Conservation of State *versus* Change of State

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Stimulated by a paper by Mirosław Zabierowski (Apeiron **17** (2010) 173-182), the axiomatic status of the notions 'state conservation' and 'change of state' in various representations of classical mechanics is considered. There are common principles of state change in non-relativistic classical and quantum mechanics.

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Introduction

In his paper [1], Mirosław Zabierowski stresses that he was not going to criticize my paper [2]. As we both are not native English speakers, I assume that there was a mutual misunderstanding. Therefore, I will concentrate on the rational kernel, the notion of state and the roles of state conservation and change of state in various representations of classical mechanics.

The notion of state is a central notion throughout physics. M. Zabierowski raises the important question of the relationship between conservation of state and change of state.

It is obvious, that both, conservation of state and change of state, build a dialectic unity as the one does not exists without the other, *cf* [3]. Nevertheless, they occur at different places in the various representations of classical mechanics.

Newton's *versus* Laplace's notion of state

Newton's axioms suggest that, for a single body, the state variable is the momentum, $\mathbf{p}$. The (stationary) states of straight uniform motion or rest are represented by $\mathbf{p}$=const$\neq\mathbf{0}$ and $\mathbf{p}=\mathbf{0}$, respectively. The change of state is quantified as $d\mathbf{p}=\mathbf{F}dt$, where $\mathbf{F}$ is an external force acting upon the body.

Euler [4] uses the same notion of state, but treats straight uniform motion and rest more separately. The state variable is the velocity, $\mathbf{v}$. The mass, m, of a body is treated to be constant. The change motion equals $d\mathbf{v}=(\mathbf{F}/m)dt$. The equation of motion [5], $dd\mathbf{r}=(\mathbf{F}/m)dt^2$, does *not* contain the state variable, $\mathbf{v}$. (For more details, see [6].)

In contrast, according to Laplace [7], the state is given through the position, $\mathbf{r}$, and velocity, $\mathbf{v}$. Using the laws of mechanics, the state can be calculated for all times, provided the initial state is known. This notion of state is usually applied in Lagrangian and also in Hamiltonian mechanics. In contrast to the Lagrangian, the Hamiltonian, H, itself is a *Newtonian* state variable, since H=const for stationary states. The change of state is described through the equation(s) of motion.

These observations are at variance with Ad 6. in [1]. "Of course, there are different formulations of mechanics equivalent to the Newtonian one. The most famous are Lagrangian and Hamiltonian formalisms." For there are essential differences between Newtonian, Lagrangian and Hamiltonian mechanics. Despite the ones mentioned above,

- the treatment of friction is quite different;
- quantum mechanics is non-classical Hamiltonian mechanics, whereas equivalent non-classical Newtonian and Lagrangean forms are not known.

Indeed, the next sentence, “They ... are founded on different principles and in different languages than the Newtonian formulation.“ (Ad 6.) effectively states the opposite of the sentence quoted above.

The advantages and disadvantages of both notions of states have have been analyzed elsewhere [8], for this, I turn to the main issue of this contribution.

Conservation of state *versus* change of state

Mirosław Zabierowski [1] continues, “Therefore in my paper not the notion of state, but the notion of the *change* („alteration”) of state (as in Newtonian formulation) is central (is a basic notion).” (Ad 6.), where “Newtonian formulation” refers to its modern understanding (Ad 5.). This is consistent within the nowadays use of Laplace’s notion of state sketched above. Here, the only conserved state is the state at rest; in all other stationary states, the state variables vary along the trajectory.

In Eulers’s representation of classical mechanics, the change of state plays a key role, too. “Science of nature is the science to explore the causes of the changes occurring at the bodies.” [4a] However, the *axiomatic* status of state conservation and of state change are *different*.

In contrast, in the ‘Principia’, both, the manner of conservation of state (Axiom 1) *and* the manner of change of state (Axioms 2 and 3), are axiomatically fixed. This makes it *impossible* to *generalize* Newton’s representation of classical mechanics *without* touching its axiomatic basis.

This impossibility is explicitly visible in Bohr’s [9] and Heisenberg’s [10] pioneering papers. Bohr claims, that the principles of state conservation are the same in classical and in atom physics, while those of state change are different. Heisenberg evokes „reinterpretation“ („Umdeutung“) rather than generalization of classical mechanics.

Now, Bohr is right w.r.t. Newton’s representation of classical mechanics, but *not* w.r.t. Euler’s one, as will be shown next. In the latter, only the description of state conservation is axiomatically fixed (see above), while that of state change depends on the situation under consideration. Due to that, Euler’s representation *can* be generalized *without* touching its axiomatic basis [6].

Common principles of state change for non-relativistic classical and quantum mechanics

More specifically, there are common principles of state change, although the representation of state and the concrete manner of state change vary from branch to branch. The corresponding variables are collected in Table 1.

System	State variable(s), Z	Non-state variables, z	External cause/ influence, F
Point-like body, Newton	momentum, $\mathbf{p}(t)$	position, $\mathbf{r}(t)$	external force, $\mathbf{F}$
Point-like body, Euler	velocity, $\mathbf{v}(t)$	position, $\mathbf{r}(t)$	external force, $\mathbf{F}$
Conservative system	Hamiltonian, $H(\mathbf{p},\mathbf{r},t)$	momenta, $\mathbf{p}(t)$; positions, $\mathbf{r}(t)$	external potential, $\partial V_{ext}/\partial t$
Schrödinger system	$\langle\psi\|\hat{H}\|\psi\rangle/\langle\psi\|\psi\rangle$	wavefunction, $\psi(\mathbf{r},t)$, $\psi^*(\mathbf{r},t)$	external potential, $\partial V_{ext}/\partial t$

Table 1. Newtonian state and non-state variables and external causes/influences for various systems.

For all systems in Table 1, up to first order in *dt*,

1. the change of the state variable, *dZ*, depends only on the external cause, *F*, but not on the non-state variable, *z*; in particular, *dZ*=0, if *F*=0;
2. the change of the state variable, *dZ*, is independent of the current value of the state variable, *Z*, itself;
3. the change of the non-state variable, *dz*, depends on the external cause only indirectly, *viz*., via the state variable;
4. the changes of the state and of the non-state variables are *in*dependent each of another
5. as soon as the external cause vanishes, the system remains in the state assumed at this point of time.

These principles represent a generalisation of Descartes' laws of elastic impact [11], Huygens' basic assumptions of motion [12] and Newton's axioms.

Summary

The conservation of energy applies throughout physics. Therefore, it can enter the axiomatic of all branches of physics. The manner of energy exchange between systems depends on their properties, however. More generally speaking, the conservation of Newtonian state variables in an isolated system is a reasonable axiom, while their change is not. Nevertheless, there are general principles of the relationships between Newtonian state and not-state variables and external causes of state change, see Table 1.

A similar unification using the Laplacean notion of state is not known to me.

"The statement that the mathematization of physics is originated in Newton's dynamics is a truism." ([1] Ad 7.) Here, Mirosław Zabierowski just completely neglects the work of Huygens.

The unification of classical physics benefits more from the original writings of Newton and Euler (and others) than from their nowadays representations (*cf* also [13]).

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