On the Sagnac effect for massive particles and some of its epistemological consequences

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The author shows in this article that a coherent description of the Sagnac effect for massive particles, which takes account of length contraction and time dilation can only be obtained with a so called "absolute" clock synchronization parameter. Any other synchronization leads to an incoherent description between the point of view of an observer on the rotating platform and a non-rotating observer. This demonstration generalises the one made by Selleri and the author for the Sagnac effect with electromagnetic waves.

Keywords: Sagnac effect, clock synchronization, special relativity, matter waves.

Introduction

From an epistemological point of view, a parameterized test theory is an essential tool to quantify the discrepancy between theory and observation. In the case of special relativity such an instrument lacked for a long time. This gap was filled in 1949 only by Robertson [1],

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who wanted to submit in a systematic way the relativity principle to the verdict of experience. He adopted in his article Einstein's clock synchronization procedure, which can be deduced from the relativity principle, but without confronting it to the experience, nor to a critical analysis and so he partially missed his objective. Mansouri and Sexl [2] have extended the test theory of Robertson, parametering explicitly all possible different synchronizations. The result has been surprising: for inertial systems moving relative to one another, there is a set of theories empirically equivalent to special relativity. Some of them admit the existence of a privileged reference frame (ether). With respect to this privileged reference frame the one-way velocity of light is constant in any direction but in the other reference frames it is not, the exception being the particular case of special relativity. All these theories distinguish themselves only by clock synchronization but all adopt time dilation and longitudinal length contraction. They explain, as well as special relativity does, the three key experiments on which this latter is based: i.e. those by Michelson-Morley [3], Kennedy-Thorndike [4] and Ives-Stillwell [5] or their modern equivalents which are respectively those of Brillet and Hall [6], Hills and Hall [7] and the Mössbauer rotor experiment [8].

Nevertheless, the equivalence of theories differing by synchronization of clocks is not valid anymore in accelerated systems. One can show as it has first been done by Selleri [9] in an essential demonstration that the Sagnac effect entails choosing an absolute synchronization[10], [11]. The Sagnac effect can be seen as the measure by means of a rotating interferometer of the difference of time for two light rays to cover the same distance in opposite directions. For example if one sends from earth surface two light rays simultaneously and parallel to the equator, one to the East and one to the West, and a system of mirrors gets them to make a round trip around the earth and then come back to their departure point, the light

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ray leaving westward will come back first. But for the terrestrial observer they have covered the same length in two different times, so he must conclude that these two light rays have not the same velocity in his reference frame and that the velocity of light is not constant in all directions.

In this article, we will extend the demonstration of Selleri and the author to matter waves, The Sagnac effect having not only been measured with photons but also neutrons [12], electrons [13] and even atoms [14]. In the next section we will make a kinematical derivation of the relativistic formula of the Sagnac effect for quantum particles in the non rotating frame, and in the following section, we will do it in the rotating frame and draw our conclusion from the comparison of these two formulas. In the next and last section we will discuss the results we have got.

Kinematical derivation of the Sagnac effect for quantum particles

The Sagnac effect, measured by means of a rotating interferometer is due to the wave side of matter. Because of particle/wave duality it can be measured for particles with non-zero mass just like neutrons or electrons. The first measure of this type was made with a neutron interferometer [12]. In this remarkable experiment a full agreement was found for the Sagnac effect between theoretical predictions and experimental measurement. Moreover the validity of the equivalence principle was tested at quantum level and the influence of gravitational field was measured on the phase of neutrons.

At theoretical level most derivations of the Sagnac effect are done in the non-rotating frame and without relativistic factors, which are beyond the precision of measurement. For completeness we derive here the Sagnac formula in a rotating and non-rotating frame for a

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massive particle taking into account factors of time dilation and longitudinal length contraction. We privilege here a kinematical approach, which allows a better understanding of the development of events in space and time, to a more abstract approach using Hamiltonian formalism. Nevertheless, a great variety of demonstrations exists. They are listed in Hasselbach's and Nicklaus' article, who realised the first measure of the Sagnac effect on electrons [13]. See also[15], [16]

Let us imagine a rotating platform (see fig. 1) in the privileged frame S_0, rotating with an angular velocity ω anticlockwise. In C at time $t_0 = 0$ a ray of particles is divided in two. For the simplicity of demonstration the particles follow a circular trajectory as it is done in numerous articles on light. One ray turns clockwise and the other anticlockwise. After a complete round trip (respective to the platform) they interfere. The interference pattern allows measuring the

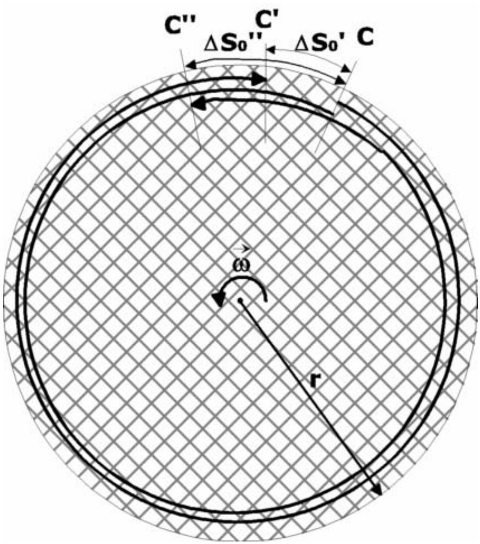

Figure 1: simplified Sagnac configuration

difference of phase and with it the difference of arrival time between the two rays. For an observer in S_0, the ray rotating clockwise arrives at its departure point on the platform at time t'_0 in C' which has moved by $\Delta s'_0$ respective to its former position. In the same way the ray rotating anticlockwise joins it at time t''_0 in C" and has moved by $\Delta s''_0$

I suppose with Hasselbach and Nicklaus that the waves associated with the particles move with a velocity u_0 in S_0. We have $\omega r<u_0<c$. Let be L_0 the length of the circle for an observer in S_0. We have then for the clockwise ray:

$$-\Delta s'_0 + L_0 = u_0 t'_0 \text{ and } \Delta s'_0 = \omega r t'_0 \tag{1}$$

and for the anticlockwise ray:

$$\Delta s''_0 + L_0 = u_0 t''_0 \text{ and } \Delta s''_0 = \omega r t''_0 , \tag{2}$$

with $L_0 = 2\pi r\sqrt{1-\frac{\omega^2 r^2}{c^2}}$. Let us eliminate $\Delta s'_0$ and $\Delta s''_0$ from $-\Delta s'_0 + L_0 = u_0 t'_0$ and $\Delta s'_0 = \omega r t'_0$ (1) and (2), we obtain easily that

$$\Delta t_0 = t''_0 - t'_0 = \frac{4A\omega\sqrt{1-\frac{\omega^2 r^2}{c^2}}}{{u_0}^2 - \omega^2 r^2} \text{ where } A = \pi r^2 \tag{3}$$

which for experimental conditions $\omega r<<u_0<<c$ reduces to the non relativistic formula:

$$\Delta t_0 = \frac{4A\omega}{{u_0}^2}$$

Otherwise in a point the phase difference $\Delta\phi$ is given by $\Delta\phi = 2\pi\upsilon\Delta t_0$ and $\lambda\upsilon = u_0$ so :

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$$\Delta\phi = \frac{8\pi A\omega\sqrt{1-\frac{\omega^2 r^2}{c^2}}}{\lambda u_0\left(1-\frac{\omega^2 r^2}{u_0{}^2}\right)} \tag{4}$$

which for experimental conditions $\omega r \ll u_0 \ll c$ gives the formula $\Delta\phi = \frac{8\pi A\omega}{\lambda u_0}$ which is the one by Hasselbach and Nicklaus.

Kinematical derivation on the rotating disk for all synchronizations

Let us write the transformations valid for any synchronization between the privileged reference frame S_0 and S moving with velocity v along the x_0 axe (see for example [2], [11]) :

$$\begin{cases} x = \frac{1}{R}\left(x_0 - vt_0\right) \\ t = s\left(x_0 - vt_0\right) + Rt_0 \end{cases} \tag{5}$$

where $R = \sqrt{1-\frac{v^2}{c^2}}$. The parameter s has nothing to do with the distance $\Delta s_0'$ used above, but describes synchronization of clocks: for s=0 we have the inertial transformations discovered by Tangherlini [17] and for $s = -\frac{v}{c^2 R}$ we find the Lorentz transformations. Let us

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find the transformation for velocities derived from (5). With $u = \frac{dx}{dt}$ and $u_0 = \frac{dx_0}{dt_0}$, we get:

$$u = \frac{1}{R}\left(\frac{u_0 - v}{s(u_0 - v) + R}\right) \tag{6}$$

For the observer on the platform the waves associated with particles have velocities u' and u" given by

$$u'' = \frac{1}{R}\left(\frac{u_0 - v}{s(u_0 - v) + R}\right)$$

$$u' = \frac{1}{R}\left(\frac{-u_0 - v}{s(u_0 - v) + R}\right) \tag{7}$$

and the time for the round trip is given by $t' = \frac{2\pi r}{u'}$ and $t'' = \frac{2\pi r}{u''}$ and so $\Delta t = t'' - t'$. After some calculation one gets:

$$\Delta t = 4\pi r R s + \left(\frac{4A\omega R}{u_0{}^2 - \omega^2 r^2}\right)R = 4\pi r R s + \Delta t_0 R \tag{8}$$

But otherwise the results for the observer on the platform must be consistent with the results for the non-rotating observer given by (3). And for any synchronization we always have

$$\Delta t = R\Delta t_0 \tag{9}$$

Comparing (8) and (9) we get s=0. It means, as for light rays, that the only synchronization parameter which allows a consistent

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description of the Sagnac effect on the platform and in the non-rotating frame is the so called absolute synchronization s=0.

Discussion of the results

This article has first been written to extend the discussion of "relativistic" Sagnac effect from electromagnetic waves to matter waves. We fundamentally obtain the same result: a coherent description of the Sagnac effect for matter waves between the point of view of the rotating and non rotating observer can only be obtained with a so called absolute clock synchronization parameter s=0, as we proved it. **This means that matter waves, as light, move relative to a medium**. In this medium, for a given frequency, these waves behave isotropically. Here the medium is the non-rotating frame. A difference to electromagnetic waves is that the medium is dispersive for these waves, as it is well known.

To know if the medium, in which the matter waves move, is the same as the medium in which electromagnetic waves move, is beyond possible conclusions of this article. Nevertheless the results of the interference experiment in a rotating frame analysed in this article, with the tool of general transformations, while leaving free the synchronization parameter s, tend to prove at least that if these mediums are different, they do not turn one relative to the other.

It is clear that all of this cannot be understood by today's theoretical physicists, who see matter waves as waves of probability of detection in a Hilbert space with an uncountable number of dimensions and having no physical reality in itself. They themselves having renounced the concept of reality. Experimental physicists may be better able to understand the ideas developed in this article. May it benefit them who understand and make something of it.

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