with the experimental value for top quark mass.

Rewriting the mathematical relation (164), we can obtain the empirical relation given by

$$\frac{m_t - (M_W + M_Z)}{m_t} = 0.0023, \tag{166}$$

which has been a motive of interest in references [28, 29].

In the approach of particle mass generation that we have introduced on this paper, the top quark has acquired its mass from the interactions with physical vacuum and the electroweak gauge bosons have acquired their masses from the charge fluctuations of physical vacuum. The common origin from vacuum for these particle masses on this approach allow us to give a theoretical explanation to the empirical mass relation given by (166). Some other works in the literature [28, 29] have also suggested that the relation (166) is perhaps more than a mere coincidence.

Conclusions

We have presented an approach for particle mass generation in which we have extracted some generic features of Higgs mechanism that do not depend on its interpretation in terms of a Higgs field. The physical vacuum has been assumed to be a medium at zero temperature constituted by fermions and antifermions interacting among themselves by means of gauge bosons. The fundamental approach describing the dynamics of this physical vacuum is the SMWHS. We have assumed that each fermion flavor in the physical vacuum is associated with a chemical potential μ_f in such manner that there is an excess of antifermions over fermions. This fact implies that the vacuum is thought to be a virtual medium having a net antimatter finite density.

Fermion masses are calculated starting from the fermion self-energy which represents fundamental interactions of a fermion with physical vacuum. The gauge boson masses are calculated from charge fluctuations of physical vacuum which are described by the vacuum polarization tensor. We have used the following general procedure to calculate these particle masses: Initially we have written one-loop self-energies and polarization tensors at finite temperature and density, next we have calculated dispersion relations obtaining the poles of fermion and gauge boson propagators, from here we have obtained the fermion and gauge boson effective masses at finite temperature and density, finally we have identified these particle effective masses at zero temperature as physical particle masses. This identification can be performed because in our approach the medium at finite density and zero temperature represents the physical vacuum.

Using this approach for particle mass generation, we have obtained masses of electroweak gauge bosons in agreement with their experimental values. A further result is that left-handed neutrinos are massive because they have a weak charge. Additionally this approach has given an explanation to an existing empirical relation between the top quark mass and the electroweak gauge boson masses.

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