New Theorem in Motional Electrodynamics

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THEOREM: The electromotive force which drives current in a homopolar generator does not depend on the magnet's rotational velocity.

At first sight, the above sentence contradicts recent experimental evidence indicating the relational nature of motional electromagnetic induction [1-14]. Since the induced motional electric field depends on the probe motion *relative* to the $\vec{B}$- sources, the proposed theorem becomes non-trivial and, as was recently pointed out [1], deserves to be rigorously demonstrated.

Keywords: Homopolar, electrodynamics, relativity.

I Introduction

A metallic disk *D* of radius *R* is spun at the angular velocity $\vec{\omega}_D$ about the z-axis in the lab. If rotation is counterclockwise we define $\vec{\omega}_D = \omega_D \cdot \hat{z}$. If rotation becomes clockwise, we get $\vec{\omega}_D = -\vec{\omega}_D \cdot \hat{z}$, $\hat{z}$ being the unit vector on z-axis. Beneath the disk, a uniform magnet *M* rotates at $\vec{\omega}_M$, as measured with respect to the lab. The induced motional electric field developed in the bulk of *D* (Fig.1) is equal to [1-14]:

$$\vec{E}_D = \left[\left(\vec{\omega}_D - \vec{\omega}_M\right) \times \vec{r}\right] \times \vec{B} \tag{1}$$

$\vec{B}$ being the magnetic field at $\vec{r}$.

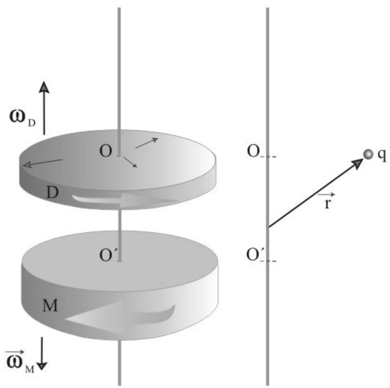

Figure 1
The radial arrows on *D* illustrate the induced electric field

The force acting on the charge *q*, located in the bulk of D, is $q \cdot \vec{E}_D$, wherein $\vec{r}$ is the position vector of q, measured from the symmetry axis. The electromotive force (emf) developed in the bulk of D is:

$$\varepsilon_D = \int_0^R \left\{\left[\left(\vec{\omega}_D - \vec{\omega}_M\right) \times \vec{r}\right] \times \vec{B}\right\}^D \cdot d\vec{r} \tag{2}$$

Wherein the above integral must be evaluated, from r = 0 to r = R, on the disk itself. The superscript D, labeling the bracket, emphasizes the fact that integration takes place on the disk. Equations (1) & (2) remain valid when applied to a spinning radial wire [1-14]. According to equations (1) & (2), a counterclockwise rotation of D at $\omega_D \cdot \hat{z}$ with the magnet stationary in the lab ($\vec{\omega}_M = 0$), is electrically equivalent to the configuration in which, with the disk at rest in the lab ($\vec{\omega}_D = 0$), the magnet is spun clockwise at $-\omega_D \cdot \hat{z}$. The above elementary relational fact was ignored since Faraday's epoch up to nowadays.

II A Spinning Closed Wire

Fig. 2-right sketches a conducting closed loop spinning at $-\omega_{loop} \cdot \hat{z}$ on a stationary uniform permanent magnet. The above configuration is electrically equivalent to the arrangement in which the magnet is spun at $+\omega_{loop} \cdot \hat{z}$, with the loop at rest in the lab (fig. 2-left).

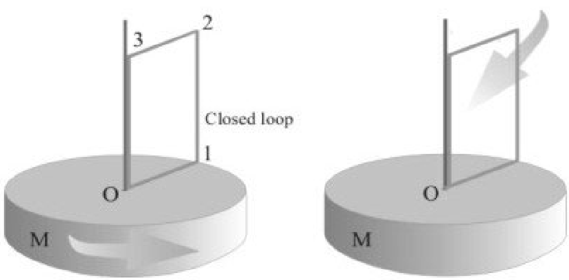

Figure 2
Current does not flow in the spinning loop

In both cases the loop is acted on by the motionally induced electric field, but current cannot flow across the loop. We can understand this elementary observational fact by splitting the whole loop in two parts: a radial wire RW (01-segment), and a closing-wire CW (1230-wire). Each separate wire, at relative motion with the magnet, becomes an *emf*-source. When the above wires are soldered together, the whole circuit behaves as two identical emf sources **connected in opposition** [1, 5].

The whole *emf* can be expressed as

$$\varepsilon = \oint_{whole\ loop} \left[\vec{E}(\vec{r}) \cdot d\vec{\ell} \right] = \varepsilon_{radial\ wire} + \varepsilon_{closing\ wire} = \varepsilon_{01} + \varepsilon_{1230} = 0 \quad (3)$$

from which follows, on account of eq. (2):

$$\begin{aligned} \varepsilon_{1230} = -\varepsilon_{01} &= -\int_0^R \left\{ \left[\left(\vec{\omega}_{RW} - \vec{\omega}_M \right) \times \vec{r} \right] \times \vec{B} \right\}^{RW} \cdot d\vec{r} = \\ &= -\int_0^R \left\{ \left[\left(\vec{\omega}_{CW} - \vec{\omega}_M \right) \times \vec{r} \right] \times \vec{B} \right\}^{RW} \cdot d\vec{r} \end{aligned} \quad (4)$$

Eq. (4) is trivially verified for the particular case in which $\vec{B} = \overrightarrow{const}$. Since here is $\left[\vec{E}(\vec{r}) \right]^{CW} = \left[\vec{E}(\vec{r}) \right]^{RW}$ and $\left[\vec{E}(\vec{r}) \cdot d\vec{\ell} \right]^{CW}$ gives a null integral along the 1-2 segment. That this cancellation occurs also in more general cases can be seen from the fact that magnetic flux entering the closed wire circuit through 01, being conserved (div **B** = 0), must emerge through 1230. Eq. (4) allows us to evaluate the *voltage integral* on the closing-wire performing straightforward calculations on the radial wire.

III Faraday Homopolar Generator

Fig. 3 sketches an actual homopolar engine patterned on fig. 1.

With the aid of an arbitrarily shaped closing-wire stationary in the lab, a circuit is closed between *O* (*r* = *0*) and the rim of the disk at the point *c* (*r* = *R*). Direct current (*DC*) flows across the whole circuit.

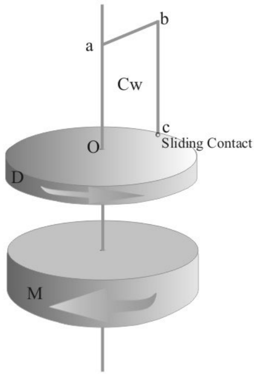

Figure 3
The essential features of the homopolar generator

The generated *DC* **has two emf sources**: the disk itself and the closing-wire, **both at relative motion with the magnet**. The whole emf can be expressed as

$$\varepsilon = \varepsilon_{disk} + \varepsilon_{closing\ wire} \tag{5}$$

Wherein

$$\varepsilon_{disk} = \varepsilon_D = \int_0^R \left\{ \left[\left(\vec{\omega}_D - \vec{\omega}_M \right) \times \vec{r} \right] \times \vec{B} \right\}^D \cdot d\vec{r} \tag{6}$$

Rotation of *D* becomes irrelevant for the evaluation of $\varepsilon_{closing\ wire}$, and the disk can be taken as being stationary in the lab for such purpose. Thanks to the analysis performed in section **II**, we get

$$\varepsilon_{closing\ wire} = \varepsilon_{cba0} = -\int_0^R \left\{ \left[\left(\vec{\omega}_{CW} - \vec{\omega}_M \right) \times \vec{r} \right] \times \vec{B} \right\}^D \cdot d\vec{r} \tag{7}$$

Inserting equations (6) & (7) in eq. (5) we get:

$$\varepsilon = \int_0^R \left\{ \left[\left(\vec{\omega}_D - \vec{\omega}_{CW} \right) \times \vec{r} \right] \times \vec{B} \right\}^D \cdot d\vec{r} \tag{8}$$

A formula in which $\vec{\omega}_M$ does not appear. QED.

IV Historical Comments

The relational (*i.e.* true relativistic) physics of homopolar motional induction, as expressed in equation (1), was only recently disclosed, despite the seminal work published by Einstein, in 1905. At the beginning of his famous paper [15] Einstein wrote:

"It is known that Maxwell's electrodynamics (as usually understood at the present time) when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends **only** on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion."

It is a curious historical fact that, during past decades, most physicists agreed with Einstein only when dealing with **translational** relative motion. **Rotations** were excluded, without known experimental evidence, from the realm of relativity [16-18].

An overwhelming bulk of experimental evidence ensures us that motional induction is, at its most elementary level (a two-body problem only involving a probe and a magnet in relative rotation), a relativistic phenomenon [19]. When, in order to close a circuit, a closing wire is required, then the whole effect only depends on the relative velocity of the involved wires.

Making use of an allegorical language, we say that relativity "moved" from the magnet/ probe pair to the probe/ closing-wire one. This seemingly curious fact was emphasized a decade ago [20].

Regretably, as far as field theory concerns, a lot of problems remains yet unsolved. Remember, for instance, the historical Barnett experiments [21] recently remarked by Kholmetskii [22]. In the above experiments a carrying current solenoid is used as source of magnetic field, instead of a permanent magnet. The outcome of Barnett experiments are unsensitive to solenoid rotation. Here we are faced with a hard problem: spinning permanent magnets become distinguishable from spinning carrying current wires, a simple fact that deserves further experimental and theoretical search. At first sight, ***B-field*** does not suffice for the full understanding of motional induction. Also the motion of its sources appears to be relevant for a coherent description on the whole problem. Crucial differences between "open-field" and "confined-field" configurations were recently reported [10].-

Acknowledgments: To Tom Phipps for critical observations. To Raúl A. Rapacioli for having improved mathematical symbolism.

References

01 A. Radovic, *Spacetime & Substance Journal* 5, Nº 1 (21), 128 (2004)

02 J. Guala-Valverde and P. Mazzoni, *Apeiron* 8, 41 (2001)

03 J. Guala-Valverde, P. Mazzoni and R. Achilles, *Am. J. Phys*. 70, 1052 (2002)

04 J. Guala-Valverde, *Physica Scripta* (Royal Swedish Acad. Sciences) 66, 252 (2002)

05 J. Guala-Valverde, *Apeiron* 11, 327 (2004)

06 J. Guala-Valverde, *Journal of Theoretics* 6-5,5; 6-6,2 (2004)

07 J. Guala-Valverde, *Spacetime & Substance Journal* 3, 140 (2002)

08 J. Guala-Valverde, *Spacetime & Substance Journal* 3, Nº 3, 140 (2002)

09 J. Guala-Valverde, *Galilean Electrodynamics* 14, 120 (2003)

10 J. Guala-Valverde, *Spacetime & Substance Journal* 3, Nº 2, 94 (2002)

11 J. Guala-Valverde, R. Blas & M. Blas, *Apeiron*, 12, 409 (2005)

12 A.K.T. Assis and D.S. Thober, *Frontiers of Fundamental Physics*, Plenum NY, 409 (1994)

13 A.K.T. Assis, *Weber Electrodynamics*, Kluwer, Dordrecht (1994)

14 R. Achilles, *Spacetime & Substance Journal* 3, Nº 5, 235 (2002)

15 A. Einstein, *Zur Elektrodynamic Bewegter Körper*, Annalen der Physik 17 (1905)

16 W.K.H. Panofsky and M. Phillips, *Classical Electricity and Magnetism*, Addisson-Wesley, New York (1955)

17 R. Feynman, *Feynman Lectures*, Addisson-Wesley, New York (1964)

18 A. Shadowitz, *Special Relativity*, Dover, NY (1968)

19 F. Rohrlich, Private communication. Letter to J.Guala-Valverde (October 2002)

20 J. Guala-Valverde and P. Mazzoni, *Am. J. Phys*. 63, 228 (1995)

21 S. J. Barnett, *Phys. Rev*. 35, 323 (1912)

22 A. L. Kholmetskii, *Am. J. Phys*. 71, 558 (2002)