Magnetic and Electric Flux Quanta: the Pion Mass

Peter Cameron cameron@bnl.gov
Brookhaven National Laboratory
Upton, NY 11973

The angular momentum of the magnetic flux quantum is balanced by that of the associated supercurrent, such that in condensed matter the resultant angular momentum is zero. The notion of a flux quantum in free space is not so simple, needing both magnetic and electric flux quanta to propagate the stable dynamic structure of the photon. Considering these flux quanta at the scale where quantum field theory becomes essential, at the scale defined by the reduced Compton wavelength of the electron, exposes variants of a paradox that apparently has not been addressed in the literature. Leaving the paradox unresolved in this note, reasonable electromagnetic rationales are presented that permit to calculate the masses of the electron, muon, pion, and nucleon with remarkable accuracy. The calculated mass of the electron is correct at the nine significant digit limit of experimental accuracy, the muon at a part in one thousand, the pion at two parts in ten thousand, and the nucleon at seven parts in one hundred thousand. The accuracy of the pion and nucleon mass calculations reinforces the unconventional common notion that the strong force is electromagnetic in origin.

Introduction

The following note revisits and expands upon material presented in three earlier notes [1-4]. It continues to address the possibility that the strong force is electromagnetic in origin [3-8].

The entry point in the present approach is the magnetic flux quantum, which is first defined in free space in terms of both electric and magnetic charges, exposing a fundamental topological anomaly in the simplest possible terms.

Consideration of flux quantization in the photon, both electric and magnetic, then results in emergence of the fine structure constant at the part-per-billion limit of experimental accuracy.

Flux quantization in the electron reveals a second broken symmetry, in this case electromagnetic rather than topological, and offers an opportunity to examine the Ward identity and the longitudinal photon.

The pion is defined as a resonant electromagnetic excitation of the electron by the photon. Resonance quality factors of the particle spectrum are plotted as a function of mass, revealing groupings that encompass fermion and boson and lepton and quark, as well as quark flavor. Unlike the lifetime groupings of MacGregor [6], the resonance groupings do not respect quark flavor.

1. The Magnetic Flux Quantum

In condensed matter the magnetic flux quantum is a fundamental constant [9] defined as

$$\Phi = \frac{h}{2e} = 2.06783367 \cdot 10^{-15}\, tesla \cdot m^2$$

The factor of two arises from electron pairing in the associated superconductor. The present note seeks to address flux quanta in free space, and therefore defines a flux quantum corresponding to a single electron

$$\Phi_{B1} = \frac{h}{e} = 4.13566733 \cdot 10^{-15}\, tesla \cdot m^2$$

The magnetic flux quantum might also be defined by Gauss's law (ignoring for now the complication of the Dirac string) as the sum of the flux passing thru the surface of a sphere which surrounds a magnetic charge

$$\Phi_{B2} = g = 4.13566733 \cdot 10^{-15}\, tesla \cdot m^2$$

where the magnetic charge in the SI Weber convention is given by the Dirac quantization condition

$$g = \frac{h}{e}$$

We can then write

$$\Phi_B = \Phi_{B1} = \Phi_{B2} = 4.13566733 \cdot 10^{-15}\, tesla \cdot m^2$$

It is interesting to note that the Gauss definition the flux quantum, while topologically distinct from the monopole, is nonetheless seen to be algebraically identical [4].

$$\Phi_B = g$$

In the simplest terms the monopole can be visualized as a point source with isotropic static radial field lines extending to infinity in three dimensions. The flux quantum is more like a spinor, a two dimensional object with static field lines extending to infinity in the third dimension. It might also be viewed as a magnetic moment with no return flux.

2. Flux Quantization in the Photon

a. The Magnetic Flux Quantum

The energy of a photon whose wavelength is the reduced Compton wavelength of the electron is

$$W_{\gamma e} = h \cdot f_e = 0.51099891 MeV \tag{1}$$

where

$$f_e = \frac{m_e}{h} = 1.23558998 \cdot 10^{20}\, Hz$$

is the electron Compton frequency.

The magnetic field associated with the magnetic flux quantum is

$$B = \frac{\Phi_B}{\pi \cdot \lambda^2} \tag{2}$$

where λ is the radius of the circle to which the quantum is confined. Assuming that the energy of (1) is stored in the magnetic field of (2), one can calculate the value of λ for which the field intensity corresponds to the photon energy

$$\lambda_{\gamma e} = \frac{\Phi_B^2}{\mu_0 \pi \cdot W_{\gamma e}} = 5.20177208 \cdot 10^{-11}\, m$$

which is of course the Bohr radius, at the nine-significant-digit limit of experimental accuracy. The ratio of this to the reduced Compton wavelength of the electron is

$$\frac{\lambda_{\gamma e}}{\lambda\!\!\!^{-}_e} = 137.0359997 = \frac{1}{\alpha} + 0.0000006$$

where the fine structure constant is taken to be

$$\alpha = 0.0072973525693 \qquad 1/\alpha = 137.03599908$$

Here the fine structure constant emerges at the nine digit limit of experimental accuracy as a result of magnetic flux quantization in the photon. In some sense the fine structure constant can be considered to define the length of the wave packet relative to the wavelength.

The energy of the photon depends on wavelength. The magnetic flux transported by the photon does not. It is quantized in units of the 'single electron' magnetic flux quantum.

In the near field it is not the photon wave length that impedance matches to the electron [3], but rather the photon wave *packet* length.

b. The Electric Flux Quantum

The electric flux quantum is not so familiar. The identity of which field (the choices are magnetic or electric) carries the photon energy is not an observable. This requires that electric flux is quantized in the photon as well [10,11]. Like the magnetic flux quantum, there are two ways in which it may be defined. The first applies the ratio of photon field intensities, $E=cB$, to the magnetic flux quantum.

$$\Phi_{E1} = c \cdot \Phi_B = \frac{h \cdot c}{e} = 1.23984188\, mV \cdot mm$$

As in case of the magnetic flux quantum, a second definition of the electric flux quantum is available, making use of Gauss's law.

3. Flux Quantization in the Electron

And here we find a puzzle. Unlike the magnetic flux quanta, the two different definitions of the electric flux quantum give differing numerical values, related by the unexpected appearance again of the fine structure constant at the limit of experimental accuracy.

$$\Phi_{E2} = \frac{e}{\varepsilon_0} = 0.0180951265\, mV \cdot mm$$

The ratio of the two electric flux quanta is

$$\frac{\Phi_{E1}}{\Phi_{E2}} = 68.5179983 = \frac{1}{2\alpha} + .0000029$$

Not only are the two values different and related by the fine structure constant, but in addition a factor of two has crept into the calculation. Thus far we have scrupulously avoided introducing any factors of two. The presence of a factor of two here is therefore thought to be a not-understood result of the physics.

Here again we find a broken symmetry, this time not topological but rather of electromagnetism. There are two electric flux quanta, and only one magnetic. What does one do with the extra electric flux quantum? How does it fit? We suggest here that it is somehow related to the difficulties regarding the Lorentz invariance of the photon and the removal of the longitudinal component via the Ward identity, and hope to address this further at some future time.

4. Flux Quantization in the Pion and Muon

We present the results so far in a simple tabular form.

$$\Phi_B = \frac{h}{e} = g = 4.1356673326 \cdot 10^{-15}\, tesla \cdot m^2$$

$$\Phi_{E1} = \frac{h \cdot c}{e} = g \cdot c = 1.2398418751 mV \cdot mm$$

$$\Phi_{E2} = \frac{e}{\varepsilon_0} = \frac{h}{\varepsilon_0 g} = 0.0180951265 mV \cdot mm$$

where the various flux quanta are defined in terms of both electric and magnetic charge.

As in equation (2) for the magnetic flux quantum, we can calculate the field strengths and energies resulting from these flux quanta when confined to the reduced Compton wavelength of the electron. The fields are

$$B = \frac{\Phi_B}{\pi \cdot \lambda\!\!\!^{-}_e{}^2} = 8.82800983 \cdot 10^9 \, tesla$$

$$E_1 = \frac{\Phi_{E1}}{\pi \cdot \lambda\!\!\!^{-}_e{}^2} = 2.64657077 \cdot 10^{18} \frac{V}{m}$$

$$E_2 = \frac{\Phi_{E2}}{\pi \cdot \lambda\!\!\!^{-}_e{}^2} = 3.86259198 \cdot 10^{16} \frac{V}{m}$$

From these fields there are two ways to calculate the associated energies. The first is via an appropriate test charge

$$W_B^{charge} = \frac{g}{\mu_0} \cdot B \cdot \lambda\!\!\!^{-}_e = 70.0252464 \, Mev \tag{3}$$

$$W_{E1}^{charge} = e \cdot E_1 \cdot \lambda\!\!\!^{-}_e = 1.02199782 \, MeV = 2m_e$$

$$W_{E2}^{charge} = e \cdot E_2 \cdot \lambda\!\!\!^{-}_e = 0.0149157568 \, MeV$$

and the second directly from the fields

$$W_B^{field} = \frac{2\pi}{\mu_0} \cdot B^2 \cdot \lambda\!\!\!^{-}_e{}^3 = 140.050493 MeV \tag{4}$$

$$W_{E1}^{field} = 2\pi\varepsilon_0 \cdot E_1^2 \cdot \lambda\!\!\!^{-}_e{}^3 = 2.04399564 MeV = 4m_e$$

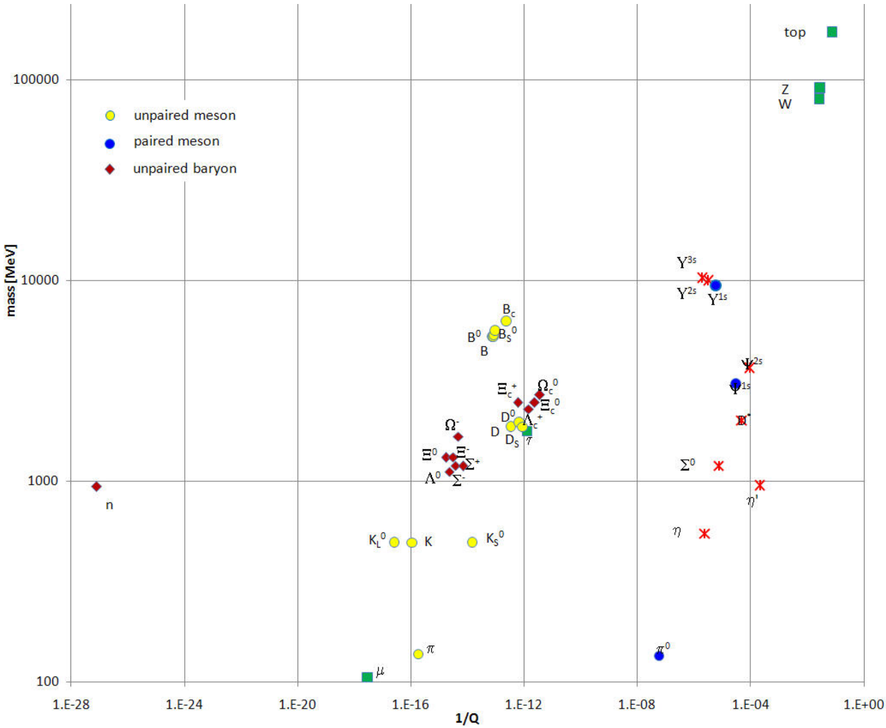
$$W_{E2}^{field} = 2\pi\varepsilon_0 \cdot E_2^2 \cdot \lambda\!\!\!^{-}_e{}^3 = 0.0298315135 MeV$$

Again there are factors of two bounding about, and now factors of four as well. At the nine significant figure limit of experimental accuracy the larger of the two electric flux quanta, when confined to the Compton wavelength of the electron, has either two (test charge) or four (field only) times the rest mass energy of the electron.

Imagine you are the photon whose wavelength is the electron Compton wavelength, cruising along in 'free space' some distance from a single free electron. Your wave packet (at one sigma?) extends some multiple of ~137 times your Compton wavelength 'size'. You feel that electron from a long way off, starting to load you as you are starting to jiggle it. If you were free to modulate your wavelength you could scan it at all frequencies, make a full transfer function measurement. As it is you can scan it only at one frequency. At this energy you jiggle it and bounce off. Your photon wave function is reasonably well matched to the quantum impedance, and exchange of energy is easy.

Now imagine that you are the photon whose wavelength is ~137 times shorter, whose *packet* length is the Compton wavelength of the electron. You just barely know the electron is there before you and your 140MeV slam into it. It rings like a bell in *pi* mode. From the figure below it can be seen that the Q of that mode is a little less than $\sim 10^{16}$ (the alert reader will note that 1/Q is plotted there, rather than Q).

Your electric and magnetic flux quanta are somehow captured and contained by that pi mode of the electron, oscillating for a few tens of nanoseconds before decaying to the muon and muon neutrino. The change in statistics [12] during the transformation from pi to mu suggests that the neutrino contains a single flux quantum. We speculate here that one can't interact with this propagating single flux quantum because it can't satisfy the combination of Maxwell's equations and angular momentum conservation during interactions with charges or fields. Hence the neutrino coupling constant is almost infinitesimal.



As can be seen from the figure, the muon Q is a little higher than that of the pion. The bosonic pi couples more easily to electromagnetic vacuum than fermionic mu. The mu mode survives a couple microseconds before the energy bounces back down to electron plus electron and muon neutrinos, the two neutrinos maintaining the fermionic character of the muon in the electron decay product.

There is some sense in which both the electron and the pion are present in the initial excited state. The pion is a boson, the electron a fermion, so (unlike the case of the muon) they interfere destructively in determining the pion rest mass.

$$m_{\pi Calc} = W_B^{field} - m_e = 139.5395 MeV$$

where the field energy of the above equation is computed in equation (4). This is the experimentally determined pi mass to two parts in ten thousand.

$$\frac{m_\pi - m_{\pi Calc}}{m_\pi} = 0.00022$$

Turning now to the fermionic muon, it interferes constructively with the electron, so that its mass is

$$m_{\mu Calc} = \frac{3}{2} W_B^{charge} + m_e = 105.55 MeV$$

where the 70MeV mass quantum is computed in equation (3). The presence of the 3/2 factor is not yet fully understood [13]. The calculated mass agrees with the experimentally determined mu mass to one part in one thousand.

$$\frac{m_\mu - m_{\mu Calc}}{m_\mu} = 0.00104$$

5. The Nucleon Mass

It has been suggested that the origin of mass is somehow related to spin [2,8]. After the neutron, the next most stable particle is the muon. If we take the muon as a platform state for the nucleon, in terms of spin-related phenomena we return here to the notion that the flux quantum is similar to a magnetic moment with no return flux, and consider the ratio of the magnetic flux quantum to the muon Bohr magneton

$$ratio_{\mu} = \frac{\Phi_B}{\mu_{\mu Bohr}} = 4.6103299 \frac{kg}{coul^2}$$

where

$$\mu_{\mu Bohr} = \frac{e \cdot \hbar}{2m_{\mu}}$$

The nucleon mass can then be calculated as

$$m_{nucleonCalc} = ratio_{\mu} \cdot e^2 \cdot \sin\left(\frac{\pi}{4}\right) = 938.8555 MeV$$

where the sin(π/4) term might be regarded as a projection operator. Taking the measured nucleon mass to be the average of the proton and the neutron, we then have the calculated nucleon mass accurate to seven parts in one hundred thousand.

$$\frac{m_{nucleon} - m_{nucleonCalc}}{m_{nucleon}} = 0.000067$$

This calculation is more speculative than those presented earlier. It is offered as much to stimulate thought as to assert a belief in the reality of the mechanism. Just the same, both the mechanism and the accuracy of the calculation are interesting.

6. Impedance Matching

The preceding material presented a brief exposition of flux quanta and their possible role in particle mass generation. With the pion mass calculation the role of electromagnetism in the strong force is made explicit. In this scheme the unstable particle spectrum (all except photon, electron, proton, and perhaps neutrino) is seen as families of resonant mode excitations of the electron by the photon. Excitation of those modes benefits from proper impedance matching [3,14-17].

The near field impedance of the photon is understood. The electron impedances are more complicated, and will be addressed in a note to follow. Here we mention that a variety of electron impedances can be derived from consideration of the two body problem and Mach's principle [2], and that it may be useful to consider parametric resonance [18] in the impedance matching.

7. The Top, W, and Z

It has been suggested [6] that the relation between the masses of the top, W, and Z is perhaps more than mere coincidence.

$$\frac{m_{top} - (m_W + m_Z)}{m_{top}} = 0.0023$$

The uncertainty in the measurement of the top mass is about 1%. Within the limit of experimental accuracy, the top mass is equal to the sum of the W and Z masses.

Turning again to the plot of 1/Q vs. mass, it can be seen that the relative Q's of top, W and Z suggest that the top is the W and Z resonating in parallel. All three have Q's of a few tens at most. They are incredibly evanescent, just barely qualify as resonances, just barely exist.

8. The Dirac Equation

It was suggested in an earlier note [4] that the electric and magnetic flux quanta might be interpreted as the two Dirac spinors in the four component bi-spinor [19,20]. This author has spent a good portion of the past year trying to understand enough of the Dirac equation to properly evaluate the possible truth of that suggestion, with neither success nor discouragement. There is hope as well that some smart Dirac equation expert will read this note and settle that question.

9. Conclusion

The physics literature outside the standard model is vast. Many of the ideas presented here exist in the literature in various forms. While the author has sought to give credit where appropriate, it is also necessary to apologize to all of the many who have advanced systems of thinking that are similar to that presented here and are not properly cited.

10. Acknowledgements

The author thanks Michaele Suisse for innumerable literature searches, and again thanks Malcolm MacGregor for the body of his work and helpful discussions.

References

[1] The most expedient route for the interested reader to become familiar with literature related to the present note is perhaps to google ‘photon impedance’. At the time of this writing such a search delivers reference [3] below as the first hit. From there one simply clicks on the ‘related articles’ link.

[2] P. Cameron, “The Two Body Problem and Mach’s Principle”, submitted to Am. Jour. Phys. (1975), in revision.

[3] P. Cameron, “Photon Impedance Match to a Single Free Electron”, this journal, July 2010.

[4] P. Cameron, “Possible Origin of the 70MeV Mass Quantum”, this journal, July 2010.

[5] M. H. MacGregor, Lett. Nuovo Cimento **4**, 1309-1315 (1970), Table V.

[6] Malcolm H. MacGregor, The Power of Alpha, World Scientific (2007).

[7] ibid., “The main theoretical idea we are trying to establish in the present work is the relevance of the fine structure constant as a coupling constant in quantum chromodynamics”, p.307.

[8] J. Perenboom and C. Peters, “Consequences of Lorentz Invariance: A Road towards First Principle Standards for Magnetic Field and Mass”, 2010 Conf. Prec. EM Measurements, Korea.

[9] all fundamental constants are CODATA 2006, excepting the fine structure constant. The value of the alpha is taken from G. Gabrielse, "Determining the Fine Structure Constant", available at http://www.itp.uzh.ch/~nurhana/PPP1_HS09/Lecture_Notes/DeterminingTheFineStructureConstant.pdf

[10] J. Perenboom and C. Peters, "The Nature of Magnetic Fields and the Flux Quantum", Yamada Conf. LX on Research in High Magnetic Fields, Jour. Phys Conf. Series 51(2006) 545-548.

[11] http://en.wikipedia.org/wiki/Quantum_LC_circuit

[12] Frank Wilczek, Fractional Statistics and Anyon Superconductivity, p.7, World Scientific (1990).

[13] The relativistic spinning sphere model of [6] has some intuitive appeal and is numerically correct, but is difficult to connect with the present view based on flux quanta.

[14] Jorge Seminario, "Quantum current-voltage relation for a single electron", J. Phys. B: At. Mol. Opt. Phys. **40** (2007) F275–F276 . In this reference one finds a simple derivation of the quantum resistance from the uncertainty principle.

[15] S.M. Girvin et al, "Quantum Fluctuations and the Single-junction Coulomb Blockade", PRL Vol.64, No.26, p.3183-3186, 25 June 1990. Uncertainty considerations arise again here in determining the onset of Pauli blocking at the quantum Hall resistance.

[16] T. Datta and R. Tsu, "Quantum Wave Resistance of Schrodinger Functions", arXiv:cond-mat/0311479v1.

[17] R. Tsu and T. Datta, "Conductance and Wave Impedance of Electrons", Prog. in Electromagnetics Res. Symp., Hangzhou, China, March 2008.

[18] B. Ya Zeldovich, "Impedance and Parametric Excitation of Oscillators", Physics-Uspekhi **51** (5) 465-484 (2008).

[19] R. Penrose, The Road to Reality, Alfred A. Knopf, New York, 2005, p628.

[20] G. A. Perkins, "The Direction of the Zitterbewegung: A Hidden Variable", Foundations of Physics, Vol. 6, No. 2, 1976.