Bose-Einstein versus Maxwell-Boltzmann distributions

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The reply by Sands & Dunning-Davies [3] to my paper [1] is acknowledged concerning the distribution functions.

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In my paper 'Are there physical systems obeying the Maxwell-Boltzmann statistics?'[1] I have written down the well-known probabilities of the 4 possible results of one fair toss of 2 fair coins according to the Maxwell-Boltzmann (MB), Bose-Einstein (BE) and Fermi-Dirac (FD) statistics. The two results

coin 1 = head, coin 2 = tail

and

coin 2 = head, coin 1 = tail

are counting as different in MB and as one and the same in BE and in FD. Then, I have quoted Gibbs [2], that the interchange of two "entirely similar particles" (Ch. XV) does not change (the "phase" of) an ensemble. Hence, these two results are *not* to be counted as different for, say, calculating the entropy of classical gases.

Furthermore, I have argued, that Gibbs' paradox in the mixing of equal classical gases is avoided, iff a permutation-invariant state function is used. Such a state function is, among others, the total momentum (following Newton), the total velocity (following Euler) and the Hamiltonian (following Gibbs), respectively. With a permutation-invariant state function, the interchange of two particles (like the two coins above) does not change the state. Therefore, a classical state function which avoids Gibbs' paradox leads to BE statistics.

I admit that the title is somewhat provocative and that there are gaps in the arguments (I hope, the text above is clearer). Moreover, it was not mentioned, that, nevertheless, the MB distribution function is a well-defined limit case of the BE and FD distribution functions. I agree with Sands & Dunning-Davies [3], that this makes the MB distribution function applicable to classical statistical ensembles. I disagree with them, that this proves the correctness of MB counting. For MB counting implies Gibbs' paradox, and since the latter is a result of incorrect counting of states, MB counting is incorrect. For classical systems, the error is vanishingly small, as the difference between the MB and BE distribution functions is so.

Sands & Dunning-Davies [3] believe, that Swendsen [4] has provided a different resolution of Gibbs' paradox than I have. The opposite is true as Swendsen has required to define the entropy not via the phase space volume, but via the probability of a macrostate. The latter is exactly what I have considered in the coin tossing. BTW, Swendsen's arguing is related to colloids solely what concerns the discontinuity in the transition from unequal to equal classical particles.

In passing, let me note, that the formulation "Boltzmann's definition of entropy, based of course on Boltzmann statistics" [3] is misleading, because the former is actually independent of the manner of counting and, thus, of the latter.

Summa summarum, permutation invariant state functions like the Hamiltonian link Boltzmann's definition of entropy as a probability resulting from state counting with the dynamics of the given system such, that Gibbs' paradox in the mixing of equal classical gases is avoided. MB counting is not permutation invariant and thus to be excluded. However, the numerical error is vanishingly small for classical systems, to which, consequently, the corresponding MB distribution function applies very well. I would like to thank D. Sands and J. Dunning-Davies for having pointed that out.

References

[1] P. Enders, *Are there physical systems obeying the Maxwell-Boltzmann statistics?*, Apeiron **16** (2009) No. 4, 555-556

[2] J. W. Gibbs, *Elementary Principles in Statistical Mechanics*, New York: Scribner 1902

[3] D. Sands, J. Dunning-Davies, *How applicable is Maxwell-Boltzmann statistics?*, Apeiron, this no.

[4] R. H. Swendsen, *Gibbs' Paradox and the Definition of Entropy*, Entropy **10** (2008) No. 1, 15-18; http://www.mdpi.com/1099-4300/10/1/15/pdf