Separating Equivalent Space-Time Theories

Stephan J. G. Gift
Department of Electrical and Computer Engineering
Faculty of Engineering
The University of the West Indies
St. Augustine, Trinidad, West Indies
Email: Stephan.Gift@sta.uwi.edu

A complete set of Equivalent Space-Time Theories based on a group of "equivalent" transformations has been studied by Selleri. This "equivalent" set of space-time theories includes Special Relativity Theory involving the Lorentz transformations and a semi-classical Absolute Space Theory based on the generalized Galilean or inertial transformations. In order to separate these theories, the light speed predictions of the complete set are compared with the relative light speed for light emanating from Io a satellite of Jupiter determined by direct calculation in the space-time framework of these theories.

Keywords: Equivalent space-time theories, special relativity theory, absolute space theory, Lorentz transformations, Inertial transformations

1. Introduction

The accepted theory of space and time is the Special Relativity Theory for which the equivalence of all inertial frames and light speed invariance are foundation postulates [1-4]. Despite this acceptance, there has been continuing interest in a semi-classical ether theory involving a preferred or absolute frame where light speed varies with movement relative to the preferred frame. Thus for example Gagnon et. al. [5] studied this semi-classical Absolute Space Theory in which light propagates isotropically in a preferred reference frame. These authors referred to this absolute space model as the Generalized Galilean Transformation since it involves the classical Galilean transformations adjusted to take into account the real effects of the Fitzgerald-Larmor-Lorentz (FLL) contractions first experimentally confirmed by Ives [6-8].

In these FLL contractions, a rod of length l_o in a preferred frame when moving with speed v relative to that preferred frame, is shortened to a length l given by

$$l = l_o\left(1 - v^2/c^2\right)^{1/2} \tag{1.1}$$

and a system of frequency f_o when stationary in the preferred frame, has a reduced frequency f given by

$$f = f_o\left(1 - v^2/c^2\right)^{1/2} \tag{1.2}$$

both changes resulting in

$$x = \gamma\left(x_o - vt_o\right), y = y_o, z = z_o, t = \gamma^{-1}t_o \tag{1.3}$$

Here x_o, y_o, z_o, t_o are the coordinates of space and time in the preferred reference frame, x, y, z, t are the coordinates in a reference frame moving at speed v relative to the preferred frame, c is the speed of light in the preferred reference frame and γ is the FLL contraction factor given by

$$\gamma = \left(1 - v^2/c^2\right)^{-1/2} \tag{1.4}$$

This Absolute Space Theory (AST) has been extensively investigated by other researchers [9-12] who have highlighted the close agreement between such a theory and Special Relativity Theory (SRT) for nearly all predicted effects. Selleri in particular [9, 13] has shown that the SRT and AST are members of a complete set of theories that differ only according to the clock-synchronization convention employed. This set is based on equivalent transformations that incorporate the experimental facts of the constancy of the two-way speed of light and clock retardation. Selleri [13] has shown that the entire set of theories makes the same predictions for several phenomena and used acceleration in an attempt to argue that the AST gives the best description of the physical world.

In order to more convincingly separate the theories, we note that the theories in the set make different light speed predictions. We therefore compare these different light speed predictions for light from Io a satellite of Jupiter detected on Earth with the light speed determined by direct calculation in the space-time framework of these theories.

2. "Equivalent" Theories of Space and Time [13]

Consider an inertial system S_o with space and time coordinates x_o, y_o, z_o, t_o in which the speed of light is c, and another inertial system S having space and time coordinates x, y, z, t which is moving at speed v relative to S_o along the x-axis. The two systems are coincident at $t_o = t = 0$. Selleri [13] has shown that the complete set of transformation laws from S_o to S is of the form,

$$x = f_1(x_o - vt_o) \tag{2.1a}$$

$$y = g_2 y_o \tag{2.1b}$$

$$z = g_2 z_o \tag{2.1c}$$

$$t = e_1 x_o + e_4 t_o \tag{2.1d}$$

where the factors f_1, g_2, e_1, e_4 can depend on the velocity v of S measured in S_o. The constancy of the two-way velocity of light [14, 15] and experimentally established clock retardation [16] reduce the generality of transformations (2.1) by requiring

$$f_1 = \frac{1}{\sqrt{1-\beta^2}} \tag{2.2a}$$

$$g_2 = 1 \tag{2.2b}$$

$$e_4 + e_1 v = \sqrt{1-\beta^2} \tag{2.2c}$$

where $\beta = v/c$. As a result, transformations (2.1) become

$$x = \frac{x_o - vt_0}{\sqrt{1-\beta^2}} \tag{2.3a}$$

$$y = y_o \tag{2.3b}$$

$$z = z_o \tag{2.3c}$$

$$t = \sqrt{1-\beta^2}\, t_o + e_1(x_o - vt_o) \tag{2.3d}$$

where e_1 is now the only unknown factor. The one-way velocity of light $c_R(S)$ relative to S in the direction of v that satisfies these conditions is given by [13]

$$c_R(S) = \frac{c}{1+\beta+ce_1\sqrt{1-\beta^2}} \tag{2.4}$$

The transformations (2.3) represent the complete set of "equivalent" theories. Different theories in the set are obtained by selecting different values of e_1 which constitute different clock-synchronization conventions. The Lorentz transformation and SRT result as a particular case when

$$e_1 = -\beta / c\sqrt{1-\beta^2} \tag{2.5}$$

and the Generalized Galilean or Inertial transformation and AST result from

$$e_1 = 0 \tag{2.6}$$

In order to determine the remaining unknown factor e_1, we directly evaluate light speed relative to S and compare the result with that predicted in (2.4) by each theory of the set.

3. Light Speed Calculation using Jupiter's Occulting Satellite Io

Following Selleri [13], let Jupiter's satellite Io be in a state of motion along the x axis of an inertial frame S_o. The Earth moves with constant speed v relative to S_o and constitutes another inertial frame S. At the time T_o measured in S_o, Io emits a light signal (occultation) when it is at the point x_{oI} as determined in S_o. At this instant let the position of the Earth as measured in S_o be $x_{oE}(T_o)$. Its equation of motion in S_o yields

$$x_{oE}(T_o) = vT_o \tag{3.1}$$

Therefore the distance d_o in S_o between Io and the Earth at the instant the light is emitted is given by

$$d_o = x_{oE}(T_o) - x_{oI} = vT_o - x_{oI} \tag{3.2}$$

Let $x'_{oE}(T_o)$ be the Earth's position in S corresponding to $x_{oE}(T_o)$ in S_o and let x'_{oI} be Io's position in S corresponding to x_{oI}. Then using the transformation (2.1a),

$$x'_{oE}(T_o) = f_1\left(x_{oE}(T_o) - vT_o\right) \tag{3.3}$$

$$x'_{oI} = f_1\left(x_{oI} - vT_o\right) \tag{3.4}$$

where

$$f_1 = \frac{1}{\sqrt{1-\beta^2}} \tag{2.2a}$$

Therefore, noting from (3.3) and (3.4) that $x'_{oE}(T_o)$ and x'_{oI} are fixed coordinates in S that are independent of the time as measured in S, the distance d'_o in S between Io and the Earth at the instant the light is emitted is given by

$$d'_o = x'_{oE}(T_o) - x'_{oI} = f_1\left(x_{oE}(T_o) - x_{oI}\right) = \frac{1}{\sqrt{1-\beta^2}}\left(x_{oE}(T_o) - x_{oI}\right) \tag{3.5}$$

From (3.2), equation (3.5) becomes

$$d'_o = \frac{d_o}{\sqrt{1-\beta^2}} \tag{3.6}$$

which can be written as

$$d_o' = \frac{1}{\sqrt{1-\beta^2}}\left(vT_o - x_{oI}\right) \tag{3.7}$$

In the right hand side of (3.7), all of v, c, T_o, x_{oI} are measured in S_o. Therefore as Selleri [13] has observed for many phenomena including length of rods moving with respect to S_o, the distance d_o' in (3.7) observed on the Earth is exactly the same for all equivalent theories independently of the factor e_1.

The time t_r in S_o when the signal reaches the Earth is given by [13]

$$t_r = \frac{cT_o - x_{oI}}{c-v} \tag{3.8}$$

From (3.8), the elapsed time Δt measured in S_o is given by

$$\Delta t = t_r - T_o = \frac{vT_o - x_{oI}}{c-v} \tag{3.9}$$

Using (3.1) and (2.1d), the time T_o' in S (i.e. on Earth) at which Io emits the signal is given by

$$T_o' = e_1 x_{oE}\left(T_o\right) + e_4 T_o = \left(e_1 v + e_4\right) T_o \tag{3.10}$$

But

$$e_1 v + e_4 = \sqrt{1-\beta^2} \tag{2.2c}$$

Therefore

$$T_o' = \sqrt{1-\beta^2}\, T_o \tag{3.11}$$

The time t_r' indicated by the clock on Earth when the signal is received is given by

$$t'_r = e_1 x_{rE} + e_4 t_r \tag{3.12}$$

where x_{rE} is the Earth's position recorded in S_o at the time of reception of the signal. Based on the equation of motion of the Earth, this position is given by

$$x_{rE} = vt_r \tag{3.13}$$

This gives

$$t'_r = \left(e_1 v + e_4\right)t_r = \sqrt{1-\beta^2}\,t_r \tag{3.14}$$

Using (3.8), the elapsed time $\Delta t'$ measured in S is given by

$$\Delta t' = t'_r - T'_o = \sqrt{1-\beta^2}\left(t_r - T_o\right) = \sqrt{1-\beta^2}\,\Delta t \tag{3.15}$$

Substituting for Δt using (3.9) gives

$$\Delta t' = \sqrt{1-\beta^2}\,\frac{\left(vT_o - x_{oI}\right)}{c-v} \tag{3.16}$$

In the right hand side of (3.16), all of v, c, T_o, x_{oI} are measured in S_o. Hence as occurs for many phenomena, the elapsed time $\Delta t'$ in (3.16) measured on the Earth is exactly the same for all equivalent theories independently of the factor e_1.

Using (3.16) for $\Delta t'$ and (3.7) for d'_o, we can now calculate the speed $c_R\left(CAL\right)$ of the received light relative to the Earth as determined from Earth. This is given by

$$c_R\left(CAL\right) = \frac{d'_o}{\Delta t'} = \frac{c-v}{1-\beta^2} \tag{3.17}$$

Consistency demands that the relative light speed $c_R\left(CAL\right)$ calculated in (3.17) be equal to the relative light speed $c_R\left(S\right)$ predicted by the set of theories in (2.4). This requires that

$$\frac{c}{1+\beta+ce_1\sqrt{1-\beta^2}}=\frac{c-v}{1-\beta^2}=\frac{c}{1+\beta} \qquad (3.18)$$

It follows from (3.18) that

$$1+\beta+ce_1\sqrt{1-\beta^2}=1+\beta \qquad (3.19)$$

which for $v \neq c$ yields

$$e_1=0 \qquad (3.20)$$

The condition $e_1=0$ corresponds to the AST which from (2.4) predicts relative light speed $c_R(AST)$ as

$$c_R(AST)=\frac{c}{1+\beta}=\frac{c-v}{1-\beta^2} \qquad (3.21)$$

the same light speed value as that calculated in (3.17). In all other theories of the set, $e_1 \neq 0$ giving predicted relative light speed $c_R(SET)$ as

$$c_R(SET)=\frac{c}{1+\beta+e_1c\sqrt{1-\beta^2}} \neq \frac{c-v}{1-\beta^2} \qquad (3.22)$$

In particular for SRT, $e_1=-\beta/c\sqrt{1-\beta^2}$ and hence SRT's predicted relative light speed $c_R(SRT)$ is

$$c_R(SRT)=c \qquad (3.23)$$

Interestingly, the one-way light speed invariance suggested by (3.23) has not been confirmed despite numerous experimental tests [17]. The condition $e_1=0$ means that the AST is the only theory of the set of "equivalent" theories that predicts a relative light speed $c_R(AST)$ which is equal to the directly calculated value $c_R(CAL)$, a value that has been experimentally confirmed [18]. All the other theories including SRT which correspond to $e_1 \neq 0$ are therefore inconsistent and hence seem unable to represent the physical world since for them $c_R(SET) \neq c_R(CAL)$. If SRT is to remain the accepted theory of space and time, this inconsistency must be removed.

4. Conclusion

In this paper, light speed predictions by Einstein's Special Relativity Theory, the Absolute Space Theory and all other members of the complete set of "equivalent" theories developed by Selleri [13] were compared with the directly calculated light speed value for Jupiter's occulting satellite Io. Of these the SRT predicts relative light speed $c_R(SRT) = c$ in accordance with its foundation light speed invariance postulate while the AST predicts relative light speed $c_R(AST) = \dfrac{c-v}{1-\beta^2}$ consistent with a preferred frame. The AST was the only theory whose prediction was in agreement with the directly calculated relative light speed value $c_R(CAL) = \dfrac{c-v}{1-\beta^2}$ corresponding to a clock synchronization parameter $e_1 = 0$. On this basis, the Absolute Space Theory with its preferred frame appears to be the best description of physical space and time.

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