Physical, metaphysical and logical thoughts about the wave equation and the symmetry of space-time

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D'Alembert's and similar wave equations are not fundamental relations, but result from a continuity equation and internal dynamics. The continuity equation is considered to be an elementary persistent element in a 'permanently changing world' (Heraclitus). Generalizing a reasoning by Euler, the principle of sufficient reason implies inertial motion in a homogeneous and isotropic space-time to be straight and uniform. For an empty as well as an homogeneously and isotropically filled universe, the principle of sufficient reason implies the universe to be spatially and temporarily homogeneous and isotropic (in agreement with Cusanus' metaphysical arguing). With EUCLIDian metric, the coordinate transformation which leads ds^2 invariant is not the Galileo, but the 'Cusanus transformation', a rotation in $\mathbb{R}^4$. The wave equation, however, corresponds to Minkowski's metric. For physical (there is no Galileo space-time) and logical reasons (asymmetry of space and time coordinates), the Galileo transformation is at most a useful approximation in problems, where Galileo invariant equations are useful.

Keywords: Conservation laws, wave equation, symmetry of space-time, Galileo transformation, propagation of electromagnetic fields

Introduction

When discussing the propagation of electromagnetic fields or the range of applicability of the GALILEO transformation, one is inevitably led to wave equations. Indeed, the physical world is full of wave processes. However, wave equations are as little axiomatically founding as NEWTON's equation of motion, $md^2\vec{r}/dt^2 = \vec{F}$, is. It is the continuity equation which is axiomatically comparable with NEWTON's Law 2 in the form $d\vec{p}/dt = \vec{F}$.

Furthermore, the symmetry of an equation of motion should be compatible with the symmetry of the space-time the motion proceeds in. In discussing this issue, inspiration is gained from metaphysical thoughts by HERACLITUS and CUSANUS and from logical reasoning by EULER, while the verification of the results is done according to physical criteria.

Thus, Section 2 encompasses, (i), the continuity equation; (ii), related wave equations; (iii), related relations like GAUSS' law and AMPERE-MAXWELL' flux law, and generalizations of them. Section 3 treats the relationship between the symmetry of space-time and the symmetry of physical laws, in particular, with respect to inertial motion and the metric tensor. Section 4 summarizes and concludes this paper.

HERACLITian foundation of the wave equation

Conservation in a world of permanent change

According to PLATO (428/427 BC - 348/347 BC), HERACLITUS OF EPHESUS (c. 535 - c. 475 BC) stated, "$T\alpha\ \Pi\alpha\nu\tau\alpha\ \rho\varepsilon\iota$" (all things are flowing) [25] [23]. He concluded this from the observation of nature as well as from the rapid changes in the society he lived in [29]. However, if all changes are irregular, science is impossible. In turn, if science exists, there *are* regularities.

Indeed, HERACLITUS' fragment 12 gives reason to doubt PLATO's and other's overestimation of the change as it states, that different waters flow onto people stepping into *the same* rivers. Although their waters change, the rivers remain the same. Therefore, HERACLITUS' understanding of change does support constancy as well. [21]

But which are the persistent elements, or regular patterns, the realization of which lies at the heart of scientific work?

According to HEGEL [23], the essence of that flowing is not the changes, but the becoming, the dialectic solution to the contradiction of being and nothing. The goal of science would thus be the understanding of the evolution laws.

Physics of conservation laws *versus* Physics of equations of motion

On the other hand, the most fundamental laws in DESCARTES', HUYGENS' and NEWTON's representations of mechanics concern the conservation of state (rest and straight uniform motion, respectively) and the change of state (speed, velocity and momentum, respectively). The motion as described through the time-dependent change of position, *eg*, in NEWTON's equation of motion, is *not* part of the axiomatic.

Later, LAGRANGE's and HAMILTON's representations of mechanics put the equation of motion at the forefront. The principle of least action has been favoured by authorities like PLANCK and virtually assumed an axiomatic status.

The results of the representations classical mechanics basing, on the one hand, on conserved quantities and, on the other hand, on equations of motions are equivalent, as first shown by Daniel Bernoulli.

However, the laws of state conservation and of state change are much more general than the equations of motions. For example, the energy conservation law holds true not only in physics, but also in chemistry. In contrast, the equations of motion are different even in different branches of physics. This has brought Bohr (1913), Heisenberg (1925) and Schroedinger (1926) to the conclusion, that the principles of state conservation are the same in classical and quantum physics, while that of state change are different, so that classical mechanics cannot be generalized in a smooth manner.

This holds true, indeed, for Newton's axiomatic, but not for Euler's one which contains only the conservation of states, not their changes. Euler's principles of state change for single bodies can be formulated such, that they apply also to non-relativistic conservative point-mechanical systems and quantum systems. I will not elaborate this in more detail, because in this paper, I'm concentrating on continuous systems.

Continuum theory

If "all is flowing", the elementary persistent elements are reflected in conservation laws of type continuity equation, while the becoming is expressed through evolution laws like inhomogeneous wave equations.

The conservation of scalar and possibly distributed quantities like the electrical charge is described through the rate equation[1]

$$\frac{dq}{dt} = \frac{d}{dt}\iiint\limits_{\Omega} \rho(\vec{r},t)d^3r = -I = -\oint\limits_{\partial\Omega} \vec{j}(\vec{r},t)\cdot d^2\vec{r} \tag{1}$$

or, if the boundary, $\partial\Omega$, is time-independent, through the equation of continuity,

$$\dot{\rho}(\vec{r},t) = -\nabla\vec{j}(\vec{r},t) \tag{2}$$

In order to solve for $\rho(\vec{r},t)$ or for $\vec{j}(\vec{r},t)$, one has to externally impress

[1] Concentrating on the wave equation, I discard any atomism. A more fundamental treatment should be possible in terms of differential forms, where metric and coordinates are not presupposed. This contribution, however, is more interested in the axiomatic status of the Galileo and Lorentz transformations, respectively.

All fields are supposed to be sufficiently smooth; all integrals are taken over simply connected domains and assumed to exists.

$\vec{j}(\vec{r},t)$ or $\rho(\vec{r},t)$, respectively.[2] The alternative is the presence of an internal dynamics as described by constitutive laws. In what follows, diffusive and relaxation processes will be considered.

Laws of motion depending on internal dynamics

Pure diffusion

In a first attempt, one may assume – as published first for heat diffusion by Fourier [19] and for matter diffusion by Fick [18] –, that the flux density is proportional to the density gradient ('Fick's 1st law').

$$\vec{j} = -D\nabla\rho \tag{3}$$

The resulting equation of motion is the diffusion equation ('Fick's 2nd law').[3]

$$\dot{\rho}(\vec{r},t) = D\Delta\rho(\vec{r},t) \tag{4}$$

Diffusion and relaxation

In order to remove the artifact of infinite speed of propagation, one may slow down the instantaneous reaction of the flow to a density gradient by means of a Maxwellian relaxation term [39].

$$\tau\frac{\partial\vec{j}}{\partial t} + \vec{j} = -D\nabla\rho \tag{5}$$

In this case, the resulting equation of motion is the equation of hyperbolic heat transfer, a damped wave equation.[4]

$$\tau\frac{\partial^2\rho}{\partial t^2} + \frac{\partial\rho}{\partial t} = D\Delta\rho \tag{6}$$

It exhibits an intrinsic speed of propagation, $c = \sqrt{D/\tau}$. Like D and τ, it is determined through internal processes and, consequently, is an internal

[2] External volume sources would appear on the r.h.s. of the continuity equation, see below. External sources at the boundary, such as electrodes, require a closed system of equations.

[3] A short historical review is given in [45]. – For simplicity, all 'material coefficients' are treated as being scalar and independent of space and time.

[4] This equation is a starting point for non-equilibrium thermodynamics [41]. It also applies to the Helmholtz-transverse component of the electrical field strength in vacuo. With an additional term proportional to ρ on the l.h.s., it is called the telegrapher's equation. This demonstrates the wide applicability of the approach described here.

parameter, *ie*, it is independent of the manner of excitation of variations of ρ, the propagation of which eq. (6) describes.

Pure relaxation

Finally, for pure relaxation, diffusion*less* intrinsic dynamics, the constitutive law reads

$$\frac{\partial \vec{j}}{\partial t} = -c^2 \nabla \rho \tag{7}$$

leading to the elementary d'ALEMBERTian wave equation.

$$\frac{\partial^2 \rho}{\partial t^2} = c^2 \Delta \rho \tag{8}$$

c persists to be an *internal* parameter; its numerical value is independent of the manner of excitation as well as of the motion of the source. Through $c = \lambda f$, it connects the *external* parameters wavelength, λ, and frequency, f; these are 'external' ones, because they depend on boundary conditions and excitation, respectively.

Inhomogeneous wave equation

In case of external volume sources, P, the continuity equation becomes

$$\frac{\partial \rho}{\partial t}(\vec{r}, t) = -\nabla \vec{j}(\vec{r}, t) + P(\vec{r}, t) \tag{9}$$

There is no physically reasonable possibility to extent the continuity equation through a *non*-local term. All remote sources, $P(\vec{r}', t')$, would sum up to

$$P(\vec{r}, t) = \int_{-\infty}^{+\infty} K(\vec{r}, t; \vec{r}', t') P(\vec{r}', t') d^3 r' dt' \tag{10}$$

where the kernel, $K(\vec{r}, t; \vec{r}', t')$, describes the propagation of the source at the source space-time point, $(\vec{r}', t')$, to the point, $(\vec{r}, t)$, under consideration. Actually, there is even no need for such an extension. For the reaction of the density, ρ, to the production rate, P, is not instantaneous, but 'fully relaxed'. As above, this will lead to a finite speed of propagation of the effect of P upon ρ.

Moreover, the constitutive law may be extended by an external current source, $\vec{J}$, as

$$\frac{\partial \vec{j}}{\partial t}(\vec{r}, t) = -c^2 \nabla \rho(\vec{r}, t) + \vec{J}(\vec{r}, t) \tag{11}$$

For the same reasons by which $P = P(\vec{r}, t)$, there are effectively only *local* current sources: $\vec{J} = \vec{J}(\vec{r}, t)$. And, again, the fact, that not $\vec{j}$ itself, but $\partial\vec{j}/\partial t$ is related to $\vec{J}(\vec{r}, t)$, leads to a finite propagation speed of the effect of $\vec{J}$ upon $\vec{j}$.

Both extensions together lead to the inhomogeneous wave equation

$$c^2\Delta\rho - \frac{\partial^2\rho}{\partial t^2} = \nabla\vec{J} - \frac{\partial P}{\partial t} \tag{12}$$

Here, the signs have been chosen such, that the transition from EUCLIDian to MINKOWSKIan space-time is most natural, see below. Since, when compared with the basic equations, the orders of the time- and space-derivates of ρ have been risen by one, those of P and $\vec{J}$, respectively, have been, too.

Secondary fields and their wave equations

MIE's approach to MAXWELL's equations

The following arguing is pioneered by MIE for electromagnetism, though it is quite general.

The conserved quantity, q, in eq. (1) can be related to a flux, $\vec{D}(\vec{r}, t)$, as

$$q = \iiint\limits_{\Omega} \rho(\vec{r}, t) d^3r = \oint\limits_{\partial\Omega} \vec{D}(\vec{r}, t) \cdot d^2\vec{r} \tag{13}$$

$\vec{D}$ represents the "excitation" of the surrounding of q.

- If ρ is the density of gravitating mass, $\vec{D}$ describes the spatial distribution of the corresponding force of gravity (the "accelerating force" in [43], Definitions). NEWTON's force law follows almost immediately. [13]
- If $\rho(\vec{r}, t)$ is the density of "free electricity", $\vec{D}$ represents the "dielectric displacement" [37][38].

By virtue of Gauss' integral theorem, again, this is equivalent to

$$\nabla\vec{D}(\vec{r}, t) = \rho(\vec{r}, t) \tag{14}$$

Then,

$$\frac{dq}{dt} = \frac{d}{dt}\oint\limits_{\partial\Omega} \vec{D}(\vec{r}, t) \cdot d^2\vec{r} = \oint\limits_{\partial\Omega} \frac{\partial}{\partial t}\vec{D}(\vec{r}, t) \cdot d^2\vec{r} \tag{15}$$

if the surface, $\partial\Omega$, is time-independent, again. Hence,

$$\oint_{\partial\Omega} \left[\vec{j}(\vec{r},t) + \frac{\partial}{\partial t}\vec{D}(\vec{r},t)\right] \cdot d^2\vec{r} = 0 \tag{16}$$

Since

$$\oint_{\partial\Omega} \left[\nabla \times \vec{H}(\vec{r},t)\right] \cdot d^2\vec{r} \equiv 0 \tag{17}$$

we have

$$\vec{j}(\vec{r},t) + \frac{\partial}{\partial t}\vec{D}(\vec{r},t) = \nabla \times \vec{H}(\vec{r},t) \tag{18}$$

$\vec{H}$ may be called the "excitation" (Mie, No. 289) of the surrounding of the "total current density" [38], $\vec{C} = \vec{j} + \partial\vec{D}/\partial t$.

- Within Maxwell's theory, this is AMPERE-MAXWELL's flux law, $\vec{H}$ being the magnetic field strength.

- By analogy, HEAVISIDE [22] has proposed a gravito-electromagnetic theory of gravity, where $\vec{H}$ is related to the gravito-magnetic field strength.[5]

It is often stated, that charge conservation is contained in the inhomogeneous MAXWELL equations.[6] This statement supposes, however, that the MAXWELL equations are more fundamental than, or at equal level with charge conservation. For metaphysical reasons (*eg*, the HERACLITian derivation above) and in view of the fact, that charge conservation persists in quantum electrodynamics, the opposite holds true.

Moreover, the resulting wave equation for $\vec{D}$ rather than for its HELMHOLTZ-transverse component, $\vec{D}_T$, and POYNTING's theorem implicitly assume, that the longitudinal field components behave in the same manner as the transverse ones. However, free electromagnetic waves are purely transverse ones, and the longitudinal photons are quite different from the transverse ones.[7]

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[5] The nowadays gravito-electromagnetic equations represent a linear approximation to EINSTEIN's field equations, see, *eg* [7] [36]. By virtue of the non-linearity of the latters, they differ by a factor of 2 from HEAVISIDE's ones.

[6] MAXWELL's original set of "20 equations for 20 variables" contain both the flux law and the continuity equation, though the current density, $\vec{j}$, means the conduction current only. I agree with HEAVISIDE, that it is compatible with MAXWELL's methodics $\vec{j}$ to include the convection current.

[7] For a more detailed discussion, see [14].

Wave equations for $\vec{D}_T$ und $\vec{H}_T$

For the calculation of $\vec{D}(\vec{r},t)$ and $\vec{H}(\vec{r},t)$, one needs an equation for $\vec{D}_T$ or $\partial\vec{D}_T/\partial t$ and $\vec{H}_L$ or $\partial\vec{H}_L/\partial t$. The derivation of them is beyond the scope of this contribution. In vacuo,

$$\vec{H}_L \equiv \vec{0} \tag{19}$$

$(\nabla\vec{H} \equiv 0)$ and

$$\frac{\partial\vec{H}_T}{\partial t} = -c_0^2\nabla\times\vec{D}_T \tag{20}$$

where c_0 is the speed of light in vacuo.

Separating $\vec{H}_T$ and $\vec{D}_T$, one obtains the inhomogeneous wave equations

$$\Delta\vec{D}_T - \frac{\partial^2\vec{D}_T}{c_0^2\partial t^2} = \frac{\partial\vec{j}_T}{c_0^2\partial t} \tag{21}$$

$$\Delta\vec{H}_T - \frac{\partial^2\vec{H}_T}{c_0^2\partial t^2} = -\nabla\times\vec{j}_T \tag{22}$$

The signs have been chosen such, that the transition from EUCLIDian to MINKOWSKIan space-time is most natural, see below.

All that refers to a CARTESian system of coordinates fixed to the material in which the transport proceeds. Tentatively, the corresponding geometry of space-time is EUCLIDian, because the theory has been developed entirely along classical lines of thought.[8] I will return to this issue below.

Generalization of $\nabla\vec{D} = \rho$

Before leaving this topic, I notice, that eq. (14) can be generalized to

$$\vec{D}_L(\vec{r},t) = -\nabla\phi_{\vec{D}}(\vec{r},t) \tag{23}$$

where $\phi_{\vec{D}}$, the scalar potential of $\vec{D}$, obeys not the POISSON equation

$$\Delta\phi_{\vec{D}}(\vec{r},t) = -\rho(\vec{r},t) \tag{24}$$

but the HELMHOLTZ equation

$$\Delta\phi_{\vec{D}}(\vec{r},t) + \kappa_{\vec{D}}\phi_{\vec{D}}(\vec{r},t) = -\rho(\vec{r},t) \tag{25}$$

where the new coefficient, $\kappa_{\vec{D}}$, depends on the application under consideration [28].

[8]For a deeper foundation, see FELIX KLEIN's 'Erlangen program' [31], according to which the geometry is determined by the transformation group of coordinates.

- In YUKAWA's classical meson theory of nuclear forces [52], $\kappa_{\vec{D}}$ is proportional to the meson mass; here, $\rho(\vec{r},t) = P\delta(\vec{r})$ and

$$\phi_{\vec{D}}(\vec{r},t) = \frac{P}{4\pi r} e^{-r/r_{\vec{D}}}; \quad r_{\vec{D}} = \frac{1}{\sqrt{\kappa_{\vec{D}}}} \tag{26}$$

  The exponential factor makes the nuclear force to be (very) short-range;

- in NEUMANN's [42] and SEELIGER's [48] theories of gravity, $\kappa_{\vec{D}}$ serves to solve the cosmological problem to make the potential of a homogeneous mass density: $\rho = const$, finite.

$$\phi_{\vec{D}} = \frac{\rho}{\kappa_{\vec{D}}} \tag{27}$$

- in systems with a high concentration of charge carriers (strong electrolytes, metals, highly doped semiconductors), the 'naked' COULOMB potential is screened to become (approximately) $\phi_{\vec{D}}$ (26), $r_{\vec{D}}$ being the linear screening length;

- in FRITZ and HEINZ LONDON's extension of MAXWELL's theory to describe superconductivity [34], $r_{\vec{D}}$ is the field penetration depth into the superconductor.

I am not aware of attempts to exploit the freedom in choosing $\kappa_{\vec{D}}$ to adjust the electromagnetic mass of an electron to the experimental value of its rest mass, or to make the electrical field energy of a point-like charge to be finite.

Generalization of $\nabla \times \vec{H} = \partial\vec{D}/\partial t + \vec{j}$

Analogously, eq. (18) can be generalized to

$$\nabla\times\vec{H} = \nabla\times\nabla\times\vec{a}_{\vec{H}} = -\Delta\vec{a}_{\vec{H}} \tag{28}$$

($\nabla \cdot \vec{a}_{\vec{H}} \equiv 0$), where $\vec{a}_{\vec{H}}$, the vector potential of $\vec{H}$, obeys not the vector POISSON equation

$$\Delta\vec{a}_{\vec{H}} = -\vec{C} \tag{29}$$

but the vector HELMHOLTZ equation

$$\Delta\vec{a}_{\vec{H}} + \hat{\kappa}_{\vec{H}} \cdot \vec{a}_{\vec{H}} = -\vec{C} \tag{30}$$

I am not aware of an application, where $\hat{\kappa}_{\vec{H}} \neq \hat{0}$. It is conceivable, however, that, within gravito-electromagnetism [7][36] , $\hat{\kappa}_{\vec{H}}$ plays a role being analogous to that of $\kappa_{\vec{D}}$ in eq. (27).

According to eq. (9),

$$P = -\frac{\partial}{\partial t}\left(\kappa_{\vec{D}}\phi_{\vec{D}}\right) - \nabla\left(\hat{\kappa}_{\vec{H}} \cdot \vec{a}_{\vec{H}}\right) \tag{31}$$

This means, that P is also the source of *another* scalar conserved quantity of density $(\kappa_{\vec{D}}\phi_{\vec{D}})$, the flux of which equals $(\hat{\kappa}_{\vec{H}} \cdot \vec{a}_{\vec{H}})$. The meaning of these relations remains to be explored.

Symmetry of laws versus symmetry of space-time

Inertial motion and the principle of sufficient reason

Let us consider a single body in the sense of EULER [16]; *ie*, the body is impenetrable and cannot exert forces upon itself. This body be located in a NEWTONian space-time ([43] Definitions, Scholium[9]).

> 1. Absolute, true, and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly...
> 2. Absolute space, of its own nature, without reference to anything external, always remains homogeneous and immovable.

Due to the principle of sufficient reason, these properties exhibit essential implications for the laws of motion.

1. If space is isotropic and, (i), the body is at rest, it remains at rest (because there is no reason for starting to move in one direction and not in the opposite one), *cf* [16]; (ii), if the body is in straight motion, it perserveres in straight motion (because there is no preference for any other direction to change to it).

2. If space is homogeneous, the behavior of the body is independent of its position; hence, it perserveres in rest or in uniform motion.

3. If space-time is isotropic and the body is in uniform motion, it perserveres in uniform motion.

[9] Newton has further developed his concepts on space and time in his *Opticks*, see [2].

4. If space-time is homogeneous and isotropic and the body is in straight uniform motion, it perserveres in straight uniform motion.[10]

In what follows, both metaphysical and physical points of view will be explored.

CUSANian space-time

ARISTOTLE (384 BC – 322 BC) [1][26] (and many others) imagined, that space is, (i), inhomogeneous insofar as the Earth represents the distinguished center of the Universe, and, (ii), anisotropic insofar as 'the ordinary things' are attracted towards the Earth, while the natural motion of the other things is flying towards the Heaven. In contrast, the bishop NIKOLAUS VON KUES (CUSANUS, 1401 – August 11, 1464) assumed, that the Lord is everywhere in the Universe in the same manner[11]. In physical words, the Universe is spatially homogeneous and isotropic [30]. The spatial geometry of the Universe is thus EUCLIDian.[12]

Accordingly, if *no* region of the Universe (in particular, *not* the Earth) is distinguished, *no* local history (in particular, *not* that of the Earth) is distinguished as well: the presence of the Lord is one and the same at all times, *all time points being equivalent* (*cf* [30] p.37). This means that the Universe is temporally homogeneous and isotropic, too.[13]

The simplest CUSANian space-time is the one, in which time is dealt with as a fourth spatial dimension. Rotations in space are extended to rotations in space-time. This requires a unique measure for the space and time coordinates through a reference velocity, V_{ref}. With $(x^1 = x, x^2 = y, x^3 = z, x^4 = V_{ref}t)$, a general rotation in space-time reads

$$x^\mu = R^\mu_\nu x'^\nu; \quad \mu, \nu = 1 \ldots 4 \tag{32}$$

where $\hat{R}$ is a 4×4 rotation matrix.

[10] Such a motion corresponds to straight lines in space-time – *cf* [33] –, and, again, there is no reason to change the motion to any other direction and not to the direction opposite to that.

[11] "For this, the universe behaves as if it would have got its center everywhere and its circumfere nowwhere, since its circumfere and its center are God, who is everwhere and nowhere." (quoted after [30], p.36)

[12] Thus, KUES' formulation, that all positions in space exhibit the same distance to the Lord [30][33], should not be taken too literally.

[13] The modern notion of this view is the 'Principle of local position invariance' [50].

The $1+1$ 'CUSANUS transformation' thus reads

$$x = \frac{x' + Vt'}{\sqrt{1 + \frac{V^2}{V_{ref}^2}}}; \quad t = \frac{t' - \frac{V}{V_{ref}^2} x'}{\sqrt{1 + \frac{V^2}{V_{ref}^2}}} \tag{33}$$

The rotation angle equals

$$\varphi = \arctan(\frac{V}{V_{ref}}) \tag{34}$$

The EUCLIDian arc length is conserved.

$$ds^2 = dx^\mu dx_\mu = d\vec{r}^2 + V_{ref}^2 dt^2 = ds'^2; \quad x^\mu = x_\mu \tag{35}$$

The CUSANian metric tensor is the 4×4 unit tensor.

$$\hat{g}^{(C)} = diag(1, 1, 1, 1) \tag{36}$$

With this metric, the spatial LAPLACE operator,

$$\Delta \equiv \nabla^2 = \frac{\partial^2}{\partial \vec{r}^2} \tag{37}$$

is invariant w.r.t. spatial rotations, and the operator

$$\frac{\partial^2}{\partial x^\mu \partial x_\mu} = \frac{\partial^2}{V_{ref}^2 \partial t^2} + \Delta \tag{38}$$

is invariant w.r.t. the CUSANUS transformation (33).

MINKOWSKIan space-time

However, the D'ALEMBERT operator in the wave equations above,

$$\Box \equiv \Delta - \frac{\partial^2}{c^2 \partial t^2} \tag{39}$$

is *not* invariant w.r.t. the CUSANUS transformation (33) but for $V_{ref} = \pm ic$. For imaginary-valued V_{ref}, the CUSANUS transformation (33) becomes the LORENTZ transformation [35][46].[14]

$$x = \frac{x' + Vt'}{\sqrt{1 - \frac{V^2}{c^2}}}; \quad t = \frac{t' + \frac{V}{c^2} x'}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{40}$$

[14] A quite similar transformation had been proposed by VOIGT in 1886 [50], where the denominator, $\sqrt{1 - V^2/c^2}$, appears as factor for the directions perpendicular to $\vec{V}$. The coefficients had been determined such that the wave equation remains unchanged. But ds^2 (41) is not invariant under VOIGT's transformation.

The invariant arc length squared (35) becomes

$$ds^2 = dx^\mu dx_\mu = -c^2dt^2 + d\vec{r}^2 = ds'^2;$$
$$\mu = 0\dots 3; \quad x^0 = ct = -x_0, \quad x^n = x_n = (\vec{r}); \quad n = 1,2,3 \quad (41)$$

the metric thus being MINKOWSKIan [40].

$$\hat{g}^{(M)} = diag(-1,1,1,1) \quad (42)$$

The d'ALEMBERT operator (39) assumes the compact and manifest LORENT covariant form

$$\Box \equiv \partial^\mu \partial_\mu = g^{(M)\mu\nu}\partial_\nu \partial_\mu \quad (43)$$

The source term of the inhomogeneous wave equation (12) fits unintentionally 'automatically' into this scheme as it can be written as 4-vector.

$$J_\mu = (cP, \vec{J}); \quad J^\mu = (-cP, \vec{J}) \quad (44)$$

The inhomogeneous wave equation (12) thus reads

$$\Box\rho = \partial_\mu J^\mu \quad (45)$$

Often, the negative of ds^2 (41) is used.

$$ds^2 = dx^\mu dx_\mu = c^2dt^2 - d\vec{r}^2 = ds'^2; \quad x^0 = ct = x_0, \quad x^n = -x_n = (\vec{r}) \quad (46)$$

It exhibits the advantage that $\tau = ds/c$ immediately provides the proper time. When starting from VOIGT's [50] question, which transformation leaves the wave equation invariant, this sign (the signature of the metric tensor) is largely a matter of taste, however.

There is no GALILEan space-time

According to the GALILEO transformation,

$$\vec{r} = \vec{r}' + \vec{V}t'; \quad t = t' \quad (47)$$

the time coordinates are absolute, independent of space: $dt = dt'$, while the space coordinates are not as they depend on the velocity, $\vec{V}$, of their origin and on time: $d\vec{r} = d\vec{r}' + V dt'$. For this, the GALILEO transformation is not a rotation in space-time, and there is no coordinate-independent GALILEan metric tensor, $g^{(G)}_{\mu\nu}$, such, that

$$ds^2 = g^{(G)}_{\mu\nu} dx^\mu dx^\nu = ds'^2; \quad dx^\mu = (d\vec{r}, dt) \quad (48)$$

In other words, there is *no* 'GALILEan space-time' with an GALILEO-invariant arc length, ds.

Indeed, space and time coordinates are treated quite differently. This may be appropriate for classical mechanics as long as time is the independent variable of motion along trajectories, $\vec{r}(t)$. However, it is *not* suitable for diffusion and wave processes, where space and time coordinates are treated on *equal* footing in the sense that both are the independent variables the fields depend on.

Last but not least, the asymmetry between spatial and temporal coordinates in the GALILEO transformation (48) is doubtful for the following reason. The coordinates of absolute space and absolute time are absolute. The GALILEO transformation thus refers to relative space and relative time coordinates. There is no reason, that the measurement of *relative* space *is* affected by *relative* time, while the measurement of *relative* time is *not* affected by *relative* space.

Summary and Conclusions

D'ALEMBERT's and other wave equations have been derived starting with the question of elementary persistence in a HERAKLITian, permanently changing world. The fundamental equations are of first order in time, as required by HUYGENS' principle [17][11]. The experiment is responsible for the decision, which quantities are conserved this way. As a matter of fact, even charge conservation can be questioned [27][32].

Continuing MIE's approach to the inhomogeneous MAXWELL equations, generalized equations for – from the points of view of the continuity equation – 'secondary fields' like the dielectric displacement and the magnetic field strength have been discussed, including proposals for new applications. In certain cases, the generalized potentials are connected with a *novel conserved quantity*.

The symmetry of the equations of motion is supposed to comply with the symmetry of space-time. The latter one has been examined, starting from CUSANUS' metaphysical point of view. The tempting direct generalization of EUCLIDian space [metric tensor $\hat{g} = diag(1, 1, 1)$] to CUSANian (EUCLIDian) space-time [$\hat{g} = diag(1, 1, 1, 1)$] is not supported by the wave equation, however – in contrast, the latter favours the LORENTZ transformation. MINKOWSKI's euphoria, "that there will no longer be space and time, but only one single thing: space-time" [40] actually applies to CUSANian rather than to MINKOWSKIan space-time. For the corresponding signs in the metric tensor are opposite, and there is a time arrow, but not a space arrow. In contrast, BALASHOV [4] argues that EINSTEINian relativity supports fourdimensionalism.

The GALILEO transformation – although being suggested by NEWTON's equation of motion [15] – is, (i), metaphysically and logically doubtful, because the (measured) relative time affects the (measured) relative space, but *not vice versa*. It is, (ii), physically doubtful, because there is no metric tensor, $\hat{g}^{(G)}$, such, that the arc length squared (48) is invariant under it.

The derivation of the LORENTZ transformation within the approach presented here cannot replace a fully dynamic approach to relativity like DIRAC's one [8][49]. Another alternative consists in the deduction of the space-time metric from the electromagnetic constitutive relations [3]. Anyway, it is necessary to pose some (metaphysical) postulates about space-time and its relationship to matter (including fields).

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