Are there physical systems obeying the Maxwell-Boltzmann statistics?

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Following an argument by Gibbs, Maxwell-Boltzmann statistics does not apply to classical gases. It applies to ordered sets of distinguishable elements, but are there such sets in physics?

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Consider 2 fair coins and the 4 possible results of one fair toss (H = head, T = tail).

case	**coin 1**	**coin 2**	**MB**	**BE**	**FD**
1	H	H	$\frac{1}{4}$	$\frac{1}{3}$	0
2	H	T	$\frac{1}{4}$	$\frac{1}{3}$	1
3	T	H	$\frac{1}{4}$		
4	T	T	$\frac{1}{4}$	$\frac{1}{3}$	0

Maxwell-Boltzmann (MB) statistics assigns to each of the 4 cases the probability of 1/4. Bose-Einstein (BE) statistics considers the cases 2 and 3 to be one and the same, and assigns to each of the 3 remaining cases the probability of 1/3. Fermi-Dirac (FD) statistics also considers the cases 2 and 3 to be one and the same and, additionally, forbids the cases 1 and 4 (Pauli ban).

This is well known and mathematically fine – what about the realization by physical systems?

In view of the Bose-Einstein condensation and the aufbau (construction) principle of atoms, I'm not going to dispute the existence of bosons and fermions, of course. What about MB statistics? According to the table above, it applies to *ordered* sets of distinguishable elements. Are there such sets in physics at all?

Usually, MB statistics is applied to classical gases. According to Gibbs ([2] Ch. XV), however, the interchange of two "entirely similar particles" does not change (the "phase" of) an ensemble. Hence, the cases 2 and 3 above are not to be counted as different for, say, calculating the entropy.

Within MB statistics, 'equal' particles can assume the same state. This is impossible for classical bodies, if the state is given through the positions and momenta (or velocities) of the bodies (Lagrange; Laplace's daemon; Gibbs). However, it is possible for classical bodies, if the state is given through their momenta (Newton) or velocities (Euler) alone. This suggests that even for classical ensembles, permutation symmetric state functions are favorable. As a matter of fact, the classical Hamiltonian is such a function; moreover, it determines the equilibrium distribution of Gibbsian ensembles ([2] Ch.I).

Generally speaking, it is hardly conceivable that in any Gibbsian ensemble the interchange of equal particles changes the physical situation (where 'equal' means equal in mass, charge, modulus of spin, etc.). Gibbs' paradox ([2] Ch. XV) shows that the correct counting for classical gases is Bose's counting yielding the BE statistics (*cf* Bach's [1] similar conclusion on the basis of de Finetti's theorem).

References

[1] A. Bach, *Indistinguishable Classical Particles*, Berlin u.s.w.: Springer 1997

[2] J. W. Gibbs, *Elementary Principles in Statistical Mechanics*, New York: Scribner 1902