

# Lorentz Driven Density Increase Results in Higher Refractive Index and Greater Fresnel Drag

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This paper builds upon an earlier paper [1] that re-derived the formulas for the physical Fitzgerald-Lorentz contraction and the Lorentz mass increase based on speed relative to the Fresnel dragged reference frame and on the isotropic speed of light in that reference frame. The acceptance of a real physical contraction and mass increase means that the density of a body, and therefore its refractive index and Fresnel drag, will also increase. It is shown that based upon this reasoning, the speed achievable for a sizeable mass (i.e. as opposed to an isolated sub-atomic particle) and a desired mass increase is further beyond the speed of light than specified in the earlier paper.

*Keywords:* Fitzgerald-Lorentz contraction, Lorentz mass increase, Fresnel dragged reference frame, isotropic, refractive index, density increase

## Introduction

In an earlier paper “Lorentz Contraction relative to Fresnel dragged reference frame explains Solid-State Michelson-Morley Experiment Null Result” [1], the formulas for the physical Fitzgerald-Lorentz contraction and the Lorentz mass increase were re-derived based on speed relative to the Fresnel dragged reference frame and on the isotropic speed of light,  $c/n$ , in that reference frame. This derivation led to the following length contraction and mass increase formulas;

$$L_M = L_R \sqrt{1 - \frac{v^2}{c^2 n^2}} \quad (1)$$

Where  $L_R$  is the length of a body when stationary in the local space reference frame where the speed of light,  $c$ , is isotropic,  $L_M$  is the length of the body in the direction of motion when moving at speed  $v$  relative to that reference frame, and  $n$  is the refractive index of the body.

$$M_M = \frac{M_R}{\sqrt{1 - \frac{v^2}{c^2 n^2}}} \quad (2)$$

Where  $M_R$  is the mass of the body when “stationary” and  $M_M$  is its mass at speed  $v$  relative to the stationary frame.

It was argued that the inner fields of matter influence its inner space and exert control over not only electromagnetic radiation within that matter but also over the electromagnetic force fields within the matter.

An assumption in the earlier paper is that the length contraction and mass increase formulas shown above can be expressed by a single refractive index,  $n$ , independent of the speed relative to a reference frame where the speed of light is isotropic for our region of

space. The acceptance of a real physical contraction and mass increase, however, means that the density of a body will increase as its speed increases. It has been shown in *Optics for Technology Students* [2] that as the density of a transparent material increases its index of refraction also increases. This means that its Fresnel drag increases, and from equations (1) and (2) we see that for a larger presently unknown value of  $n$ , the length contraction and mass increase will be less than that specified in the earlier paper for a value of  $n$  determined prior to attaining high speeds. If the mass increase is less for a given speed,  $v$ , then we can achieve a higher speed for a specified desirable mass increase than that predicted in the earlier paper. The analysis which follows determines the unknown moving value of  $n$  and the speed achievable for a specified mass increase. The density increase is somewhat similar to that which occurs during the development of a shock wave in air.

## Determination of Refractive Index for High Speeds

In the earlier paper, the value for  $n$  in equations (1) and (2) was taken to be the value that is currently measured (e.g.  $n=1.5$  for silica) without regard for the value of  $v$ . Since these equations hold for any value of  $n$  they also hold for its unknown moving value which we shall call  $n_M$ . Let the rest volume,  $V_R$ , at  $v=0$  for a given rectangular mass with side lengths  $L_{R1}$ ,  $L_{R2}$ , and  $L_{R3}$ , be given by;

$$V_R = L_{R1}L_{R2}L_{R3} \quad (3)$$

where  $L_{R1}$  is parallel to the velocity  $v$ . Then the moving volume,  $V_M$  is given by;

$$V_M = L_{R1}L_{R2}L_{R3}\sqrt{1-\frac{v^2}{c^2n_M^2}} \quad (4)$$

Evidently this volume change would hold true for any shape since infinitesimal volumes may be used above and then summed together to represent any odd shaped volume. The moving mass,  $M_M$  is given by;

$$M_M = \frac{M_R}{\sqrt{1-\frac{v^2}{c^2n_M^2}}} \quad (5)$$

The moving density,  $\rho_M$ , is then given by;

$$\rho_M = \frac{M_M}{V_M} = \frac{M_R / L_{R1}L_{R2}L_{R3}}{\left(1-\frac{v^2}{c^2n_M^2}\right)} \quad (6)$$

But the top of equation (6) is the rest density,  $\rho_R$ , so our density relationship is;

$$\rho_M = \frac{\rho_R}{\left(1-\frac{v^2}{c^2n_M^2}\right)} \quad (7)$$

As mentioned in the introduction, it has been shown by Robert O. Naess in “*Optics for Technology Students*” [2] that as the density of glass increases its index of refraction also increases. Appendix A of this text book plots points for refractive index vs. specific gravity for 200 types of optical glass at a wavelength of 587.6 nm. Although the points are somewhat scattered most of them fall within a well defined narrow band which can be approximated by a straight line. The

straight line is given in terms of the density of glass,  $\rho_G$ , and the density of water,  $\rho_W$ , by;

$$n = \frac{0.1168}{\rho_W}(\rho_G) + 1.21141 \quad (8)$$

For our purpose we shall write this in the following general form and drop the glass subscript “G”, but it must be remembered that for actual calculations made later with these values, we are working with approximations that are specifically for glass in the data range available (i.e.  $n=1.5$  through  $n=1.9$ ).

$$n = k_1\rho + k_2 \quad (9)$$

Where  $k_1$  and  $k_2$  are constants for the slope and y intercept respectively which we will tentatively consider to represent any substance. Using our straight line equation, we can express the refractive index in terms of the density for the rest and moving cases.

$$n_R = k_1\rho_R + k_2 \quad (10)$$

$$n_M = k_1\rho_M + k_2 \quad (11)$$

Substituting equation (7) into equation (11) we have;

$$n_M = k_1 \frac{\rho_R}{\left(1 - \frac{v^2}{c^2 n_M^2}\right)} + k_2 \quad (12)$$

From equation (10) we have;

$$\rho_R = \frac{n_R - k_2}{k_1} \quad (13)$$

Substituting equation (13) into equation (12) gives;

$$n_M = \frac{n_R - k_2}{\left(1 - \frac{v^2}{c^2 n_M^2}\right)} + k_2 \quad (14)$$

After a bit of algebra, equation (14) can be put in the following form;

$$n_M^3 - (n_R) n_M^2 - \left(\frac{v^2}{c^2}\right) n_M + \frac{k_2 v^2}{c^2} = 0 \quad (15)$$

With specified values for  $n_R$ ,  $v$ , and  $k_2$ , this cubic equation can be solved for  $n_M$ . Note that for  $v \ll c$ ,  $n_M \approx n_R$  as it should. Note also that  $k_1$  has been eliminated so the solution is independent of the slope of the refractive index vs. density straight line approximation. Equation (15) may be solved graphically by setting it equal to  $y$ , plugging in various values for  $n_M$ , plotting the curve, and finding the 3 places where  $y=0$  (i.e. finding the 3 roots). To determine which of these roots is the answer we desire, consider that once a body starts moving relative to the vacuum reference frame where the speed of light is isotropic for our region of space, its velocity relative to the dragged reference frame is always greater than zero so there will always be a contraction and mass increase and therefore density increase relative to the dragged frame. This means that there will always be an increase in refractive index, so we know that for the real solution  $n_M$  must be greater than  $n_R$ . Thus any roots where  $n_M \leq n_R$  can be eliminated. An EXCEL program was written to plot equation (15) for any set of specified input conditions, but before presenting results we seek an exact real solution. The plot will then simply be used as a cross check on the result.

## Moving Refractive Index and Achievable Speed as Function of Desired Mass Increase

Equation (5) can be rewritten as

$$\frac{v^2}{c^2} = n_M^2 \left( 1 - \frac{M_R^2}{M_M^2} \right) \quad (16)$$

Using equation (16) to substitute for  $v^2/c^2$  in equation (15) and rearranging terms we have;

$$n_M^2 \left( \frac{M_R^2 n_M}{M_M^2} + k_2 - \frac{k_2 M_R^2}{M_M^2} - n_R \right) = 0 \quad (17)$$

One solution to this equation can be obtained by setting the portion in brackets to zero and then solving for  $n_M$ .  $n_M$  is then given for a specified desirable mass ratio,  $M_M/M_R$ , by;

$$n_M = k_2 + \frac{M_M^2}{M_R^2} (n_R - k_2) \quad (18)$$

Equation (16) can be rewritten as;

$$v = n_M c \sqrt{1 - \frac{M_R^2}{M_M^2}} \quad (19)$$

Once  $n_M$  is found using equation (18) we can then find  $v$  using equation (19). Then we can plot equation (15), for many values of  $n_M$  using our value of  $v$ , to cross check this root and estimate the other 2 roots. Note that for a specified mass ratio of 1.0 equation (18) gives  $n_M=n_R$  and equation (19) gives  $v=0$  as they should.

On page 80 of the earlier paper we said that for  $n = 1.5$ , even at  $v = 1.4c$ , the mass ratio,  $M_M/M_R$ , is only 2.79. But now, if we specify a mass ratio of 2.79 and set  $n_R = 1.5$  in equation (18), we find that  $n_M =$

3.457823419 (where  $k_2 = 1.21141$  as specified earlier). Using these values for  $n_M$  and mass ratio in equation (19) then gives  $v = 3.228083267c$ . This shows that because a Lorentz driven density increase is now accounted for, we should be able to achieve speeds well above  $v = 1.5c$  without the mass ratio approaching infinity.

Now we can check our solution by plugging  $v = 3.228083267c$ ,  $n_R = 1.5$ , and  $k_2 = 1.21141$ , into equation (15) and then evaluating this equation for many different values of  $n_M$ . An EXCEL program was written to accomplish this and the results are plotted below in Figure 1.0. Note that the curve crosses the x axis in 3 places at the 3 roots where the value of the equation is zero. A detailed printout of the values (not shown here) shows that one solution falls between  $n_M = -3.15$  and  $n_M = -2.9$ , another falls between  $n_M = 1.1$  and  $n_M = 1.35$ , and the 3<sup>rd</sup> falls between  $n_M = 3.35$  and  $n_M = 3.6$ . As mentioned earlier, for a real physical solution we must have  $n_M > n_R$ . Only the 3<sup>rd</sup> solution satisfies this condition and its bounding values,  $n_M = 3.35$  and  $n_M = 3.6$ , span our earlier exact solution,  $n_M = 3.457823419$ , as they should. As one final check, all values, including  $n_M = 3.457823419$ , were plugged into equation (15). The result is a value extremely close to zero (i.e. 8 places to the right of the decimal point are zero), providing great confidence that our mathematical solution is correct. The many decimal places were used simply to assure a correct mathematical solution. We should not lose sight of the fact, however, that as mentioned earlier the solution is approximate because the data used were approximate and were specifically for glass in the data range available (i.e.  $n=1.5$  through  $n=1.9$ ). The value of  $k_2$  may vary for different substances and require adjustment even for glass.



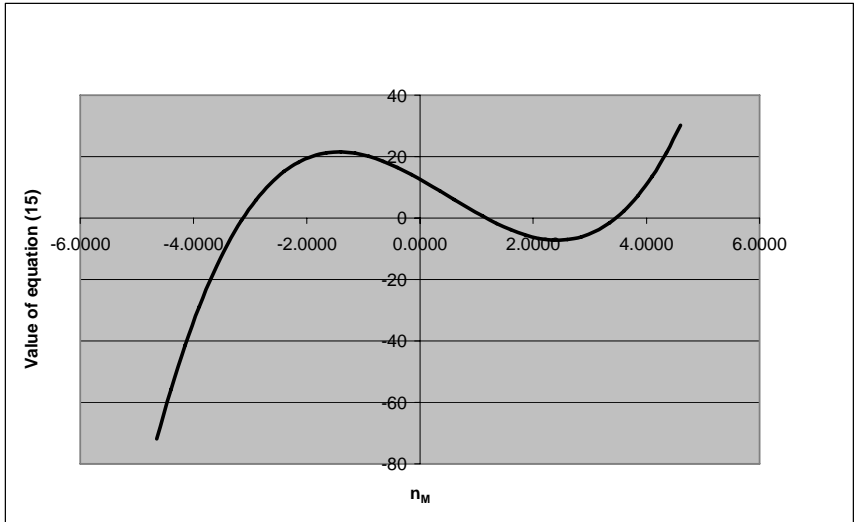


Figure 1.0 Value of Equation (15) for Various Values of  $n_M$

## Conclusion

The speed achievable for a sizeable mass (i.e. as opposed to an isolated sub-atomic particle) and a desired mass increase is further beyond the speed of light than specified in an earlier paper [1], due to a Lorentz driven density increase and greater Fresnel drag. With a specified mass ratio of 2.79, for example, an isolated sub-atomic particle could achieve a speed of about  $0.93c$ , while a sizeable piece of glass could achieve a speed of about  $1.4c$  based on reasoning in the earlier paper, and a speed of about  $3.22c$  based on reasoning in this paper.

One way to substantiate the reasoning in both this paper and the earlier paper is to prove that Einstein's assumption in his following comment on the Fizeau experiment is incorrect; "In accordance with the principle of relativity we shall certainly have to take for granted

that the propagation of light always takes place with the same velocity  $w$  with respect to the liquid, whether the latter is in motion with reference to other bodies or not”. A positive result from the group light speed experiment, proposed in “Fresnel Drag vs. Einstein Velocity – a Case for Further Investigation” [3], would prove that Einstein’s assumption is incorrect.

## References

- [1] D. Wagner, “Lorentz contraction relative to Fresnel dragged reference frame explains Solid-State Michelson-Morley Experiment Null Result”, *Apeiron*, Vol. 16, No. 1, January 2009, pages 70 - 81
- [2] R. O. Naess, *Optics for Technology Students*, p 27 and Appendix A, (Prentice-Hall, Inc., Upper Saddle River, New Jersey, 2001)
- [3] D. Wagner, “Fresnel Drag vs. Einstein Velocity – a Case for Further Investigation”, *Galilean Electrodynamics*, Vol. 19, No. 3, May/June 2008, pages 43 - 50