Time Dilation and the Concept of an Objective Rest System

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The case of two rockets approaching each other with constant velocity is used to underscore a common misconception in relativity theory. Since each rocket is an inertial system (IS), it has been argued that a clock located on either one of them must be running faster in each case than an identical clock on the other (symmetry principle). This conclusion is based on the Lorentz transformation (LT) of Einstein's special theory of relativity (STR). There has never been an experimental verification of this prediction, however. On the contrary, experiments with atomic clocks carried onboard airplanes and rockets have demonstrated that their rates can be computed in a simple manner just by knowing their speed relative to the Earth's non-rotating polar axis. After accounting for gravitational effects, it has always been found that the rates of these clocks are slower than those of identical counterparts located on the polar axis. These results can be predicted quantitatively by defining an objective rest system (ORS), the Earth's polar axis in the present case, from which to uniquely

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apply Einstein's time dilation formula for clocks moving relative to it. On this basis it is concluded that measurement is objective (rational) rather than symmetric, and that the LT therefore does not correctly predict the rates of atomic clocks on a sufficiently general basis. A key result of the ORS formulation is that the lifetime of meta-stable particles on the Earth's surface measured by an observer moving at high speed relative to them is *shorter* than the value he finds for the identical particles when they are at rest in his laboratory. More generally, this approach underscores the advantages of introducing a set of physical units that is dependent on the state of motion of the observer.

Keywords: relativistic Doppler effect, rational system of units, First Law of Thermodynamics, alternative Lorentz transformation (ALT), muon decay, clock-rate parameters

I. Introduction

Symmetry plays a key role in Einstein's special theory of relativity (STR) [1], starting with the idea that two observers in relative motion to one another each have the feeling of being at rest. Experiment has nonetheless demonstrated in a variety of ways that the rates of clocks are slowed when they are accelerated and that this effect is by no means symmetric. This phenomenon has come to be known as the Twin or Clock Paradox. Numerous attempts have been made to explain this result, often claiming that one needs to go beyond STR to understand how such a fundamentally asymmetric relationship occurs, as for example in the work of Born [2]. Taylor and Wheeler [3] have argued instead that the Twin Paradox can be explained entirely within the framework of STR by introducing a pair of identical rockets moving at a constant speed in opposite directions in order to carry out the necessary timing procedures. The key point in both explanations is that the relativistic symmetry principle is somehow rigorously upheld at all times when the clocks are in the constant-velocity phases of their respective journeys, specifically that each clock is running slower than the other during these periods.

In this discussion it is important to see how the experimental timing results obtained with clocks carried onboard airplanes [4] and rockets [5] have been successfully analyzed in actual practice. The entire journey is viewed through the eyes of a single observer (reference clock) located at one of the Earth's poles. At each stage the clocks on the airplanes (rockets) are assumed to be running slower by a factor of $\gamma(u)=\left(1-\frac{u^2}{c^2}\right)^{-0.5}$ (after correction for the red shift effect on clock rates) than their identical counterpart on the polar axis, where u is the speed of the airplanes relative to this reference point and c is the speed of light in free space. Moreover, clocks located elsewhere on the Earth's surface are also assumed to run slower than the reference clock. In this case the rotational speed of the Earth at a given latitude is used in applying Einstein's formula. As Phipps [6] has pointed out, the whole procedure takes place without invoking any kind of discontinuity in the clock rates such as are required in the explanations that insist upon adherence to the relativistic symmetry principle [2,3].

The issue that clearly needs to be resolved is why the observed timing results can only be successfully predicted when the speed u inserted in Einstein's time dilation formula is taken *relative to a specific rest frame*, contrary to what would otherwise be assumed on the basis of STR. Hafele and Keating justified this choice in their original work [4] by noting that the airplanes are non-inertial during the course of the experiments and therefore are unsuitable for direct application of Einstein's formula. In the following discussion it will be argued that the assumption that the Earth's non-rotating polar axis is an IS is neither correct nor essential for the accurate prediction of the relative clock rates on the airplanes.

II. Comparative Timings: Symmetric or Objective?

Consider two rockets E and W traveling in opposite directions (Fig. 1) but with equal and constant speeds relative to their departure point, so that both can be assumed to be IS. If Einstein's formula is valid for E,

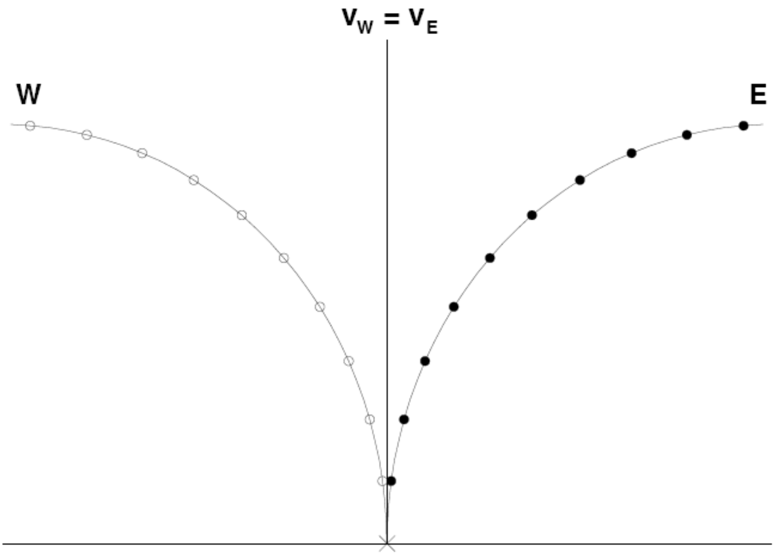

Fig. 1: Diagram of two rockets leaving the same position in a gravity-free region of space. Their speed relative to the departure position is the same for both at all times, even though the respective directions of velocity are always different. The symmetric relationship of their trajectories indicates that the rates of their respective onboard clocks are always the same, however. This remains true even for the termini of the trajectories shown, in which case the rockets are both a) inertial systems (each traveling at constant velocity) and b) in relative motion to one another at that point.

it follows that its onboard atomic clock is running $\gamma(u)$ times faster than an identical clock on W, where u is the relative speed of the two airplanes. On the other hand, if one makes the same assumptions about W, it follows from its perspective that it is E's clock that is running γ times slower. This analysis is clearly *inconsistent* with the Hafele-Keating procedure [4] in their experiments, according to which it must be assumed that the speeds to be used in applying Einstein's formula for each clock are v_{E} and v_{W}, respectively, i.e. *their speeds relative to the departure point* in Fig. 1. Let us further assume that the two airplanes eventually change directions and return to the original airport and land at exactly the same time. Finally, let us also assume that their respective trajectories during the entire flight are exact mirror images of one another, also as indicated in Fig. 1.

By construction, there is no interaction that could possibly affect the clocks on the two airplanes in a different manner since their trajectories are equivalent to one another by symmetry (the airport is also assumed to be inertial). As a result, the two onboard clocks must show exactly the same time after their arrival (assuming they were appropriately synchronized prior to their departure). This is exactly what is expected from the Hafele-Keating procedure [4] described above, but it conflicts with the predictions of STR since the latter asserts that each clock runs slower than the other during at least the constant-velocity phases of their respective journeys. It is true that one could claim that there is something extraordinary about the non-inertial phases when the airplanes change directions to return to the airport that somehow erases any distinction between the two clock readings upon arrival there. The symmetry implied in Fig. 1 makes this an extremely implausible explanation for what must occur at the end of the flight, however. Moreover, one could at least in principle *start* both onboard clocks at the *precise moment when constant* *velocity is reached* and stop them later *when they begin their respective return journeys to the airport*. The elapsed times shown on the two clocks must be the same during this portion of their journeys because of the symmetric relationship of their trajectories, despite the fact that each airplane is an IS during this period.

The point to be emphasized is that although no theoretical argument can be negated entirely by the above experimental results, there comes a point where one is justified in looking for *alternative* explanations for what has transpired that do not require some discontinuous, as yet unobserved, effect [2,3,6] to resolve the Clock Paradox. The procedure actually employed in the Hafele-Keating experiment [4] assumes that the rate of an atomic clock changes *continuously* according to a well-defined prescription which leads to results which are in quantitative agreement with observation. At any one time one only needs to know the altitude h of the airplane on which a given clock is located and its speed u relative to a fixed position on the Earth's polar axis in order to compute the amount of time dilation. After correcting for the gravitational red shift effect (for which h is required), the elapsed time τ on the airborne clock is computed as follows:

$$\tau = \frac{\tau_0}{\gamma(u)}, \tag{1}$$

where τ_0 is the corresponding elapsed time on the reference clock on the polar axis.

The interpretation is that the clock on the airplane simply runs at a different rate than the reference clock, and that the ratio of these two rates depends solely on the relative speed u. An observer on the airplane would not be able to notice any change in clock rate based on purely *in situ* measurements for the simple reason that all natural clocks in his rest frame slow down by exactly the same amount. In effect, *his unit of time has changed without him noticing it.* Clearly implicit in this relationship is the assumption of *remote simultaneity*, which is to say that a given event occurs at exactly the same time for observers on the airplane and at the reference point. Their respective clocks will disagree on the numerical value of the elapsed time in any given case, but always in the same proportion as long as the relative speed stays constant. The same assumption is critical for the accuracy of the Global Positioning System (GPS) methodology. In that case an atomic clock is put onboard the satellite which has been "pre-corrected" prior to launch so that it will run at exactly the same frequency as an identical clock left behind on the Earth's surface. This would be a useless exercise were it not for the fact that events such as the emission of light signals occur simultaneously for a hypothetical observer on the satellite as for his companion on the ground [7]. The great practical success that this navigation system has enjoyed in the past few decades is perhaps the strongest evidence for the reliability of the assumptions made in the Hafele-Keating timing procedure. The clear indication is that their computations of elapsed times *are correct for each stage of a given airplane's journey, not simply for the entire round trip back to the airport of origin*.

The Hafele-Keating timing procedure is also consistent with another basic principle that is thought to be inoperable in STR. Measurements have been carried out [4] for atomic clocks on two airplanes and also for those located on the Earth's surface. The above formula is found to hold in each case and this allows direct comparisons of these timing results. The ratio of elapsed times τ_1 and τ_2 for two such clocks moving with speeds u_1 and u_2, respectively, relative to the same reference point is obtained by eliminating τ_0 in eq. (1) in each case, with the result:

$$\frac{\tau_1}{\tau_2} = \frac{\gamma(u_2)}{\gamma(u_1)}. \tag{2}$$

The above formula is verified to within experimental accuracy in the Hafele-Keating experiments [4], consistent with the discussion given above in connection with the rockets in Fig. 1. As a result, it can be concluded that the timing measurements satisfy the principle of objectivity or *rationality of measurement* (PRM [8]) as well as the principle of remote simultaneity. There is simply a *conversion factor* that can be used to translate the timing results for one clock into those for the other for any given event. It isn't just a matter of perspective as to which clock is running slower than the other and by how much. Moreover, the above formulas can be applied on a quite general basis to obtain comparisons of timing results, independent of whether any of the clocks conform to the definition of IS at the time the measurements are made or not. There also is no reason to believe that the symmetry principle of STR holds for values of other physical properties on this basis. The indication is that the PRM [8] also applies to the measurements of lengths, energies and inertial masses and any combination thereof [9]. Measurement is *objective* rather than symmetric.

III. Objective Rest System

Perhaps the most intriguing question raised by the timing experiments carried out on circumnavigating airplanes [4] and rockets [5] is why the speed of the various clocks to be inserted in Einstein's time dilation formula must be measured relative to a particular rest frame, namely the Earth's non-rotating polar axis. The answer given in the work of Hafele and Keating was that, unlike the airplanes and most positions on the Earth's surface, the polar axis constitutes an IS and that this is critical because the prescriptions of relativity theory only apply when the observer is moving at constant velocity. The fact is that the Earth is always moving at high and variable speed in an elliptical orbit around the Sun, however, and therefore its polar axis does not truly qualify as an IS. The discussion of Sect. II also shows that there are difficulties in trying to explain the results of timing experiments when several IS are present in a given experimental arrangement. One is therefore justified in looking for a different explanation as to why the polar axis plays such a specific role in making these determinations.

It helps to imagine the situation if the Earth were not rotating about its axis. All objects on the Earth's surface would therefore be moving at the same speed relative to any reference frame and clocks located anywhere on it would be running at the same rate according to Einstein's theory. This state of affairs would certainly be consistent with the Hafele-Keating empirical formula given in eq. (1) as well. Once rotation begins, the situation would change and the clocks would begin to slow down by varying amounts depending on their latitude. The rotation would not affect clocks located on the polar axis, however, which would make them an obvious reference for determining the amount of time dilation elsewhere on the Earth's surface. In this discussion we are interested solely in the effects of relative motion on the clocks. Gravitational effects can be easily accounted for once we know the altitudes of the clocks, and let us assume in the following that this is always done. It seems equally plausible to assume that if an airplane or rocket leaves the Earth's surface, the rates of their onboard clocks would be similarly slowed and could also be accurately computed just by knowing their speed relative to the polar axis. Indeed, since we assume that gravitational effects are always accounted for, we can just as well take the specific point of reference to be the Earth's center of mass in determining the value of the relative speed to be inserted in Einstein's formula for a given clock. Nothing changes in this scenario if we assume that the Earth is constantly changing its speed relative to other objects with the passage of time, so this removes any necessity of assuming that the polar axis is a perfect IS.

The center of the Earth serves as an *objective rest system* (ORS) from which to judge the relative rates of all clocks that have been studied in the previous experiments with airplanes and rockets [4,5]. In accordance with the latter experience, one only needs to know the altitude of a given clock and its speed relative to the ORS to make a quantitative prediction of its rate on the basis of eq. (1). The PRM [8] then allows us to compare rates of any two such clocks, consistent with eq. (2). All such results are found to agree to within experimental error with actual observations. Why should the Earth's center of mass be the ORS in this case? It certainly plays a unique role in other aspects of life in our part of the universe. A plumb line from any of the clocks studied points to it directly regardless of how fast they're moving or how far away they are. The assumption is that clocks near the surface of other planets would have a different ORS.

This raises other questions about which ORS should be used in a given determination. The case of a rocket flying from the Earth to the Moon serves as an interesting test case. The decision can be based entirely on the aforementioned plumb line. As long as the latter points to the Earth, one can expect that its center of mass continues to act in this capacity. As soon as the point is reached along the journey where the plumb line shifts to the Moon's center of mass, however, the choice of ORS would then change to this point in space.

The path that a given clock takes to reach its current velocity is immaterial in making such determinations. The First Law of Thermodynamics holds that energy is a state function and is therefore path-independent [10]. Since light frequency is directly proportional to energy by virtue of Planck's radiation law, it follows that the same independence holds for clock rates as well. On this basis the rates of clocks belonging to different ORS can be obtained by combining appropriate γ factors in eq. (1). For example, the ratio of the rates of clocks located on the rocket in the present example and on the surface of the Sun can be predicted as follows based on a series of relative speeds, that of the rocket relative to the Moon (u_1), the latter's speed relative to the Earth (u_2) which serves as its ORS, and finally the relative speed (u_3) of the Earth to the Sun. The product, $\gamma(u_1)\gamma(u_2)\gamma(u_3)$, tells us how much slower the rocket's clock is running than its counterpart on the Sun based on the time dilation effect alone. The corresponding gravitational effect can also be obtained quite simply by computing the appropriate red shift factors for the two clocks, which only requires knowledge of the positions of the two clocks in relation to neighboring masses [9]. In other words, excluding the red shift from consideration, the clock on the rocket runs slower than its reference clock on the Moon, which in turns runs slower than its reference on the Earth, etc. Just knowing the relative speed of the clock on the rocket to that on the Sun is not sufficient for this purpose because they don't have the same ORS in this arrangement.

The above computational procedure is a logical extension of that used by Hafele-Keating in their experiment [4]. It allows the time dilation effect for a given clock to be predicted by knowing the relative speed of its ORS at any time. The same principle is used in the GPS methodology to determine the amount that satellite clocks need to be pre-corrected prior to launch. The total amount of time dilation can then be computed by integrating over the entire path followed by the clock, again exactly as was done in the airplane experiments. The same approach can also be used to continuously predict the transverse Doppler shift of a moving light source, as was done in the rocket experiments of Vessot and Levine [5], since this second-order effect is directly proportional to the amount of time dilation. It remains to be shown why such an objective and rational procedure is actually perfectly consistent with relativity theory, contrary to what is claimed in STR.

IV. The Alternative Lorentz Transformation

The main justification for the symmetric interpretation of STR stems from the Lorentz transformation (LT) for the time and spatial coordinates of two (primed and unprimed) IS moving with a constant relative speed u with respect to one another, where x and x' are parallel to the direction of relative motion:

$$\mathrm{d}x = \gamma\left(\mathrm{d}x' + u\,\mathrm{d}t'\right) \tag{3a}$$

$$\mathrm{d}t = \gamma\left[\mathrm{d}t' + \left(\frac{u}{c^2}\right)\mathrm{d}x'\right]. \tag{3b}$$

It is well known that these two equations can be inverted by simply switching the respective primed and unprimed designations and replacing u by $-u$. This result invariably has been interpreted to mean that an observer in either IS can employ eqs. (3a-b) with equal validity, but it has been demonstrated above that this position is untenable.

The concept of an ORS allows one to take a different view of the apparently symmetrical nature of eqs. (3a-b). The LT, as its predecessor, the Galilean transformation, expresses the fact that an object moving on a train or ship appears to have traveled a different distance to a stationary observer (in the ORS) than to a counterpart moving with the vehicle. Einstein [1] argued with the LT that the corresponding elapsed time is also different for the two observers, since otherwise it would be impossible in his view to explain how the speed of light can be the same for both of them. He foresaw that the clocks employed by the two observers might run at different rates, however. Specifically, he predicted that clocks run γ times more slowly on the moving object than they do in the ORS.

It has been shown in recent work [7,11], however, that the experience with the GPS technology is in direct contradiction to the LT. It demonstrates in a most convincing way that events are simultaneous for clocks on satellites and their identical counterparts on the surface of the Earth, contrary to the unequivocal predictions of the LT. It is possible to satisfy Einstein's second postulate without abandoning the principle of absolute simultaneity of events, however. One has to give up Lorentz invariance to do this, but not the relativity principle, Einstein's first postulate. The space-time transformation given below (the alternative Lorentz transformation or ALT [7,11]) satisfies the condition of light-speed constancy but it also insists on the absolute simultaneity of events for observers in relative motion:

$$\mathrm{d}x = \eta\left(\mathrm{d}x' + u\mathrm{d}t'\right) \tag{4a}$$

$$\mathrm{d}t = \mathrm{d}t', \tag{4b}$$

where $\eta(\mathbf{u}\bullet\mathbf{v}') = \left(1 + \frac{\mathbf{u}\bullet\mathbf{v}'}{c^2}\right)^{-1} = \left(1 + \frac{u\mathrm{d}x'}{c^2\mathrm{d}t'}\right)^{-1}$.

The ALT is obtained from the LT of eqs. (3a-b) by multiplying both of its equations by η/γ on the right-hand side, thereby leaving the value of the speed $\frac{\mathrm{d}x}{\mathrm{d}t}$ of the object unchanged and insuring that Einstein's second postulate is also satisfied in the new set of equations. Simultaneity is guaranteed by the second of these equations, which is the same as in the original Galilean transformation. It is important to recognize, however, that *the unit of time must be the same* for both the primed and unprimed observer in the ALT (the same holds true for the corresponding unit of distance). In GPS terminology, this means that the time (t') on the "pre-corrected" clock on the satellite must be compared with that (t) on the Earth's clock since they both run at the same rate. In other words, *just because the time on the* ***uncorrected*** *clock on the satellite is not the same as on the Earth's clock does not mean that events are not simultaneous for them.* Because of the time-dilation effect on clocks, one has to change the unit of time (period of the onboard clock) on the satellite so that it agrees with that used on the Earth's surface in order to demonstrate that events occur simultaneously for observers in the two locations.

The most significant aspect of the ALT in the present context is what it does *not* do. For example, there is no indication whatsoever that the phenomenon of time dilation should occur on this basis, unlike the case for the LT and its eq. (3b). It therefore does *not* lead one to believe that there must be a symmetry principle that makes it impossible for two observers who are at rest in different IS that are in relative motion to one another to agree on whose clock is running slower than the other. It also doesn't say that all one has to know is how fast one IS moves relative to the other in order to compute the amount of time dilation. *Instead, the ALT simply leaves open the possibility that the unit of time may differ from one rest frame to another*. It leaves it up to experiment to determine exactly what is required in order to compute the exact amount of the time dilation in a given case, completely independent of whether either the object or the observer is at rest in an IS or not. The experiments carried out with circumnavigating airplanes [4] indicate that this objective can be achieved in an unambiguous manner by first identifying a rest frame that fulfills the conditions of an ORS in any given situation.

V. Distance Variations and the Speed of Light

Both the LT [1] and the ALT [7,11] are derived under the assumption (as verified experimentally in refs. [12-14], for example) that the speed of light has the same constant value for all observers in any IS, independent of the velocity of the source (Einstein's second postulate). In the simplest case for which an object moves along the direction of relative motion of the two IS, one obtains the following relation (relativistic velocity transformation or VT) between the two observed speeds $\mathrm{v}=\frac{\mathrm{d}x'}{\mathrm{d}t'}$ and $\mathrm{v}=\frac{\mathrm{d}x}{\mathrm{d}t}$ by direct substitution from *either* eqs. (3a-b) or eqs. (4a-b):

$$\mathrm{v}=\frac{\left(\mathrm{v}'+u\right)}{1+\frac{\mathrm{v}'u}{c^2}}. \tag{5}$$

For the special case of $\mathrm{v}'=c$, it is found that $\mathrm{v}=c$ as well, independent of the relative speed u of the two IS. In Sect. II, it has been pointed out that the LT is contradicted by experience with the GPS technology [7], however, since the latter relies explicitly on the existence of the absolute simultaneity of events for observers in relative motion whereas the LT rules this out on a general basis. The ALT, on the other hand, is perfectly consistent with absolute simultaneity and also avoids the claim that each observer's clock is slower than the other's when both are at rest in different IS [8], contrary to what must be assumed based on the LT. The question that needs to be considered next is what the ALT tells us about measurements of distance made by the same two observers.

The key point is that the ALT also satisfies Einstein's second postulate and that this has consequences when, because of the time dilation phenomenon, two observers in relative motion disagree on the unit of time (period of their clocks) each uses in measuring the speed of light. For concreteness, let us assume that the observer O in the ORS finds that a light pulse travels a distance L over a time interval T. The question is then what values an observer M who has been accelerated on a rocket ship to speed u relative to the ORS measures for these two quantities. We know from the airplane experiments [4] and GPS that the corresponding elapsed time for M will be *shorter*, $\mathrm{T}(\mathrm{M})=\frac{\mathrm{T}}{\gamma}$, because his clock runs slower than O's by this factor ($\gamma > 1$). In order for M to also obtain a value of c for the speed of light, it follows that he must measure the distance traveled by the light to be $\mathrm{L}(\mathrm{M})=\frac{\mathrm{L}}{\gamma}$, i.e., the ratio of the distance traveled by the light to the corresponding elapsed time must be the same for both observers.

Does this mean that distances have been contracted on the rocket ship? *No, quite the opposite*. It means that the standard measuring rod (meter stick) carried onboard the rocket must have *increased* in length by a factor of γ. A measurement consists of making a comparison between the length of a given object and that of a standard. If M's meter stick were *shorter* than the standard length used by O, he would necessarily measure the distance traveled by the light to be larger than O's value and hence not agree with him on the corresponding value of the light speed. As pointed out in ref. [7], the above experiment can be carried out on the basis of *exclusively local* measurements on a GPS satellite. One simply has to assume that the "pre-corrected" atomic clock runs at exactly the same rate on the satellite as its identical counterpart on the Earth's surface, that is, faster than the uncorrected clock on the satellite once one has taken proper account of the effects of gravity (red shift) in these determinations.

The main conclusion from the above discussion is that the lengths of objects *expand* rather than contract at the same time that clocks moving with the object slow down. In both cases there is a physical change in the properties of the object. It is not reciprocal/symmetric, i.e., simply depending on one's vantage point. *The length expansion is also the same in all directions.* Otherwise, light would not propagate isotropically for both O and M. Upon decelerating, objects contract and clock rates increase in the same proportion. These conclusions are all perfectly consistent with the ALT of eqs. (4a-b) [7,11], but they come in direct conflict with the LT because of its assumption of the non-simultaneity of events [1].

VI. Muon Decay

The decay of atmospheric muons offers an additional example for testing the consistency of the ORS interpretation of relativity theory. In this case it is clear that the ORS is Earth's polar axis and that the muons created by cosmic rays in the upper atmosphere are in motion relative to it. The fraction of muons that remain after a time interval T has elapsed is determined by the ratio $\frac{T}{\theta}$, where θ is the exponential decay lifetime. *All observers must agree on the value of this fraction because the number of muons is a relativistic invariant.* The lifetime of the muons measured *in situ* is known to be $\tau = 2x10^{-6}$ s. The observer (O) on Earth measures their speed to be u and thus [15] $\theta = \gamma\left(u\right)\tau$ from his vantage point. If the distance traveled by the muons on their way to the Earth's surface is L, then he measures the corresponding elapsed time to be $T = \frac{L}{u}$, so that $\frac{T}{\theta} = \frac{L}{u\gamma\tau}$. According to the discussion in Sect. V, the "moving" observer M in the rest frame of the muons measures a smaller value for the distance traveled $\left(\frac{L}{\gamma}\right)$ because his measuring rod (meter stick) is larger $(\gamma > 1)$. He finds that the speed with which the Earth is approaching is also u, however, because his clock runs γ times more slowly than O's. Since he is at rest with respect to the muons, M measures their lifetime to be $\theta = \tau$ in his system of units. He therefore measures the ratio $\frac{T}{\theta}$ to be $\frac{\frac{L}{\gamma u}}{\tau} = \frac{L}{u\gamma\tau}$, the same value as observer O at rest in the ORS, as required.

The decay of muons *in the ORS* can be treated similarly. In this case O measures the lifetime of the muons to be $\theta = \tau$, whereas the elapsed time is the same as before, $T = \frac{L}{u}$. The fraction of muons remaining at the end of this time interval is thus given by the ratio $\frac{T}{\theta} = \frac{L}{u\tau}$ in the ORS. Observer M again finds that the elapsed time is $\frac{L}{\gamma u}$, but since his clock is running *slower* than that at rest in the ORS, he measures the lifetime of these muons to be *smaller* than τ, i.e.,

$\theta = \frac{\tau}{\gamma}$. He thus finds the key ratio $\frac{\mathrm{T}}{\theta}$ to be $\frac{\frac{\mathrm{L}}{\gamma u}}{\frac{\tau}{\gamma}} = \frac{\mathrm{L}}{u\tau}$, the same value as for O, again as required.

In both of the above examples, it has been assumed that the rate of the clocks in M's rest frame is γ times slower than that of the ORS clocks. The underlying physical assumption is that the particles that produced the muons were initially at rest in the ORS before being accelerated by the cosmic rays in the Earth's upper atmosphere. If these muons (or other meta-stable particles) are observed from a third rest frame Q, the ratio of the rates of the clocks in the rest system of the decaying particles and that of the ORS will still be γ because all measurement is rational according to the PRM [8]. The agreement on the $\frac{\mathrm{T}}{\theta}$ values discussed above is completely independent of this clock-rate ratio, however. If Q's clock runs R (instead of γ) times slower than those in the ORS, for example, this means that he would nonetheless measure the relative speed of the approaching Earth to be u but the distance traveled to be $\frac{\mathrm{L}}{\mathrm{R}}$. He would find the lifetime of the atmospheric muons to be $\theta = \frac{\gamma\tau}{\mathrm{R}}$. The $\frac{\mathrm{T}}{\theta}$ ratio for Q would thus be $\frac{\frac{\mathrm{L}}{u\mathrm{R}}}{\frac{\gamma\tau}{\mathrm{R}}} = \frac{\mathrm{L}}{\gamma u\tau}$, the same as for both O and M in the same experiment, so that all three observers would agree on the fraction of meta-stable particles remaining at the time of their arrival at the Earth's surface.

VII. Clock-rate Parameters

The main conclusion of the above discussion is that that clocks slow down upon increasing their speed relative to their ORS and that analogous changes occur for other physical quantities such as lengths and energies of objects co-moving with the clocks [9]. These changes have no effect on the speed of light, however, which can be looked upon as the standard or unit of velocity in each rest frame. Einstein was the first to show [16] that the speed of light does change as it moves through a gravitational field, but it is possible to avoid such complications by restricting one's attention to objects in a gravity-free region of space [9,17], and this course will be followed in the discussion below.

An important objective of relativity theory is to develop a quantitative procedure for predicting how clock rates vary from one rest frame to another. In Sect. II it has been shown that identifying an objective rest system (ORS) for a given rest frame can be quite useful for accomplishing this goal, but clearly any conclusions reached on this basis must be verifiable by experiment. The relativistic Doppler effect [18-19] is probably the most effective experimental technique for providing the necessary information about clock rates to accomplish this task.

The basic idea is then to define what we will refer to as a clock-rate parameter for each rest frame. To begin with, it is necessary to designate a standard laboratory S, which for convenience will be located somewhere on the Earth's surface, for which the local clock-rate parameter is defined to be $\alpha_S = 1$. It is completely immaterial that the state of motion of this laboratory and its position in a gravitational field is constantly changing by virtue of its orbit around the Sun as well as its rotation about the Earth's axis. One simply needs a well-defined reference system from which to compare the clock rates in any other rest frame in the universe. It is necessary to fix a starting time ($t = 0$), but the choice of an event that corresponds to this value is also completely arbitrary. In order to satisfy the principle of absolute simultaneity, it is necessary to assign a null value ($t' = 0$) for the time of the same event to all other clocks in the universal system.

The transverse Doppler effect [18,19] allows one to measure the ratio of clock-rate parameters directly for any two rest frames O and M as:

$$\frac{\alpha_{\mathrm{M}}}{\alpha_{\mathrm{O}}} = \frac{\lambda^{\mathrm{M}}(\mathrm{O})}{\lambda^{\mathrm{O}}(\mathrm{O})} = \frac{\nu^{\mathrm{O}}(\mathrm{O})}{\nu^{\mathrm{M}}(\mathrm{O})}. \tag{6}$$

It is assumed thereby that the light source is located in the rest frame of M, whereby $\lambda^{\mathrm{M}}(\mathrm{O})$ is the wavelength and $\nu^{\mathrm{M}}(\mathrm{O})$, the frequency, of the light measured by O. Each of the *in situ* quantities, $\lambda^{\mathrm{O}}(\mathrm{O})$ and $\nu^{\mathrm{O}}(\mathrm{O})$, is equal to the corresponding wavelength $\lambda^{\mathrm{S}}(\mathrm{S})$ and frequency $\nu^{\mathrm{S}}(\mathrm{S})$ measured for the same atomic line in the rest frame of the standard laboratory S. Finally, a correction for the gravitational red shift needs to be applied when a given rest frame is at a different gravitational potential than S [9], similarly as has been done for the rates of the atomic clocks in the Hafele-Keating experiments [4]. The parameters so defined therefore only take into account "kinetic" effects on clock rates [17], that is, those that result exclusively because of acceleration of a given clock relative to its ORS.

It is also not necessary according to the above definition that either O or M be an ORS. The value of α_{O} for a laboratory O on the surface of the Earth can be computed from its speed u_{O} relative to the (non-rotating) polar axis (ORS). It is $\gamma(\mathrm{O})$ times larger than the clock-rate parameter α_{P} for the ORS. The value of α_{P} can be calculated by knowing the speed u_{S} of the standard laboratory relative to the ORS (in which case $\alpha_{\mathrm{P}} < 1$).

Once the values of the clock-rate parameters have been established for two different rest frames, it is possible to predict the ratios of all measurements of physical quantities for the corresponding observers. If $\frac{\alpha_{\mathrm{M}}}{\alpha_{\mathrm{O}}} = \mathrm{R}$, for example, it means that a lifetime measured by M to be τ will be $\mathrm{R}\tau$ when measured by O. It does not matter where the object of the measurement is located (assuming appropriate gravitational corrections have been applied) or what its speed relative to either observer is, this ratio will always be the same. The same holds true for wavelengths or other distances. They will be R times as large for O as they are for M [9,17]. The ratio of their measured energies of a given object will also be equal to R. Note that R can be less than unity. In this case O's measured values for lifetimes, lengths and energies will always be smaller than M's and in the same ratio in each case. The ratios of the measured values for other physical quantities can be obtained quite simply by inspection of their units. For example, since the unit of angular momentum is J s, the corresponding ratio of O's value to M's will be R^2, i.e. one factor of R for energy and one for time [17].

In effect, these definitions amount to having *a unit for each quantity that is dependent on the state of motion of the observer* [17]. They are also dependent on the time of measurement as a result. The values measured *in situ* will not depend on the state of motion, however, because $\mathrm{R} = 1.0$ in this case ($\alpha_{\mathrm{M}} = \alpha_{\mathrm{O}}$). If an observer's unit for a given quantity is larger than his counterpart's, then he will obtain *smaller* values for his measurements than the other does. The above arguments tell us what these ratios will be in any given case. This formulation stands in stark contrast to the long-established position that "everything is relative," that each observer thinks that the other's clock is running slower than his, or that his meter stick is shorter than the other's. Instead, "everything is proportional" is the rule in relativity theory, with the numerical values one determines depending in a perfectly well-defined manner on the rates of standard clocks in a given state of relative motion.

It is important to see that the role of the inertial system is somewhat diminished in the above formulation of relativity theory. If the observer O is subjected to some external force, he can still use the ALT *on an instantaneous basis* to relate his measured values for time and distance for the moving system to the corresponding results that would be obtained if the system were at rest. It must simply be assumed that O's clock rate is constantly changing as he is accelerated, that is, α_O is not constant in this case. This is an important distinction, allowing the ALT to be employed on a far more general scale than the space/time transformation (LT) of conventional STR. One must simply know the instantaneous values of the clock-rate parameters for both the observer's rest frame and that of the moving object at the time the measurements are carried out.

VIII. Conclusion

The special theory of relativity (STR) has been shown to be deficient on several grounds. First, a counter-example has been found to demonstrate that it is not always possible to apply it successfully to obtain the relative rates of clocks located in different IS. Two rockets approaching each other with constant velocity (see Fig. 1) both qualify as IS, but once one accepts the principle of remote simultaneity it is a logical impossibility for the clocks on each of them to be running faster than those on the other. Indeed, if the rockets leave the same point in a gravity-free region of space with the same speed v but in opposite directions, symmetry demands that their clocks slow down by exactly the same amount.

To apply the theory consistently in this situation, it is necessary to assume that there is an objective rest system (ORS) from which Einstein's time dilation formula can be correctly applied. In the example shown in Fig. 1, it is the point in space from which the two rockets depart. An observer who is at this point can determine the clock rates on both rockets, finding each of them to slow down by a factor of $\gamma(\mathrm{v})$ relative to the rate of his own clock. In the Twin Paradox it is always the stay-at-home twin who is co-moving with the ORS. He predicts correctly that the clock carried by the moving twin has slowed down, but the latter will come to a wrong conclusion based on STR if he doesn't take into account the fact that he has undergone acceleration relative to the ORS. In the experiments with atomic clocks carried out onboard airplanes [4] and rockets [5], the ORS is the Earth's center of mass. More generally, it seems reasonable to conclude one can use a plumb line from a given object to determine the location of its ORS.

There is a related question of how the measurement of length is affected by relative motion. STR holds that the lengths of objects on moving systems are contracted by a factor of γ *along the direction of relative motion*, but not at all in a perpendicular direction. Moreover, the situation is claimed to be symmetric, i.e. each observer thinks that the contraction takes place in the other IS. As with the timing measurements discussed first, it is seen that such predictions are unavoidable if one bases them on the Lorentz transformation (LT) of STR. The LT is contradicted in other ways as well, as for example by its claim that events can be non-simultaneous for observers in relative motion. The Global Positioning System (GPS) technology relies successfully on the opposite assumption, namely that the time of emission of a light signal is exactly the same for observers on a satellite and on the Earth's surface and that one only has to adjust the rate of one of their clocks to demonstrate this on a quite general basis. There is an alternative Lorentz transformation [7,11] that retains all the good features of the original LT (satisfaction of Einstein's two postulates) while at the same time ensuring that the principle of absolute remote simultaneity of events holds for all observers. The ALT is perfectly compatible with the concept of an ORS, simply requiring that both observers use the *same unit of time* (and distance) in comparing their respective space-time measurements. Since the unit of distance is proportional to the period of a standard clock in a given rest frame, it follows from the ALT that *isotropic length expansion* accompanies time dilation, not the asymmetric length contraction predicted by the LT. As with the simultaneity principle, the GPS technology allows one to verify this conclusion on the basis of local measurements of distance directly on the satellite [7].

The ALT makes it possible to formulate relativity theory in terms of a simple principle, namely that the values of measured quantities obtained by observers in relative motion will always occur in definite ratios of their respective clock-rate parameters defined in Sect. VII. Measurement of Doppler-shifted light frequencies or wavelengths allows one to determine the key ratio $R = \frac{\alpha_M}{\alpha_O}$ of these parameters, as defined in eq. (6) via the transverse Doppler effect. The units of time, length and energy/inertial mass for a given observer O are directly proportional to the clock-rate parameter α_O in his rest frame (after making appropriate gravitational corrections [9,17]). The numerical values for his measurements are then inversely proportional to the size of the unit for a given quantity. For *in situ* measurements, $\alpha_{\mathrm{M}} = \alpha_{\mathrm{O}}$, so $R = 1$ by definition for any rest frame. This means that all quantities that are measured *in situ* are the same for any observer, regardless of his/her state of motion or position in a gravitational field. The ORS concept enables one to clearly distinguish between different IS, with the result that it is no longer necessary to claim that both the stationary and accelerated observer will measure a slower rate for the other's clocks or a smaller length for the corresponding measuring rods. Instead, one can return to the ancient principle [8] of the objectivity/rationality of measurement (PRM) that holds that the only reason two observers can disagree on the result of a given measurement is because they employ different standard units in expressing their results.

References

[1] A. Einstein, "Zur Elektrodynamik bewegter Körper", *Annalen der Physik* **322(10)** (1905) 891-921.

[2] M. Born, *Einstein's Theory of Relativity*, Dover Publications (New York, 1962) pp. 354-356.

[3] J. F. Taylor and J. A. Wheeler, *Spacetime Physics*, Freeman (San Francisco, 1966) p. 95.

[4] J. C. Hafele and R. E. Keating, "Around-the-World Atomic Clocks: Predicted Relativistic Time Gains", *Science* **177** (1972) 166-168.

[5] R.F.C. Vessot and M. W. Levine, "A Test of the Equivalence Principle Using a Space-Borne Clock", *General Relativity and Gravitation* **10(3)** (1979) 181-204.

[6] T. E. Phipps, Jr., *Old Physics for New*, Apeiron (Montreal, 2006) p. 133.

[7] R. J. Buenker, "The Global Positioning System and the Lorentz Transformation", *Apeiron* **15(3)** (2008) 254-269.

[8] R. J. Buenker, "*Relativity Theory and the Principle of the Rationality of Measurement,*" submitted for publication.

[9] R. J. Buenker, "Gravitational and Kinetic Scaling of Physical Units", *Apeiron* **15(4)** (2008) 382-413.

[10] T. E. Phipps, Jr., *Old Physics for New*, Apeiron (Montreal, 2006) p. 166.

[11] R. J. Buenker, "Simultaneity and the Constancy of the Speed of Light: Normalization of Space-time Vectors in the Lorentz Transformation", *Apeiron* **16(1)** (2009) 96-146.

[12] A. Michelson and E. W. Morley, "Relative motion of earth and luminiferous Ether", *American Journal of Science* **34** (1887) 333.

[13] R. J. Kennedy and E. M. Thorndike, "Experimental Establishment of the Relativity of Time", *Physical Review* **42(3)** (1932) 400-418.

[14] C. Braxmaier, H, Müller, O. Pradl, J. Mlynek,and A. Peters, "Test of Relativity using a cryogenic optical resonator", *Physical Review Letters* **88(1)** (2002) 010401.

[15] D. S. Ayres, D. O. Caldwell, A. J. Greenberg, R. W. Kenney, R. J. Kurz, and B. F. Stearns, Comparison of π^+ and π^- Lifetimes", *Physical Review* **157(5)** (1967) 1288-1292.

[16] A. Einstein, "Über den Einfluß der Schwerkraft auf die Ausbreitung des Lichtes", *Annalen der Physik* **340(10)** (1911) 898-908.

[17] R. J. Buenker, "*The Relativity Principle and the Kinetic Scaling of the Units of Energy, Time and Length,*" submitted for publication.

[18] H. E. Ives and G. R. Stilwell, "An Experimental Study of the Rate of a Moving Atomic Clock", *Journal of the Optical Society of America* **28(7)** (1938) 215-219 (contd. in **31(5)** (1941) 369-374); G. Otting, "Der quadratische Dopplereffekt", *Physikalische Zeitschrift* **40** (1939) 681-687; H. I. Mandelberg and L. Witten, "Experimental Verification of the Relativistic Doppler Effect", *Journal of the Optical Society of America* **52(5)** (1962) 529-535.

[19] W. Kuendig, "Measurement of the Transverse Doppler Effect in an Accelerated System", *Physical Review* **129(6)** (1963) 2371-2375.