Through the Looking-Glass to the Möbius Strip

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A question of mirror symmetry of the universe is considered in terms of the relationship between observer and the observed system. By including observer into the system, the handedness becomes a purely relative property; thereby the system represents both left and right spatial orders. Topologically interpreted, this logical conclusion suggests that universe represents a multiply connected space of the properties related to Möbius strip.

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One of the most intriguing cases of human experience is connected with looking at the mirror. This, otherwise trivial activity, gives us a possibility of reflection that goes far beyond the practical use, and reveals the obscure side of the obviousness. Almost everyone experienced a weird feeling trying to identify him/herself with the figure reflected in the mirror. Whereas the right side becomes left and the left side becomes right, one may ask: what does exactly the change consist in? Does it make any difference to be composed according to the spatial order of that observed in the mirror if the whole world changes in the same way?

Most of physical objects accessible to observation, beginning from particles, are "chiral", that is to say are characterized by "chirality" or "handedness". In short, handedness is a property that distinguishes a given object from its mirror image. The spatial inversion (P-symmetry) transforms the object to its mirror image by flipping the sign of one, three or (in the general case) uneven number of spatial coordinates. Some of the physical "odd" quantities (e.g. position, force vector, velocity vector, helicity) change upon P-symmetry, wile the other "even" quantities (e.g. mass, energy,s time) remain unchanged. The object "preserves" parity if, in spite of changing its odd quantities, it remains physically indistinguishable from its mirror equivalent. A widely discussed problem in physics sounds: is the parity always preserved? Up to the quite recent times, one supposed that, at the level of fundamental laws, the nature does not distinguish between the right and left handedness. In other words, one supposed that all the fundamental interactions were invariant under P-symmetry. Therefore, the discovery of the parity violation predicted [1] and then detected in experiments [2] became a real surprise. It appeared eventually that nature, in a very narrow domain of weak interactions, is handedness-sensitive.

Logically, there are two ways to restore the lost symmetry, both intensively investigated in the last decades. The first one consists in coupling parity with other fundamental symmetries; exploring this way led to conclusion that, in all probability, only the combined symmetry CPT is always satisfied. The second way (that aims at preserving the parity by itself) postulates the existence of a spatially inverted matter (known as mirror matter or Alice matter) that would represent the handedness opposite to that of ordinary matter. The Alice matter is expected to interact with the ordinary matter through gravity and the photon-mirror photon kinetic mixing interaction. This involves certain observational predictions tested in the ongoing or planned experiments, in particular connected with dark matter theories [3,4].

There is, however, still another way to save parity that, to a certain degree, is neutral with regard to the question of a hypothetical "hidden mirror sector". We aim to show that, even in the case of very small abundance of mirror matter in the universe (as some theories predict), or its entire absence, the parity remains preserved in a certain described manner. Although based on pure logic, this way leads however to some definite observational predictions.

The key feature of this approach consists in including the observer to the observed system. It has been known "since ever" that any effective searches of the nature require experiments. This, of course, involves the presence of an observer provided with abilities of perceiving and interpreting. However, not until quantum mechanics it became evident that the meaning of the observer is much more essential than one previously thought. The standard examples illustrating this new inalienable role of observer are: the double slit experiment, the Heisenberg uncertainty principle and the Schrodinger's cat thought experiment (in general, the collapse of wave function). It is still a matter of dispute whether the concept of an observer is necessarily connected with (human) consciousness. However, no matter how this dispute ends (if it ends whenever), the main conclusion is: *The observer is always a part of experiment.* Hence: *The observer cannot be excluded from the observed system.* This, of course, refers to the quantum level of cognition, which means that, for instance, including Newton to his observation of the falling apple would be senseless. However, the quantum level is not a separate point of view directed on a certain cutaway of the nature only, but, on the contrary, it should be comprehended as a fundamental level of cognition. What we suggest here is that the question of parity should be also (i.e. apart from its other aspects) considered from that fundamental point of view. Thereby, the parity-violation in weak interactions, as well as the relation of parity to other symmetries, is a separate question, different from the problem considered herein.

Whatever is the role of the human consciousness, we shall not insist on identifying the observer with human being. The important is that we treat the observer as a point of reference located within the system, and that the system, in an ultimate case, extends on the whole universe.

Thinking of the observer in terms of “detecting machine” enables to realize the logical relationship between the observer and the observed object. Say, we have a very simple device serving to detect the handedness (wrong or correct) of the typewriting. It could be a kind of a “sensitive” plastic band with type-pattern applied on it. If this pattern, as put against the typescript, does not fit to the handedness of typewriting a signal turns on. Say now, we substitute the typewriting with the mirror-reflected one but, at the same time, we also change the type-pattern on the band in the same manner. Then, of course, the device recognizes again the typewriting as correct. Such a behavior of the device exactly corresponds with the general correlation between the observer and the observed system. No matter how sophisticated the device is (even as a human being, after all) it is however unable to detect the change of the handedness if it subjects this change too.

There are good reasons to think of the handedness problem in terms of an actual cognitive framework determining our reception of “reality”. Admittedly, we are able to imagine the whole surroundingus-universe in a shape of mirror reflected model, “different” from “our” universe. Any chiral object may have its mirror equivalent (fictitious or real), in some regard different from the original. For instance, the earth organisms assimilate the “right” sugar but do not assimilate the “left” one. Nevertheless, if a thought operation concerning the parity inversion involves us as the observers, we are deprived of a criterion enabling to settle which of the two opposite versions of the universe we consist and observe. In other words, in spite of the illusion of thinking of separate things, the end-point of such experiment is indistinguishable from its starting point.

Let us explain it again using the example of parity violation in the beta decay of cobalt-60. What we observe there is that the “left” decay is different from the “right” decay. Let us describe (purely conventionally) this difference naming the left decay “warmer” and the right one “colder”. Say next, we (as observers) are “left”, which means that our handedness is (again conventionally) “concordant” with the “warmer” decay and “discordant” with the “colder” one. Now, let the pair of “left” and “right” cobalt-60 samples be spatially inverted (we may observe it in the mirror). Of course, we observe that parity is still violated, but now the “right” decay is “warmer”, so that our handedness is concordant with the “colder” decay. Eventually, let the spatial inversion include us (the observers) either. In result of that, our handedness becomes concordant with the “warmer” decay, as it was in the beginning. Hence, “left” and “right” appear the purely relative descriptions.

Since the “right” and “left” universes appear identical except their handedness, they both may consist a pair of “twin universes” inhabited by “twin observers”. If one insists, however, on ascribing the term “universe” to the whole physical reality, then the above solution would mean that the universe splits for two copies described by the opposite handedness. However, a solution of the “handedness paradox" implying the existence of two separate systems is not the only possible one and, considering the Occam's razor, probably not the best one. The opposite handedness has not to be necessarily comprehended in terms of different "real" properties, irreducible to each other. It can be, as well, conceived as the purely relative property of a single one system, described by the superimposing space orders. We pronounce for such a version of the "split universe" solution, and suggest it may have cosmological reference connected with topology.

Considered as topological model, the universe can be either simply connected (as e.g. the sphere) or multiply connected (as e.g. the torus). Each of the presumed geometries of the universe, i.e. spherical, hyperbolical or flat, may represent different types of the multiply connected spaces. Such topologies reconcile the condition of the finite volume with the infinite (in principle) space accessible to observation, filled however mainly with ghost images. Different experimental programs based on the assumption of multiply connected topology of our cosmic space have been intensively conducted in the last years, in particular based on the data obtained from Wilkinson Microwave Anisotropy Probe (WMAP) [5,6]. The main argument speaking for the multiconnected topology refers to the observed lack of big structures (exceeding 60°) in the cosmic microwave background (CMB).

The Möbius 3-torus (three-dimensional equivalent of Möbius strip, obtained by twisting and gluing together the opposite faces of a cube) is the simplest example of the multiply connected, three-dimensional space (topological manifold) that gives a topological solution to the handedness paradox. Despite different number of dimensions, the Möbius 3-torus shares its topological and geometrical properties with the Möbius strip. First, it has zero Gaussian curvature, which matches well the observations of the universe's mean curvature at global scale [7]. Besides, topologically, it is a non-orientable space, which signifies the impossibility of defining the described spatial order (handedness) within that space taken as a whole. On the other hand, it is a topological example of the nontrivial bundle, which means that locally it looks like a "normal" orientable space, in which the Alice objects may exist apart from the ordinary ones.

While considering the topological properties of Möbius-like space connected with handedness, one should distinguish between the intrinsic and extrinsic dimensional number of a given space. This difference refers, respectively, to the number (n) of dimensions accessible for the observer located within a given space, and to n+1 dimensions, accessible for the observer from the "outside". Assume a sheet of paper of infinitesimal thickness representing the model of the two-dimensional space (surface). While embed (and perceived) in three dimensions, the surface has two sides. Perceiving two sides of the surface relates to its extrinsic dimensional number 3. Meanwhile, the surface's intrinsic dimensional number is 2, which means that surface has only one "side". In the case of Möbius strip, the question of sides is somewhat confusing since two sides perceived locally in the three-dimensional space form globally the one side. Nevertheless, such a depiction that perceives two sides of Möbius strip locally, still refers to the extrinsic dimensional number 3. One can easily cover the whole paper model of Möbius strip with chiral figures (such as the letter L, for instance) without running into difficulties connected with the consistent space orientation. However, if one replaces the opaque paper model with the one made of a transparent material, the chiral figures marked on such a strip "serve" both sides equally, likewise single glasses in the stained glass window. This corresponds to the intrinsic dimensional number 2. The handedness of a given chiral figure located within the Möbius strip becomes then both "right" and "left". In consequence, covering the whole Möbius strip with chiral figures becomes impossible, which exactly defines it as the non-orientable space.

Although the Möbius 3-torus seems to reconcile the handedness paradox with the demand of the universe's singleness, yet there are reasons to base the universe's topology on the dodecahedron rather than on cube. In analogy to the 3-torus obtained by gluing the opposite pairs of faces of a cube, the Poincaré sphere is obtained by gluing together (identifying) the opposite pentagonal faces of dodecahedron, using a twist of the angle 36° to match the opposite faces. Some CMB data obtained from WMAP seem to indicate the correlation of temperature at this angle [8,9]. It is noteworthy, however, that construction of the Möbius-like Poincaré sphere is even more simple. The opposite faces of dodecahedron (pentagons) are twisted of 180°, which is exactly the angle needed to twist a face while gluing it with the opposite one, to get the Möbius space.

In the "normal" simply or multiconnected space, a chiral object cannot be mapped to its mirror image by rotations and translations. However, as translated throughout the "whole" Möbius space (which means that translation takes an advantage of the space properties connected with multiconnected topology) the object will return from the opposite direction with the handedness opposite to the original one. As referred to cosmological observations, this involves specific predictions. In particular, provided the fulfillment of other cosmological conditions required for detecting the multiconnected topology of the universe (roughly connected with the size of the universe and the rate of expansion), one should expect that sequential (i.e. coming alternately from opposite directions) images of cosmic objects, or the structures identified within CMB, would have the opposite handedness. This is just what we suggest Alice should find there.

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