Precursors of force fields in Newton's 'Principia'

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The "accelerating" force in Definition VII of the 'Principia' is essentially equal to the field strength in the modern treatment of Newtonian gravity. Thus, one needs not to postulate the existence of charges and fields when treating electromagnetism, because these are planted already in Newton's representation of mechanics.

Keywords: Newton's 'Principia', force fields, unity of classical physics

Usually, the modern notion of field is attributed to Faraday and Maxwell. These crucial contributions are not to be disputed here. From the point of view of the unity of classical physics, however, it is favorable to have got notions like 'charge' and 'field' within classical mechanics, too. In this Letter I wish to show, that at least precursors of these notions are indeed present in Newton's foundation of classical mechanics [1].

"**DEFINITION I.** The quantity of matter is the measure of the same, arising from its density and bulk conjunctly.

> Thus air of double density, in a double space, is quadruple in quantity; in a triple space, sextuple in quantity. The same thing is to be understood of snow, and fine dust or powders, that are condensed by compression or liquefaction; and of all bodies that are by any caused whatever differently condensed. I have no regard in this place to a medium, if any such there is, that freely pervades the interstices between the parts of bodies. It is this quantity that I mean hereafter everywhere under the name of body or mass. And the same is known by the weight of each body; for it is proportional to the weight, as I have found by experiments on pendulums, very accurately made, which shall be shewn hereafter."

In modern language, "quantity of matter" means mass and equals density times volume ("bulk").

"DEFINITION II. The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjunctly.

> The motion of the whole is the sum of the motions of all the parts; and therefore in a body double in quantity, with equal velocity, the motion is double; with twice the velocity, it is quadruple."

In modern language, "quantity of matter" means momentum and equals velocity times mass ("quantity of matter", *cf* Def. I above).

"DEFINITION VI. The absolute quantity of a centripetal force is the measure of the same proportional to the efficacy of the cause that propagates it from the centre, through the spaces round about.

> Thus the magnetic force is greater in one load-stone and less in another according to their sizes and strength of intensity."

In other words,

1. The force between distant bodies is propagated from its source to the surrounding space;

2. The force between distant bodies is proportional to a body-dependent factor which depends on "size" (volume) and "intensity" ('density'); in modern language, $m = \int \rho dV$.

"DEFINITION VII. The accelerative quantity of a centripetal force is the measure of the same, proportional to the velocity which it generates in a given time.

> Thus the force of the same load-stone is greater at a less distance, and less at a greater: also the force of gravity is greater in valleys, less on tops of exceeding high mountains; and yet less (as shall hereafter be shown), at greater distances from the body of the earth; but at equal distances, it is the same everywhere; because (taking away, or allowing for the resistance of the air), it equally accelerates all falling bodies, whether heavy or light, great or small."

In other words,

3. The force between distant bodies depends on the distance, it decreases with increasing distance.

4. The force of gravity accelerates all bodies in the same amount, independently of their size and total mass.

"**DEFINITION VIII.** The motive quantity of a centripetal force is the measure of the same, proportional to the motion which it generates in a given time.

> Thus the weight is greater in a greater body, less in a less body; and, in the same body, it is greater near to the earth, and less at remoter distances. This sort of quantity is the centripetency, or propension of the whole body towards the centre, or, as I may say, its weight; and it is always known by the quantity of an equal and contrary force just sufficient to hinder, the descent of the body."

In other words,

5. The quantity of a centripetal force is proportional to the change of momentum ("motion", see Def.II above) per time unit. Law 2 is an immediate generalization thereof.

6. The weight is proportional to the (gravitating) mass.

Newton continues,

> "These quantities of forces, we may, for brevity's sake, call by the names of motive, accelerative, and absolute forces; and, for distinction's sake, consider them with respect to the bodies that tend to the centre; to the places of those bodies; and to the centre of force towards which they tend; that is to say, I refer the motive force to the body as an endeavour and propensity of the whole towards a centre, arising from the propensities of the several parts taken together; the accelerative force to the place of the body, as a certain power or energy diffused from the centre to all places around to move the bodies that are in them; and the absolute force to the centre, as endued with some cause, without which

> those motive forces would not be propagated through the spaces round about; whether that cause be some central body (such as is the load-stone, in the centre of the magnetic force, or the earth in the centre of the gravitating force), or anything else that does not yet appear. For I here design only to give a mathematical notion of those forces, without considering their physical causes and seats."

This is one of the several places in the 'Principia' [1], where Newton urges the reader to consider his description of forces to be purely mathematical. It thus is incorrect to connect the physical meaning of 'action-at-distance' with Newton.

Further,

> "Wherefore the accelerative force will stand in the same relation to the motive, as celerity does to motion. For the quantity of motion arises from the celerity drawn into the quantity of matter; and the motive force arises from the accelerative force drawn into the same quantity of matter. For the sum of the actions of the accelerative force, upon the several particles of the body, is the motive force of the whole. Hence it is, that near the surface of the earth, where the accelerative gravity, or force productive of gravity, in all bodies is the same, the motive gravity or the weight is as the body: but if we should ascend to higher regions, where the accelerative gravity is less, the weight would be equally diminished, and would always be as the product of the body, by the accelerative gravity. So in those regions, where the accelerative gravity is diminished into one half, the weight of a body two or three times less, will be four or six times less."

In other words,

7.

$$accelerative\_force : motive\_force = velocity : momentum = const : m \tag{1}$$

8.

$$\int accelerative\_force\, dm = motive\_force \tag{2}$$

9. The accelerative force is the same for all bodies;

10.
$$weight \sim accelerative\_force \times mass \tag{3}$$

Finally,

> "I likewise call attractions and impulses, in the same sense, accelerative, and motive; and use the words attraction, impulse or propensity of any sort towards a centre, promiscuously, and indifferently, one for another; considering those forces not physically, but mathematically: wherefore, the reader is not to imagine, that by those words, I anywhere take upon me to define the kind, or the manner of any action, the causes or the physical reason thereof, or that I attribute forces, in a true and physical sense, to certain centres (which are only mathematical points); when at any time I happen to speak of centres as attracting, or as endued with attractive powers."

The fact, that Newton stresses the purely mathematical character of his explorations a second time after only one paragraph underpins my remark above.

Of course, one has to be extremely careful with the interpretation of historic writings, in order not to artificially introduce modern insights. For this, I would like to conclude here only the following.

- Newton divided the forces between distant bodies into a factor depending on the bodies and a geometric factor, the latter representing "the propagation of the force into space";
- the forces are two-body central forces, where both bodies enter in a symmetrical manner;
- Newton noticed similarities between magnetic and gravitational forces;
- $$accelerative\_force = motive\_force : mass \tag{4}$$
  thus, since the 'motive force' corresponds to the nowadays common force (dp/dt), the 'accelerative force' corresponds to the nowadays notion of *field strength*.

Notice, that the notion of field strength does not solve Newton's problem of lacking a physical mechanism for the propagation of the force (field). This may be one reason for that he did not develop it (the 'accelerative force') in more detail.

In summary, a careful analysis of the 'Definitions' in Newton's 'Principia' reveals that the basic ingredients of classical field theory are planted already there. This demonstrates Newton's ability and will of most tenacious thinking. On the other hand, in such a pioneering work, one cannot expect, that notions not being essential for its main topics are clearly and rigorously presented. Nevertheless, Newton's formulations bear a huge benefit for the foundation of classical physics in the sense of Hertz's program: to represent classical mechanics such, that the other branches can be derived from it [2].

References

[1] I. Newton, *Mathematical Principles of Natural Philosophy*, London 1729; http://gravitee.tripod.com/definitions.htm

[2] P. Enders, *Towards the Unity of Classical Physics*, Apeiron 16 (2009) 22-44