Photon Impedance Match to a Single Free Electron

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It is not surprising that consideration of impedance matching the photon to the electron, or more specifically to the quantum of resistance at the length scale defined by the mass and angular momentum of the electron, has been long ignored in quantum electrodynamics. Conceptually the development of QED preceded the discovery of ‘exact quantization’ and the associated von Klitzing constant by many decades. Additionally, the relevance of the resistance quantum to photon interactions with a single free electron has only recently begun to be appreciated. In this note we offer a simple presentation of such an impedance match, briefly discuss the unexpected emergence of the fine structure constant from these simple first principles, and suggest how the procedure can be inverted to deliver a first principles calculation of the mass of the electron.

Introduction

The power of the fine structure constant in ordering particle lifetimes has been clearly demonstrated [1,2]. This ordering breaks down at the zeptosecond boundary [3], or equivalently at the reduced Compton wavelength of the electron. One might infer from this ordering that the strong force is electromagnetic in origin [4]. One might further consider that the smallest stable units, the quanta, of possible resonant electromagnetic structures exist where the ordering breaks down, defining the eigenmode structure at the zeptosecond boundary. It then becomes interesting to impedance match the photon to the quantum of resistance [5-9] at this length scale, at the wave scale defined by the mass and angular momentum of the free electron. From these first principles the dimensionless numerical value of the fine structure constant emerges as the argument of the near field wave impedances. This suggests the existence of additional eigenmodes of resonant electromagnetic structures, not at the reduced Compton wavelength of the electron, but rather ~137 times smaller, at the scale of the 70MeV platform state [10].

The Impedance Match

The match is straightforward. The impedance of free space is

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.73031346 \ \ ohm$$

where μ_0 and ε_0 are the free space magnetic and electric permeabilties [11].

The near field impedance is, of course, scale dependent. Here we choose the scale at which relativistic quantum field theory becomes essential, the scale defined by the reduced Compton wavelength. Ignoring the phases for the moment, the near field electric and magnetic wave impedances [12] can be calculated in multiples of the reduced Compton wavelength of the electron

$$\lambdabar_e = \frac{\hbar}{m_e \cdot c}$$

and a scale parameter r that permits to plot these impedances as a function of $\lambdabar_e$.

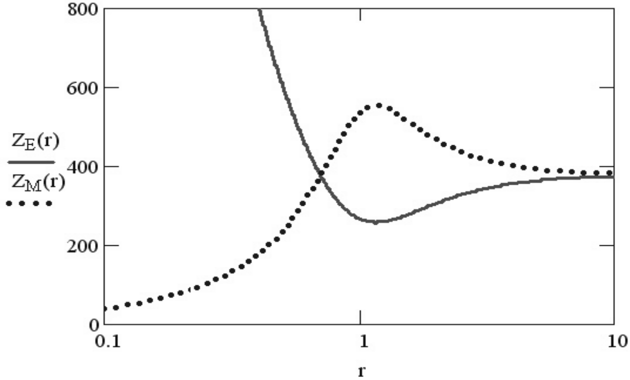
$$Z_E(r) = Z_0 \cdot \frac{\left|1 + \frac{\lambdabar_e}{i \cdot r \cdot \lambdabar_e} + \frac{\lambdabar_e^{\,2}}{(i \cdot r \cdot \lambdabar_e)^2}\right|}{\left|1 + \frac{\lambdabar_e}{i \cdot r \cdot \lambdabar_e}\right|}$$

$$Z_B(r) = Z_0 \cdot \frac{\left|1 + \frac{\lambdabar_e}{i \cdot r \cdot \lambdabar_e}\right|}{\left|1 + \frac{\lambdabar_e}{i \cdot r \cdot \lambdabar_e} + \frac{\lambdabar_e^{\,2}}{(i \cdot r \cdot \lambdabar_e)^2}\right|}$$

Such a plot is shown in the following figure, where r=1 corresponds to the reduced Compton wavelength of the electron. The sole crossing of the impedances is at the RMS of the reduced Compton wavelength.

$$Z_E\left(\sin\left(\frac{\pi}{4}\right)\right) = Z_B\left(\sin\left(\frac{\pi}{4}\right)\right) = 376.73031346\, ohm = Z_0$$

At this crossing the impedances again return to Z_0, the impedance of free space.



Series Near Field Wave Impedance

We seek to match the wave impedance to the quantum impedance [5-9], here taken in series to be twice the quantum Hall resistance.

$$2R_H = \frac{2h}{e^2} = 5.162561512 \cdot 10^4 ohm$$

Taking the value of the scale parameter r to be the fine structure constant α , the corresponding near field wave impedance is

$$Z_{series}(\alpha) = Z_E(\alpha) + Z_M(\alpha) = 5.162561518 \cdot 10^4 \ ohm$$

At the limit of experimental accuracy the quantum Hall resistance is identical to the near field series impedance of the electron at the scale of the reduced Compton wavelength times the fine structure constant, at the scale of the 70MeV platform state, at the classical radius of the electron.

In this context the fine structure constant emerges as a property of the scale of space as defined by impedance matching the quantum and near field wave impedances for the mass and angular momentum of the electron.

Parallel Near Field Wave Impedance

The parallel impedance is

$$Z_{parallel}(\alpha) = \frac{1}{\dfrac{1}{Z_E(\alpha)} + \dfrac{1}{Z_M(\alpha)}} = 2.7491339055\,ohm$$

The meaning of this not immediately obvious. However, if one takes the ratio of these impedances one finds that

$$\alpha^2 \frac{Z_{series}(\alpha)}{Z_{parallel}(\alpha)} = 1.000000011$$

With the assertion that the fine structure constant is relevant "as a coupling constant in quantum chromodynamics" [4], it might be that this two-powers-of-alpha ratio is operative in the mode coupling of the resonant electromagnetic structures that results in the striking four-powers-of-alpha lifetime desert between the "…slow (unpaired) single quark decay channels and much faster (paired) quark-antiquark radiative decay channels…" [13].

The Electron Mass

In the preceeding paragraphs, the fine structure constant emerges from the space scale at which the near field wave impedance is matched to the quantum impedance for the mass and angular momentum of the electron. The fundamental constants used in this calculation are:

- impedance of free space – defined by the electric and magnetic permeabilties
- electron Compton wavelength – defined by the angular momentum quantum, the electron mass, and the speed of light

- quantum impedance – defined by the charge and angular momentum quanta

In summary, the fundamental constants used in the matching calculation are

$$e, h, m_e, c, Z_0$$

The fine structure constant is defined as

$$\alpha = \frac{e^2}{2\varepsilon_0 hc}$$

or alternatively [14] as

$$\alpha = \frac{e^2}{2h} Z_0$$

which of course is

$$\alpha = \frac{Z_0}{2R_H}$$

This suggests that one can take the fine structure constant as given (as we do for instance in QED) and backsolve the impedance equations to calculate the mass of the electron (this we don't do in QED), again at the limit of experimental accuracy.

Conclusion

It is not impossible that there appear here tentative first steps in sketching out some sort of dynamic electromagnetic structure that exists in the laboratory rest frame, something akin to Maxwell's equations for the massive particle. If one accepts this as a platform state it is similarly not impossible that such a sketch might shed light on the masses and eigenmode structures of additional members of the particle spectrum. The behaviors of the magnetic and electric flux quanta [15-17] are particularly interesting in this context.

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