The Dark Universe Riddle

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In this work we review some of the theoretical efforts and experimental evidences related to Dark matter and Dark energy problems in the universe. These dilemmas show us how incomplete our knowledge of gravity is, and how our concepts about the universe must at least be revised. Mainly on the Wilkinson Microwave Anisotropy Probe (WMAP) fifth year data indicates that more than 90% of the total energy density of the universe is dark. Here we discuss the impact of these phenomena imprint on gravitational and quantum field theory's standard history. Moreover, we point out some recent and upcoming projects on Cosmology intended to shed light on these problems.

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Introduction

Our understanding of the universe has dramatically changed in the past decades. Cosmology has actually become an experimental based discipline with remarkable development and high level of confidence when regarding measurements. Ever since the interest on cosmology has grown, the scientific community began to realize that there is a lot more to the universe than it meets the eye. Indeed, a sought-after consistent physics theory must be a successful combination between theoretical predictions and phenomenology, and efforts from several research groups have been made in this direction. Moreover, in recent decades, there were an amazing development of physical theories, some of them beyond the standard frameworks of *Quantum field theory*(QFT) and *General relativity* (GR); due to the lack of a proper solution by the mainstream theories, new theories have contributed to the most intriguing and exciting scenarios physics has ever seen. For instance, a vast new multidimensional world dominated by superstrings and/or branes has been developed, and more recently, Brane-worlds, became the ultimate quest to describe nature. This review aims at some capital problems in modern Astrophysics and Cosmology concerning the dark matter and dark energy problems, and drawing some attention to the hierarchy problem between gravitation and gauge interactions. However, this note does not aim to be a complete account of these problems and it is far from being due to its large influence and complexity, and we restrict ourselves to the critical discussion of the most important points in respect to some basics theoretical and phenomenological issues.

The paper is organized as follows: the second section briefly discusses the unification of the fundamental interactions and the understanding of gravitation within theories of spin-2. The underlying point is that the "dark problems", as we call them, require an understanding on how gravity interacts with strong, weak and electromagnetic fields in nature. Moreover, the subsequent sections refer to the discussion of dark problems *per se*. Therefore, the third section discusses *Dark matter* from its original motivation in the early 1930's, with studies on galaxies and clusters of galaxies movements, all the way to the importance of recent collected data and its effect on unification theory candidates.

In addition, in the fourth section, we discuss the *Dark energy* problem [1, 2] consisting in an unclustered component of negative pressure related to the accelerated expansion of the universe. In particular, this problem includes in itself another problem discussed in this article: the *Cosmological Constant*. At first neglected by Einstein himself as his Greatest blunder, the cosmological Constant has been regarded as an odd solution for the dark energy problem. And in the last section, we make brief comments on some recent and upcoming projects, whose main purpose is to detect effects related to dark matter and dark energy in the universe. The final comments are in the conclusion section.

Hereafter, for the sake of notation, we use capital Latin indices that run from 1 to 5, Greek indices are counted from 1 to 4 and small case Latin indices run from 1 to 3 and the index 4 refers specifically to the time coordinate. The colon and semicolon refer to ordinary derivative and covariant derivative, respectively.

Remarks on the quest for unification of the fundamental interactions

> *Although Einstein's theory of the gravitational field is the most widely accepted theory of gravitation, it is rather disconcerting to note that Einstein's theory appears to be strikingly different from the present theories of the electromagnetic field and the meson fields*(...) S.N. Gupta[3]

After the success of GR in 1916, bringing depth to Newton's gravitational theory, theoretical efforts to merge gravitation and electromagnetism into a unique scheme of unification dramatically increased over the following decades, after all the electromagnetic force was the only gauge interaction conceived at that time. Since gravity does not match the gauge interactions as posed by the hierarchy problem of the fundamental interactions, that is, the quantitative difference between electroweak and Planck scales $\left(m_{pl}/m_{EW} \sim 10^{16}\right)$ based on the coupling constants measurements, we find necessity in briefly revising some proposals in a manner of realizing how knowledge of gravity was developed and how strange its behavior can be mainly on the current dark matter and dark energy problems.

Weyl and Kaluza-Klein theories

To do so, we have to look back upon the first half of the XX century. In the end of the 1910's, Weyl's theory [4] was the first attempt to unify gravity and electromagnetism. He altered Riemann's geometry with a non-vanishing metric condition

$$g_{\mu\nu;\rho} = -2A_\rho g_{\mu\nu} \quad , \tag{1}$$

where $g_{\mu\nu}$ is the metric tensor and A_ρ is the 4-vector electromagnetic potential. Hence, one can find the connection

$$\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2}g^{\mu\sigma}(g_{\lambda\sigma,\nu} + g_{\sigma\nu,\lambda} - g_{\lambda\nu,\sigma}) + g_{\mu\sigma}(g_{\lambda\sigma}A_\nu + g_{\sigma\nu}A_\lambda - g_{\lambda\nu}A_\sigma) \ ,$$

and write the action

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left\{\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + ({}^*R)^2\right\} \ , \qquad (2)$$

where k_μ is an arbitrary vector. The $F_{\mu\nu}$ tensor is given by

$$F_{\mu\nu} = k_{\mu,\nu} - k_{\nu,\mu} \ , \qquad (3)$$

which obeys the contracted Bianchi identities and it can be related to Maxwell tensor. In addition, the modified scalar curvature ${}^*R$ is given by

$${}^*R = R + 6k^{\ \mu}_{;\mu} - 6k_\mu k^\mu \ , \qquad (4)$$

where R is the usual scalar curvature.

The main problem of the proposal was that Weyl built his theory in a curved space-time geometry and due to GR, Poincaré's symmetry from electromagnetism was replaced by a diffeomorphism group of the space-time, which led to a non-gauge invariance theory of coordinate transformations. Essentially, it means that any gauge and any solution of the equations are not valid to all observers, making it incompatible with electromagnetism, which is firmly based on experimental grounds. For instance, in Weyl's theory, a bar moving under influence of the electromagnetic field could have its length changed, which is clearly incompatible with observations [5, 6]. In a geometrical sense, the parallel transport in Weyl's geometry could modify both direction and the length of a vector in this space.

Accordingly, still in 1919 (actually, the paper was published only in 1921), motivated by the previous Weyl's work, Kaluza [7] proposed a five-dimensional theory where electromagnetism was compactified in an extra-dimension small circle S_1 built at each point over the Minkowski space-time M_4. Actually, this was not a new address, Nordström [6, 8] in 1914 had already made an attempt to unify electromagnetism to gravity considering the "cylinder" condition which means that it can be easily expanded in Fourier modes and proved by Einstein and Bergman proposal in which the metric of the *bulk* $\mathcal{G}_{AB}$ was given by

$$\mathcal{G}_{AB} = \sum_n \mathcal{G}^0_{AB}(x) e^{\frac{in\pi\phi}{c}} \ ,$$

where ϕ is the coordinate related to the extra dimension. Thus, for instance, let be a scalar field ϕ we can write the periodic condition $\phi(x) = \phi(x + 2\pi R)$, where R is the radius of cylindrical extra-dimension. Hence, one can determinate discrete n values for the momenta as

$$p = \frac{n}{R} \ , \ n = 0, 1, 2... \tag{5}$$

The set of states is currently called nowadays as the tower of Kaluza-Klein modes. In addition, Kaluza used the topology $M_4 \times S_1$ of the 5-dimensional space-time with the *ansatz*

$$\mathcal{G}_{AB} = \begin{pmatrix} g_{\mu\nu} + A_\mu A_\nu & A_\mu \\ A_\nu & \phi \end{pmatrix} \ , \tag{6}$$

where $\mathcal{G}_{AB}$ was the 5-dimensional Riemann metric, and $g_{\mu\nu}$ and g_{ij} are components of spin-2 and spin-1 particles respectively; both are described from the point of view of a 4-D observer where the g_{55} component can be regarded as a non-massive scalar field ϕ.

In 1926, Klein proposed that Kaluza's theory was only valid in a quantum regime of order of 10^{-33}cm (the Planck length), which corresponds to the Planck energy of 10^{19}Gev in addition to the condition of cylindricity besides stating that g_{55} is a constant with $\phi = 1$. Despite the fact that the fifth dimension provides an uneasy feeling, the Kaluza-Klein theory was very suggesting because *a priori* it unified gravity and electromagnetism with the lagrangian

$$\mathcal{L} = R\sqrt{\det(g_{\mu\nu})} + \frac{1}{4}\,\mathrm{tr}\ \left(F_{\mu\nu}F^{\mu\nu}\right) \quad , \tag{7}$$

where $F_{\mu\nu}$ is the Maxwell tensor. However, Klein's proposal imposed serious constraints on the theory, which should predict a massive and detectable particle according to the solution of Klein-Gordon's equation, but it never happened [9]. Some efforts for saving the theory were made in subsequent decades with the Kaluza-Klein non-abelian approach.

In 1926-27, Fock [10] and London [11] pointed out that Weyl's theory could be relevant in the quantum context *if dissociated from gravity*. Thus, based on Weyl's theory, they brought up the concept of a local *gauge* transformation, e.g

$$\Psi'(x) = \exp\left[i\xi(x)\right]\,\Psi(x),\ \ \Psi'^{*}(x) = \exp\left[-i\xi(x)\right]\,\Psi^{*}(x) \quad ,$$

where $\Psi(x)$ and its conjugated $\Psi^{*}(x)$ are regarded as complex functions of a wave field, and $\xi(x)$ is a phase coordinate-dependent parameter. Even though the theory had seem to be successfully retrieved, it came at odds with isospin [12] observations in the strong field scale. The isospin presented a *global* symmetry, called after $SU(2)$ symmetry group, in contrast with the local gauge in

Weyl's theory. Hence, the internal gauge transformations gave a final strike on Weyl's theory as a theory of unification.

Gupta's and ADM scheme

Another interesting approach was made in 1954 by Gupta [3], whose original intention was to study spin-2 fields. He proposed a theorem which establishes that the spin-2 fields in a Minkowski space-time can be described by an Einstein-type system of field equations. His motivation was the study of a linear massless spin-2 field in the Minkowski space-time, a theory first conceptualized by Pauli and Fierz [13]. Gupta's new theorem was very attractive because it showed a remarkable resemblance with the linear approximations of Einstein's equations for the gravitational field. In this sense, he linearized Einstein's equations in Minkowski flat space-time with an infinite number of terms in the Lagrangian density. Thus, the linearized Einstein's equations could be written as

$$\epsilon^{\alpha\beta}\frac{\partial^2 g^{\mu\nu}}{\partial x^\alpha \partial x^\beta} = \tau_0 \Theta^{\mu\nu} \ , \tag{8}$$

$$\Theta_{\mu\nu;}{}^{\nu} = 0 \ , \tag{9}$$

where τ_0 is a constant and $\epsilon^{\alpha\beta}$ is a set of quantities given by

$$\epsilon^{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 \end{pmatrix} \ . \tag{10}$$

It is important to note that $\Theta_{\mu\nu}$ is the symmetrical energy-momentum pseudotensor, where the supplementary condition $g_{\mu\nu;}{}^{\nu} = 0$ applies. A similar situation occurs on Maxwell's theory

$$\Box^2 A_\mu = -(1/c) j_\mu \; , \tag{11}$$

with the Lorentz gauge

$$A_{\mu\ ,\mu} = 0 \; , \tag{12}$$

where A_μ is the electromagnetic potential and j_μ is the current four-vector. Thus, according to Gupta, the same rationalization can be applied to a spin-2 field such that

$$\begin{aligned} &\Box^2 U_{\mu\nu} = \tau_0 \Omega_{\mu\nu} \; , \\ &U_{\mu\nu;}{}^{\nu} = 0 \; , \\ &\Omega_{\mu\nu;}{}^{\nu} = 0 \; , \end{aligned} \tag{13}$$

where $U_{\mu\nu}$ is a real symmetrical tensor, τ_0 is a coupling constant and $\Omega_{\mu\nu}$ is a conserved symmetrical tensor, which can also be written as $t_{\mu\nu}$. The $t_{\mu\nu}$ tensor represents the tensor for the gravitational field energy-momentum plus $T_{\mu\nu}$ tensor, which represents the tensor for the energy-momentum of the gauge interactions. The same equation can also be derived from a variational principle, leading to the Lagrangian density

$$\mathcal{L} = -\frac{1}{2}\frac{\partial U_{\mu\nu}}{\partial x_\lambda}\frac{\partial U_{\mu\nu}}{\partial x_\lambda} + f_1 + f_2 + ... \quad , \tag{14}$$

where the infinite terms $(f_1, f_2, ...)$ compose the energy-momentum $t_{\mu\nu}$. Even further, if we consider only gravitation, one can write a set of Einstein-type equations

$$U_{\mu\nu} - \frac{1}{2} U u_{\mu\nu} = \alpha \tau_{\mu\nu} \; , \tag{15}$$

where the symbols $u_{\mu\nu}, U_{\mu\nu}$, and U can be regarded as a metric-type tensor, a Ricci-type tensor and a scalar-type tensor respectively. Clearly, this new geometry was a copy of Riemann's geometry with a metric and a curvature associated to it.

The shortcoming of this scheme is that Gupta assumed that the physical quadridimensional spacetime was flat what induced a geometrical inconsistency by the metric tensors $g_{\mu\nu}$ and $u_{\mu\nu}$. Moreover, in 1970 Deser [14] showed a generalization of Gupta's theorem suggesting that it could be possible to apply such mechanism to Yang-Mills' theory in a manner to be derived from a similar argumentation. In 1978, Fronsdal [15] proposed a generalization of the theorem for arbitrary *spins* of massless fields. As far as we know, at least over the last decade, there is no trace in literature of a work based on such theorem, except in [16] which made an approach to deal with strong gravity and spin-2 fields in order to associate extrinsic curvature, which in differential geometry is responsible for measure the divergence or convergence of the normal vector with respect to the surface, to a fundamental spin-2 field in nature.

Another interesting attempt was made in 1962 by Arnowitt, Deser and Misner (ADM) [17]. The ADM theory was based on the attempt of making a canonical quantization of gravitation in a manner to deal with quantum fluctuations of 3-dimensional hypersurfaces. The three-plus-one dimensional decomposition of the Einstein field leads to the line element decomposition

$$ds^2 = -N^2 dt^2 + \left(N^i dt + dx^i\right)\left(N^j dt + dx^j\right) g_{ij} \; , \tag{16}$$

where the time component is given by

$$\bar{g}_{44} = N^i N^j g_{ij} - N^2 \; , \tag{17}$$

and

$$\bar{g}_{4j} = N^i g_{ij} \ , \ \bar{g}^{\,44} = -(N)^{-2} \ , \tag{18}$$

$$\bar{g}^{ij} = g_{ij} - \frac{N^i}{N^2} N^j \ , \ \sqrt{\det(\bar{g}_{\mu\nu})} = N\sqrt{\det(g_{ij})} \ . \tag{19}$$

where the overbar indicates a four-dimensional quantity. The N is the lapse function and N^i are the components of shift vector field. They are Lagrange multipliers and determine, for instance, the deformation of a three-dimensional space-type hypersurface σ at time t to another hypersurface σ' at time $t+dt$ in a space time $M_4 \times \sigma$. Moreover, from the $(3+1)$-decomposition of the Einstein-Hilbert action

$$S = \int dt \int_\sigma dx^3 (\pi^{ij} \dot{g}_{ij} - N^i \mathcal{H}_i - N\mathcal{H}) \ , \tag{20}$$

where the dot means time-derivative, $\pi^{ij} = \sqrt{\det(g_{ij})}(k^{ij} - g^{ij}k)$ and k^{ij} is the extrinsic curvature projected on the σ surface. Thus, one can obtain the super-momentum and super-Hamiltonian constraints on g_{ij} and π^{ij}

$$\mathcal{H}_i = -2\pi^j{}_{i;j} = 0 \ , \tag{21}$$

and also

$$\mathcal{H} = (\det(g_{ij}))^{-1/2} g_{ij} g_{kl} (\pi^{ik}\pi^{jl} - \frac{1}{2}\pi^{ij}\pi^{kl}) - \sqrt{\det(g_{ij})} R = 0 \ , \tag{22}$$

where R is the three-dimensional scalar curvature.

The formulation came to fail due to the arbitrary diffeomorphism transformations, which imposed a constraint on the Poisson brackets structure. Basically, the Poisson brackets do not propagate covariantly which still remains as a keen obstacle to quantization of gravity even in other theories. Otherwise, Dirac took into account of this problem stating that the general covariance was the main shortcoming for this method of trying quantization. According to him, it was only possible when one chose normal direction of propagation [19]. A good review of Dirac's method can be found in ref.[20]. Recently, adapting to the brane-world models, inspired on ADM formulation some authors [21] demonstrated that quantization of gravity is possible due to the confinement of the diffeomorphism transformations, which do not "leak" into the bulk, where the brane is embedded, but a lot of work is yet to be developed in order to close this argument.

Non-abelian Kaluza-Klein, two-tensor metric theories and other comments

In addition, in 1965 [6] the non-abelian approach to Kaluza-Klein's theory was developed. In this new theory, the space-time was defined by the topology product $M_4 \times B_N$, where B_N is a compact inner space. This geometry was the solution to Einstein's equations on $(4+N)$-dimensions. The metric *ansatz* was given by

$$\mathcal{G}_{AB} = \begin{pmatrix} g_{\mu\nu} + g_{ab}A^a_\mu A^b_\nu & A_{a\mu} \\ A_{\mu a} & g_{ab} \end{pmatrix} \quad , \tag{23}$$

where $\mathcal{G}_{AB}$ is the Riemannian metric in N-dimensions and A^a_μ is the Yang-Mills' potential, i.e, the connection associated to a $SU(N)$ symmetry. The g_{ab} component can be regarded as a Killing's form of Lie's algebra for the gauge groups, and (a, b) indices are Lie indices of $SU(N)$ group. Hence, the outcome lagrangian was

$$\mathcal{L} = R\sqrt{\det(g_{\mu\nu})} + \frac{1}{4}\,\mathrm{Tr}\,\left(F_{\mu\nu}F^{\mu\nu}\right) \quad , \tag{24}$$

where $F_{\mu\nu} = [[D_\mu, D_\nu], D_\mu]$, $D_\mu = \mathbf{1}\partial_\mu + gA_\mu$ and g is a coupling constant.

Although having a interesting structure, the non-abelian Kaluza-Klein theory needed to be submitted to experiments. When applied to the experimental observations, the theory fell through on the fermionic chirality due to the prediction of a huge fermionic mass which was at odds with the observed helicity at electroweak scale. Besides, there were also inner theoretical inconsistencies such as the size of the B_N space and the definition of the "ground state" (flat, deSitter or Anti-deSitter) of the theory, as pointed out by Abbot and Deser in 1982 [22]. In a manner to give a proper solution to the fundamental problem, in 1983 Rubakov and Shaposhnikov [23] proposed a bidimensional model where the gauge interactions would be confined under a potential well. They did not succeed. The B_N space was not observed because it had a different scale of order of Planck scale which created another observational limitation. Nevertheless, this original work can be regarded as the former inspiration for the current brane-world models. The theory endured until 1984, when observational inconsistences were discovered, even so, several studies based on Kaluza-Klein theory and its reinterpretation have been made in several works [24, 25, 26, 27].

In 1970, based on Gupta's theorem and the photon-meson-ρ model, Isham, Salam and Strathdee [28] idealized a theory in which the spin-2 field was an effective field for short distances. It was assumed that the existence of a new tensor field $f_{\mu\nu}$ or a f-field that should describe a spin-2 massive particle called *f-meson* would couple directly to the hadronic matter. The hadrons and leptons were regarded as high and low energy interacting particles, respectively. Moreover, Einstein's graviton g-field would describe leptons that couple to hadronic matter only through a f-g mixing term. As an application of Gupta's theorem, the f-g theory was built on the 4-dimensional Minkowski space-time.

Criticizing this scheme, in 1973 Aichelburg [29] stated that it was impossible to build a theory with two metric tensors in the same space-time without losing causality. On their defense, some authors [31, 30] assured that to make an unified theory, the adoption of two metric tensors is required to deal with atomic and gravitational phenomena. However, it was proven that the f-meson was a resonance of a quark bound state with short lifetime deprived from a fundamental meaning. Nonetheless, in the same year, Dirac [31] proposed another theory using two metrics. He retrieved Weyl's theory by adapting it to his hypothesis of Large numbers, originally proposed in 1938 [32].

Dirac's large numbers proposal attributed a time-dependence to the gravitational constant, such that $G \approx \frac{1}{T}$ and T is the age of the universe differently from GR, where, hereafter, the gravitational constant G has a fixed *three-dimensional* value in accordance with the Newtonian theory. Moreover, with the same definitions of eq.2, Dirac added a scalar field to Wely's action and found the following functional

$$S[\phi, k_\mu] = \frac{1}{2} \int d^4x \sqrt{-g} \left\{ \frac{1}{2} F_{\mu\nu} F^{\mu\nu} +^* R\phi^2 + \alpha \phi_{*\mu} \phi^{*\mu} \right\} \ , \tag{25}$$

where α is a dimensionless constant. Thus, the original Weyl's theory was modified by replacing the term $({}^*R)^2$ by ${}^*R\phi^2$ [31, 33].

The $\phi_{*\mu}$ term is the co-covariant derivative of ϕ and is defined as

$$\phi_{*\mu} = \phi_\mu + \phi k_\mu \; . \tag{26}$$

For many authors, the large numbers are only the result of numerological coincidence, until now without any experimental indications. Even so, such hypothesis is still considered by some authors [34, 35, 36] mainly of the strong influence of the cosmological constant problem and hierarchy problem of the fundamental interactions have imprinted in the recent years.

In addition, still in 1973, besides of Dirac-Weyl's and *f-g* theories, Rosen [30] proposed also the *bimetric* theory. This theory was built on Minkowski flat space-time with two symmetric tensors. The so-called $\Gamma_{\mu\nu}$ tensor would describe properties of space-time and was interpreted as a second rank tensor of spin-2. The $g_{\mu\nu}$ tensor was interpreted as a gravitational potential tensor and was responsible for making the interaction between gravitation, matter and other fields. As a criticism to GR, Rosen's bimetric theory does not provide such singularities as *black-holes* and, moreover, it can provide a gravitational energy momentum-tensor. Hence, when applied to cosmology, the universe predicted does not have an initial singularity as the *big-bang* being closed in space (closed curvature) and eternal in time [37, 16]. The main shortcoming of the theory is that provides a dipole gravitational waves, instead of a quadrupole modes like GR, which is at odds with the binary pulsar PSR1913+16 measurements. As well known, the binary pulsar PSR1913+16 has proven to be a valuable tool to test alternative gravitational theories [38]. This fact made the theory lose any theoretical interest until now.

On the other hand, it is instructive to point out that between 1930 and 1970, there was an intensive development on theories concerning the unification process dissociated from gravitation. The main contestant was the Yang-Mills scheme. In spite of some difficulties, as how to allocate all the particle groups in families, which would require an adoption of other sort of symmetries in a Grand Unified Theory (GUT) proposal, the standard model of gauge interactions unified electromagnetism, weak and strong interactions in the group $U(1) \times SU(2) \times SU(3)$.

In summary, the coupling of gravity to other fields constitutes a hard task due to the lack of understanding of what gravity really is. The problem persists because we still do not have a definite quantum gravity theory, despite of the advent of M-theory and brane-worlds. As stated by Misner [39], gravity does not behave as a gauge theory, that is, there is a qualitative issue that makes gravity different from other gauge interactions. As we are going to show throughout the next sections, dark matter and dark energy problems, essentially, as far as we conceived these days, both being effects of gravitation, aggravate this difference and require a deeper insight on the meaning of gravity and how it interacts with others fields constituting an additional barrier to an effective unified field theory.

Dark Matter phenomenology

When the universe became "dark"

Our standard knowledge of gravitation, formation and evolution of the universe is based on *General relativity.* It has a fundamental assumption that describes the universe in a reasonable manner: the so-called *Cosmological principle*. In short, the cosmological principle states that in large scales, based on redshift surveys up to 100Mpc (1Mpc$\sim 3,08 \times 10^{24} cm$) and the measurement of the galaxy 2-point correlation function [40], the observable universe (approximately 3000Mpc) can be regarded as homogeneous and isotropic. However, considering cosmological distances lesser than 100Mpc, inhomogeneity takes place. On this scale, galaxies, cluster and superclusters of galaxies have greater importance, which the Newtonian theory supposedly could be applied with a reasonable level of confidence, completing studies of kinematics and dynamics of these objects.

On the other hand, since the beginning of the 20th century, there has been observed some astrophysical problems regarding rotation of galaxies and clusters of galaxies. For instance, in the end of the 1920's J Oort [41] pointed out the differential rotation of the Milk-way, that is, the velocity in the core of galaxy was bigger than the velocity in its outskirts. In subsequent studies, Oort [42] noted that in the outer parts of galaxies, stars were moving faster than predicted by Newtonian gravity, suggesting that an additional force should exist to maintain stars orbiting one galaxy. Actually, what is really observed is the velocity of regions of hydrogen clouds which varies on the distance in respect to the core of the galaxy.

A similar situation appeared to occur at cluster scales. In 1933, F. Zwicky [43] noted that the galaxy velocities in the COMA cluster of galaxies were also moving faster than the velocities predicted by Newton's gravitational theory. The velocity of the cluster, as well as its stability could not be justified by only taking into account its visible mass. He found a total mass approximately 400 times bigger than the expected, considering the number of galaxies and clusters' luminosity. Zwicky named it the *missing mass problem*.

Surprisingly, the problem pointed out by Zwicky was almost forgotten by the scientific community until the 1970's, when it was retrieved by Rubin, Ford and collaborators [44, 45, 46, 47, 48], who obtained experimental evidences to support Zwicky's observations. They studied the path described by stars in galaxies where the function between the velocity of a star and its distance from the center of the galaxy is usually called *rotation curve* as shown in fig.1. The studies of rotation curves play an important contribution to our understanding of the formation of the galaxies, particularly in spiral galaxies where the galactic disks are observed [47, 48, 49]. In short, they discovered an unusual high speed of stars on the edge of spiral galaxies, completely contradicting Kepler's theory, where a slow down scenario is expected. Thus, *we should also add mass to maintain the galaxy's stability*, in the same way that it should be done to galaxies' clusters. When we study such motions, away from the core of galaxy, it is verified a discrepancy between the observed velocity and the theoretical prediction. Moreover, the same anomaly also appears in the elliptical galaxies case [50, 51, 52, 53]. It is important to note that Zwicky's proposal runs in the Newtonian context. In other words, Newton's theory should hold true also in galactic scales. Hence, the solution for the missing mass problem came with the idea of adding ordinary baryonic mass to the systems studied as a manner

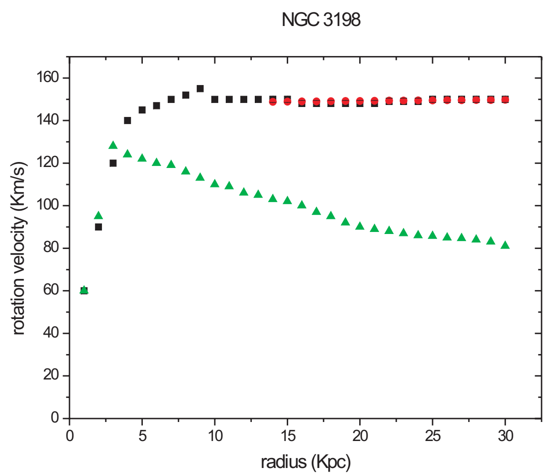

Figure 1: A standard representation of the rotation curve problem [54] in the NCG3198 galaxy. Note the discrepancy between the observed velocity (squared) and the theoretical prediction (triangles) away from the core of galaxy.

to preserve Newton's theory. We can understand baryonic matter as the common matter composed of elementary particles(quarks and leptons) from the standard model of particle physics.

Another intriguing effect is the Pioneer anomaly [55, 56]. The two space probes, Pioneer 10, launched on March 2nd 1972 to visit Jupiter, and Pioneer 11, launched on April 5th 1973 to Saturn, which they are under influence of an *unexplained* constant acceleration directed towards the sun with constant approximate value of $a = (8 \pm 1.3) \times 10^{-10} m.s^{-2}$. Surprisingly enough, this phenomenon is compatible with the numerical value of acceleration constant of the MOdified Newtonian Dynamics (MOND) [57, 58], proposed by Milgrom in the beginning of the 1980's. In principle, this deviation should had been explained by a gravitational field produced by GR, but it was not. Among other explanations for the Pioneer phenomenon, *a local effect, at solar system level* of dark matter also has been proposed to explain such anomalous behavior of the space probes [59] at odds with other works which state that dark matter in the solar system is not related to Pioneer anomaly [60] at all. However, with the improvement of observational devices, dark matter reveals a much more complex problem decisively affecting the way we perceive the universe.

Can we understand Dark Matter?

Our sense about dark matter changes as experiments become more and more effective. As discussed before, according to Zwicky's observations, dark matter consists of a sort of matter which neither absorb, nor emit light, or any electromagnetic radiation in any frequency bandwidth whatsoever. Its presence can be revealed through its gravitational field, or eventually, by weak interaction. The main mechanism of trying to identify dark matter concerns gravitational micro-lensing effects. These effects play an important role on mapping and measuring subtle luminosity distortions of objects on the space-time background. This can only be achieved through usage of advanced spectrographical and optical telescopes, such as NASA's Chandra X-ray Observatory [61], and Canadian-France-Hawaii Telescope (CFHT) [62] in Hawaii.

Recently, two merging phenomena in giant clusters gained substantial attention for providing the first sought-after apparently *direct* evidence of dark matter based on weak lensing, x-ray and visible optics astronomy. These measurements were collected by the Chandra X-ray Observatory on the so-called *Bullet cluster* [63] 1E0657-558, which essentially consists of two clusters forming a bullet-like structure tied to galaxies but moving through the intercluster plasma. Specifically, the center of mass of two spherically symmetric dark matter halo does not match the center of mass obtained by alterations of the gravitational force law with respect to the Newtonian theory.

On the other hand, the Abell 520 cluster (MS0451+02) observed a dark core [64], according to observations of CFHT, where dark matter does not appear to be anchored to any other galaxy, but in the intercluster plasma. Note that we presented two situations and two different outcomes. The silver line is pinpointing under what conditions and constraints dark matter justify such anomalous behaviors as those mentioned above. Nonetheless, due to the lack of a proper explanation, one can appeal that these evidences indicate an odd existence of dark matter, whatever that is [65].

Dark matter candidates

In a manner of trying to understand the nature of dark matter, two main approaches have been considered: first, the *Hot Dark Matter* (HDM) model, which states that when the galaxies were first formed, dark matter is composed of relativistic particles ($kT >> mc^2$). This odd reference to "temperature" refers to the energy levels of these particles and how fast they travel. The main candidates are neutrinos and the hypothetic Strongly Interactive Massive Particle (SiMPs) [66, 67]. But the quantity required for neutrinos [68] is larger than observed, so they can not lead alone to formation of any large scale structure. Moreover, relativistic particles do not clump, due to their high speeds, which is at odds with observations about the universe structure. Until now, constraints on HDM kept it away as an odd solution for the dark matter problem.

The second approach is the *Cold Dark Matter* (CDM) hypothesis, which consists of non-relativistic particles. The main candidates are the *Weakly Interacting Massive Particles* (WIMPs) [69] mainly represented by the supersymmetric particles called *neutralinos*. In addition, the *Massive Compact Halo Objects* (MACHOs) are just generic denominations to massive cosmic bodies, such as massive planets as Jupiter, or/and distributed in a spherical halo, orbiting the galaxy itself, far away from the stars. It is important to point out that MACHOs are composed of baryonic matter. Moreover, the quantities required for solving the rotation curve problem, or the large structure formation, are much bigger than the observations of MACHOS in the universe. These components have been observed with help of gravitational microlensing effects, but only in very small amounts far beyond the large quantity needed.

Another candidate to CDM is the Axion [70], which is a hypothetic spin-0 particle, originally postulated by Peccei-Quim's theory to solve CP problems in quantum chromodynamics. It was regarded as a candidate because it has a very small mass $\left(10^6 - 10^2\ eV.c^{-2}\right)$ and zero electric charge. However, current debates on the influence of dark matter on large structure formation suggest that cold dark matter is related to a gas of generical Wimps due to some of dark matter's properties as long-lived and stable particles and regarded as the main character to formation of large scale structure.

Dark matter at cosmological scales

Today's sophisticated astrophysical experiments tell us that dark matter cannot be understood only taking into account the local rotation curves problem or slow moving objects in clusters and colliding clusters. If one asks for the origins of dark matter, we end up in the early universe, when dark matter was supposed to break the cosmological homogeneity of baryons, thus creating large structures as observed today. In this context, the natural starting point to study dark matter is its gravitational field.

In the cosmological scale, dark matter seems to be consistent with the standard FLRW model, but only recently the cosmic microwave radiation data collected by WMAP indicated that most of dark matter content must be cold, and must be of *non-baryonic nature*, i.e, out of the standard model of particle physics. As suggested in some works [70, 71, 72], cold dark matter plays a serious role in baryogenesis process and could induce to a small overdensity in the primordial universe after the initial inflation, which could give birth to an extra gravitational attractive force, leading to a large scale structure formation by the growth of perturbations and gravitational instability. According to WMAP data, dark matter is a dominant component in the early universe representing more than 60% of the total energy density.

As it happens, dark matter seems to be the "spark" that unrolled the formation of large structures. Otherwise, if we consider the HDM hypothesis, it produces a "top-down" scenario, where large structure objects (clusters and superclusters) are formed before small structures, due to the fact that relativistic particles do not clump. Nevertheless, there are strong evidences of the existence of young galaxies [73] of order of 500 or more celestial objects, which were formed at least 13 thousand million years after the big bang [74]. This is a stunning fact because according to the big-bang theory they would not have enough time to be formed. This notion is at odds with galaxies evolutionary timescales. Other works state that a Giant Boson star [75] could be a more reliable model that provide a solution to, for instance, the flatness of the rotation curves and halo formation to the detriment of the CDM model. Therefore, either some exotic particles must be considered, or else an adequate gravitational theory should be devised.

As shown in fig.2, on the fifth year of WMAP observations revealed a new distribution of the composition of the universe [76, 77], which shows that the thermal radiation is about $2.73K$; also no substantial amount of anti-matter was found. It also revealed that 23% of the universe is composed of Dark Matter, 72% of Dark Energy and only 4.6% of visible or baryonic matter (H$\sim$ 75%, He$\sim$ 25% and trace amounts of heavy elements). Moreover, the 23% total energy density of the universe related to Dark matter is mostly of cold (non-relativistic) and non-baryonic nature. More specifically, the analysis of the power spectrum (fig.3) indicates that a theory of gravity based essentially on the properties of baryonic matter would produce a lower third peak [76, 78] and it would be incompatible with the universe formation. These evidences contributed to the acceptance of the CDM models to the structure formation. Today, the most accepted phenomenological models are also based on CDM proposal, for instance, the Λ and X-CDM models. For further information see [71, 79, 80, 81].

Alternatives to Dark matter

The most simple model of dark matter consists on the so-called

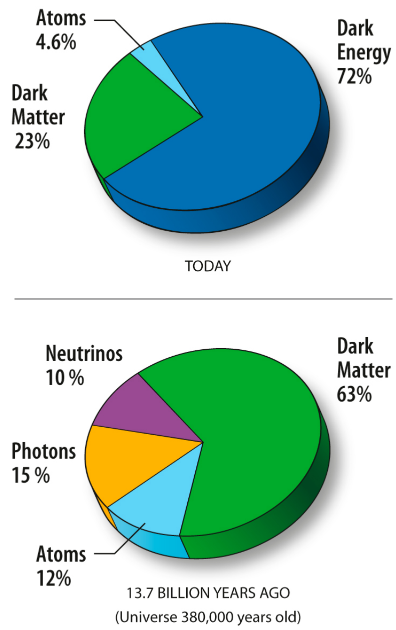

Figure 2: The universe content today and in the early universe according to the fifth year of WMAP measurements[77].

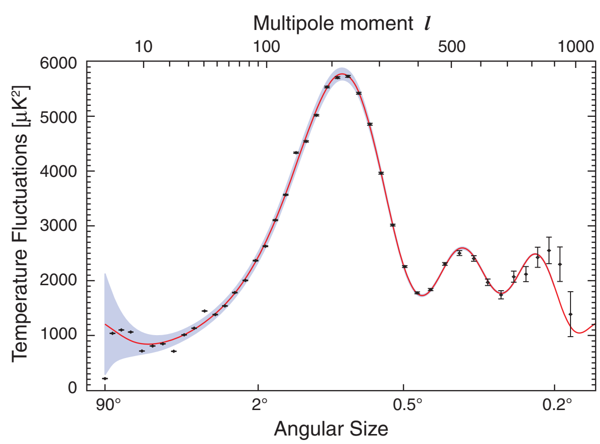

Figure 3: The WMAP fifth year cosmic microwave background power spectrum, which shows a improvement on the measurement of the third acoustic peak[77, 78].

Dark Matter Halo hypothesis, in which a galaxy would be embedded in a dark matter bulk which is extensively used in simulations of dynamics of universe [82]. This concept of a dark matter bulk has its origins in the end of 20's and mid 30's due to Oort and Zwicky's observations. Hence, it was entirely based on Newton's theory satisfying a particular symmetry and boundary conditions. The term bulk here is used only to try to explain where the missing mass would be allocated or how further it is extended, being a proposal of how to recover a gravitational pull that should exist. Since dark matter interacts with ordinary matter essentially

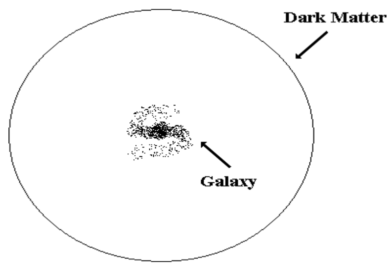

Figure 4: The Dark matter bulk [83].

by gravity, and that is assumed to be of Newtonian nature, most present dark matter models depend upon Newton's theory and, therefore, are gauged by Newtonian gravity paradigms. Thus, a previously defined gravitational theory must be postulated before the analysis of CMBR power spectrum experimental results can be more conclusive. Indeed, all simulations and comparisons are estimated with respect to Newtonian forces derived from the Newtonian gravitational potential. Even so, the recent data suggests that dark matter, whatever it may be, induces serious constraints to gravitational theories based only on baryons or in other words the inertia concept like MOND.

MOND [57, 58] has received a substantial attention in recent years and it has been backed by a theory in which Poisson's equation for the Newtonian gravitational field is replaced by the equation

$$< \nabla, \mu \left(\frac{|\nabla \Phi|}{a_0} \right) >= 4\pi G\rho \ \ , \tag{27}$$

where $\mu\left(\frac{|\nabla\Phi|}{a_0}\right)$ is a function to be adjusted to the specific type of galaxy, a_0 is an acceleration constant with magnitude $a \sim 9 \times 10^{-10} m.s^{-2}$, Φ is the Newtonian gravitational field and ρ is the energy density of the baryonic matter source. Recently, a relativistic theory of MOND called TensorVectorScalar or TeVeS [84, 85] has been developed. This relativistic model includes tensors, vectors and scalar fields in a manner of providing an alternative cosmology and in some sense generalizing the original MOND to cosmological scales.

On the other hand, it seems obvious that General Relativity regarded as the correction to Newton's theory would be a natural candidate to deal with the curve rotation problem and the dark matter problem. The standard argument against the effectiveness of GR is that gravitation is much stronger in the core of galaxy than in its external points and that is where GR would hold and should provide the required correction. However, it agrees with it precisely where Newton's theory holds. This is due to the huge concentration of mass in the core of the galaxy and it produces a spherically gravitational field but beyond that region, the gravitational field becomes sufficiently weak to be taken over again by its Newtonian limit. As well known, Schwarzschild's solution is an exact spherical solution of Einstein's equations and with correct assumptions we can derive Newton's gravitational potential. Therefore, the gravitational field should be Newtonian everywhere else in the galaxy.

If we want to look further, we can extend our analysis to *the Parametric Newtonian Approximation* of GR, and find a gravitational potential which decays by the law $\left(1/r^3\right)$ [86]. Unfortunately this law is not consistent with observations leading to a rapid decay of the rotation curve away from the core. This rationalization implies that the exact solution to Einstein's equations can be disregarded under dark matter context. This is due to that the lack a proper justification why general relativity cannot be used in the dark matter problem. For instance, we present some argumentation: first, Newton's theory does not describe a strong gravitational field like those observed at the galaxy cores [87, 88]. Secondly, the weak gravitational lensing used to detect the presence of dark matter in clusters cannot be described with Newtonian gravity. These two evidences suggest that if the predominant gravitational field in galaxies and clusters is due to dark matter, then the dismissal of general relativity in favor of Newtonian gravity is not completely justified.

These attempts to explain dark matter have motivated the emergence of many others gravitational theories, like, for instance: (1) Adding a scalar field to Einstein's equation, in such a way that the scalar-tensor theory corrects the Newtonian limit [89]; (2) Modifying the concept of time in general relativity, so that the Newtonian limit of the theory differs from the original Newton's theory [90, 91]; (3) Adding a cosmological constant with appropriate sign [92]; (4) Including higher order curvature terms in the gravitational variational principle [93]; (5) quantum cosmology based on *a priori* stochastic process considering the universe as a pregeometric system [94];(6) Several brane-world models and variants have been considered in the hope that more general braneworld equations of motion may provide the correct velocity curves

[95, 96, 97, 98, 99, 100]. Nevertheless, the problem with the constraints provided by observations are still too subtle and difficult to deal with. From the theoretical point of view, in a self-consistent manner capable of taking into account both cosmological and local effects of dark matter, a sought-out dark matter model is still an ideal.

The Dark Energy problem

In order to obtain more insight on the dark energy problem, it is instructive to attain some remarks on its history before we discuss the problem itself. Nowadays, the current theoretical debate about dark energy problem is related *mainly* to the cosmological constant as we are going to point out in the next subsections.

Einstein's dilemma and the cosmological standard model

After proposing his equations in the end of 1915, Einstein concluded that they could provide a non-permanent or static universe. But at that time the universe was bounded by western philosophical thoughts [101], which stated the eternity of the universe without beginning or end.

In accordance with this philosophy and the lack of any precise experimental data about the conditions of the universe, Einstein was compelled to modify his original theory by introducing a new term $\Lambda > 0$ in order to obtain static solutions. In 1917, Einstein proposed that the equations could be written as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \ , \tag{28}$$

by using the spherical metric $ds^2 = dt^2 - R^2(d\chi^2 + \sin^2\chi d\theta^2 +$

$\sin^2\chi\sin^2\theta d\phi^2)$, where R is the constant radius of a 3-D sphere, χ runs from 0 to π, and $c = 1$. Hence, one could generate a static dust-dominated universe visualized as a perfect fluid with constant density $\rho = \Lambda(8\pi G)^{-1}$, radius $r = (8\pi G\rho)^{-1/2}$ and mass $M = 2\pi^3 r^3\rho = \frac{\pi}{4}\Lambda^{-1/2}G^{-1}$.

Starting from these seminal results, Einstein stated that the inertia of a body could be "induced by its mass but not determined by it" [102] in accordance with Mach's principle. This principle was originally proposed by Mach [103] in 1883, which consisted in a relativity of concepts of inertia at odds with the Newtonian concept of absolute space and time. According to Mach, the inertia of a body was generated by the influence of the entire mass of the universe on the body. In fact, despite the fact that GR does not fulfill all the requirements of Mach's principle, due to the equivalence principle of GR, even so Einstein believed that by introducing a cosmological term he would be able to solve this question [104].

Still in 1917, de Sitter presented a new result by adding the cosmological constant to Einstein's equation in vacuum ($T_{\mu\nu} = 0$) with the line element

$$ds^2 = \frac{1}{\cosh^2 Hr}[dt^2 - dr^2 - H^{-2}\tanh^2 Hr(d\theta^2 + \sin^2\theta d\phi^2)]\ , \tag{29}$$

where $H = \sqrt{\Lambda/3}$ in a spherical quasi-static universe with radius $R = 3\Lambda^{-1}$. Weyl [105] and Eddington [106] checked independently that in the de Sitter's universe, two arbitrary test particles could repel each other. This fact was the first theoretical evidence of a possible expanding universe [102].

In 1922, Cartan [107] demonstrated that the most general expression of Einstein's tensor was guaranteed by adding a term multiplied by the metric, in accordance with Bianchi's identities. Hence, the existence of Λ is a consequence of the "imprecision" of Riemann's geometry with respect to the shape of the objects. Thus, the dismissal of Λ is only justified by one of the following arguments: symmetry, or a observational data constraint. Nevertheless, the proposal of a static or quasi-static universes started to fail in the subsequent periods.

The first strike on the cosmological term came along in the same year by Friedmann. Friedmann [108] published a paper in which he demonstrated a dynamical solution *without* the cosmological constant by assuming a homogeneous and isotropic universe. The element line proposed was

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] , \qquad (30)$$

where $a(t)$ is *a priori* an unknown function of time and k is a constant. When applied to cosmology, $a(t)$ is the scale factor which can describe the distance between comoving observers as a function of time, and $k = 0, \pm 1$, which corresponds to the spatial curvature of the universe . Hence, the model predicts three possibilities for the geometry of the universe: $k = 0, -1$ or $+1$, which corresponds to a flat (asymptotically expansion), parabolic (contracting universe) and hyperbolic (eternal expansion) universe, respectively which depends on the total mass of the universe. The same results were obtained independently by A. Walker [109] and H. Robertson [110], and with contributions of G. Lemaître [111]. It turned out to be the well-known Friedmann-Lemaître-Robertson-Walker (FLRW) metric.

It is important to point out that the Lemaître model of the universe consisted of an intermediate solution. It began in a static Einstein universe and led to a vacuum solution of the expanding deSitter universe. The universe was originated by what he named the *primeval atom*, launching a primitive idea of what we now know as the *big-bang* model.

Before we proceed further, it is instructive to attain some aspects of the FLRW model. First, Friedmann equation can be reproduced by taking the energy-momentum tensor of a perfect fluid in comoving coordinates

$$T_{\mu\nu} = (p+\rho)U_\mu U_\nu + p\, g_{\mu\nu}, \quad U_\mu = \delta^4_\mu\ , \tag{31}$$

where U_μ is the 4-velocity, ρ is the total density of all matter-energy contribution and p is the pressure of the perfect fluid. Thus, using the local conservation law $T_{\mu 4;\,\mu} = 0$, we obtain

$$\dot\rho = -3\frac{\dot a}{a}(\rho + p)\ , \tag{32}$$

where the dot represents time derivative. Secondly, by solving Einstein equations with the FLRW metric, we can find the spatial components the Raychaudhury acceleration equation taking into account the cosmological constant Λ

$$\frac{\ddot a}{a} = -\frac{4}{3}\pi G(\rho + 3p) + \frac{\Lambda}{3}\ . \tag{33}$$

Therefore, from eq.(32) and eq.(33), we can obtain the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a} = -\frac{8\pi G}{3}\rho + \frac{\Lambda}{3}\,, \tag{34}$$

where $a(t)$ is the scale factor of the universe. The geometrical meaning of k as the spatial curvature is given by the time component (44) of the Einstein equations. The Friedmann equation describes the dynamics of the universe and its validity at all times.

Alternatively, according to the experimental observations at present, one can write the equivalent form

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\,[\Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_M\left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda + \Omega_k\left(\frac{a_0}{a}\right)^2]\,, \tag{35}$$

or in terms of redshift z

$$H^2 = H_0^2\left[\Omega_R(1+z)^4 + \Omega_M(1+z)^3 + \Omega_\Lambda + \Omega_k(1+z)^2\right]\,, \tag{36}$$

where $\frac{a_0}{a} = 1 + z$, $H_0 = \left(\frac{\dot{a}}{a}\right)_0 = 75 \pm 10\ km.s^{-1}.Mpc^{-1}$ is the present value of Hubble parameter, i.e, the rate of expansion of the universe at present. The cosmological parameter Ω is defined as $\Omega = \rho(\rho_{crit})^{-1}$, where ρ is the energy density and $\rho_{crit} = 3H^2(8\pi G)^{-1} \sim 10^{-29}g.cm^{-3}$. One can think of the critical density as the minimal scape velocity when we calculate the rocket problem in mechanics. However, in cosmology, the critical density is regarded as the minimal amount of energy density to maintain a homogeneous and isotropic universe. Hence, if $\rho > \rho_{crit}$, or $\Omega > 1$, then $k = +1$, which gives a closed universe; on the other hand, $\rho < \rho_{crit}$, or $\Omega < 1$, implies $k = -1$, an open universe, and finally, when $\rho = \rho_{crit}$, or $\Omega = 1$, we have a flat universe $k = 0$. According to CBMR observations, the *total* cosmological parameter Ω, or simply, Ω_{tot}, varies as $0.98 \leq \Omega_{tot} \leq 1.08$ which suggest that we live in an approximately flat universe [112, 113].

Considering all contributions to the content of the universe, we can normalize the equation to one and obtain

$$\Omega_{tot} = \Omega_R + \Omega_B + \Omega_{CDM} + \Omega_\Lambda = 1. \tag{37}$$

As it happens, $\Omega_R = 8\pi G\rho_r(3H^2)^{-1}$ corresponds to the radiation contribution of the universe of order of 5×10^{-5} with energy density $\rho_r \sim a^{-4}$; and $\Omega_M = 8\pi G\rho_m(3H^2)^{-1}$, related to the rest mass density $\rho_m \sim a^{-3}$, is the content of non-relativistic matter which we have separated into two parts: baryonic matter $\Omega_B \sim 0.046$, and the cold dark matter $\Omega_{CDM} \sim 0.23$. If we reconsider the cosmological constant and impose that it plays the role of dark energy, we have its contribution as $\Omega_\Lambda = 8\pi G\rho_\Lambda(3H^2)^{-1} \sim 0.72$ and $\Omega_k = 8\pi G\rho_k(3H^2) \sim 0.0 \pm 0.1$, which represents the contribution of the geometry or curvature of the universe with energy density $\rho_k \sim a^{-2}$.

In fig.5, it is shown the possible solutions provided by the FLRW model with the values of the spatial curvature parameter k and the cosmological parameter Ω. In addition, we point out some experimental facts which contributed decisively for the dismissal of a cosmological term at that time. For instance, the measurements of redshift of galaxies by Slipher [106] in 1924, as well as Hubble's pioneer observations of homogeneity and isotropy [114] in large scales and redshift [115] in 1929. It all provided acceptance of the FLRW model. Compelled by these facts, Einstein removed the cosmological constant, stating that it was the *biggest* blunder of

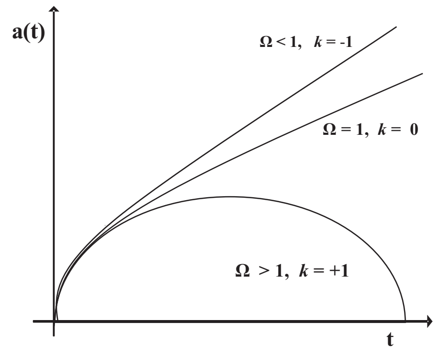

Figure 5: Evolution of the scale factor in different scenarios according to the values of the spatial curvature and cosmological parameters, k and Ω, respectively. Actually, with the accelerated expansion of the universe more cosmological scenarios have been currently proposed.

his life.

In 1964, the detection of CMBR by Penzias and Wilson [116] marked an important moment for observational cosmology, confirming and stating the FLRW model as the standard model of cosmology. On the other hand, improvement of observations, such as the *COsmic Background Explorer* (COBE) in 1992 [117, 118], and more recently, its successor, WMAP, has appointed some drawbacks in the model as, for instance, the presence of the well-known anisotropies of the CMBR, as well as the small temperature fluctuations on large scales of the order of $\frac{\Delta T}{T} \sim 10^{-5}$ [119], which cannot be explained rigourously by the standard model without an adjustment of the mechanism [120, 121].

The development of observational experiments has reconsidered the cosmological constant firmly in the last decade, which has revealed that Λ does not vanish with the effective value $|\Lambda_{eff}| \sim 10^{-47}\ GeV^4$. A non-vanishing cosmological constant is compatible with the ancient globular clusters, reconciling with the matter density observed [122, 123, 124]. Otherwise, the value of the cosmological constant, if regarded as a vacuum energy (as proved by the Casimir effect [125, 126, 127]), is at odds with the theoretical value of the quantum energy density predicted by QFT. Indeed, the situation has been aggravated by the decisive observations of the accelerated expanding universe in 1998.

The accelerated universe

The first evidences of an accelerated expansion of the universe was obtained from Hubble Space Telescope (HST) of current type Ia supernovae (SNIa) in 1998 [1, 2], in agreement with Chandra observations [61]. Moreover, it was sustained by measurements of the cosmic microwave background (CMB) anisotropies [128] and the large scale structure data [129]. The data suggested the existence of a density energy component unclustered that fulfill 72% [76] of the universe with negative pressure driving the universe to an accelerated phase of expansion, that is, a repulsive effect of gravity. This effect was the so-called *Dark Energy*, which is corroborated by 250 independents astronomical observations events in supernovae [41, 130].

In principle, the first interpretation for the acceleration of the universe is given by the FLRW standard model. According to eq.33, we can get two important conditions: first, the *strong* energy condition: when $(\rho + 3p) > 0$, then $\ddot{a} < 0$, i.e, the gravity decelerates the expansion of the universe. Second, the *weak* energy condition: when $(\rho + 3p) < 0$, with negative pressure $p < 0$, then gravity accelerates the relative expansion between the structures of the cosmos. This last condition is compatible with the experimental observations of supernovae regarded as standard candles (stellar objects that are used to infer distances based on its luminosity). In a manner of trying to deal with the observations, the cosmological constant has been reconsidered as a dark energy component. The presence of such term induces to an existence of a repulsive gravity in universe and, in fact, turned out to be the most simple option to deal with. The main debate now concentrates on the perturbation of the Friedmann equations with Λ as in eq.(34) or, besides the context of GR, Λ plus a correction term

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a} = -\frac{8\pi G}{3}\rho + \frac{\Lambda}{3} + \ (correction\ term)\ . \tag{38}$$

In the following, we present some of proposals to solve the dark energy dilemma.

Dark Energy, the Cosmological Constant problem and some alternatives

In fact, the dark energy problem is related to the foundations of gravitational cosmological theory and it has stimulated a demand for gravitational models and theories. In this subsection, amidst other proposals, we make general comments of some current proposals to modify GR so it fits on the assumption of extra-dimensional models. Most of these proposals try to solve both dark energy and cosmological constant problems. Since the cosmological constant can be regarded as dark energy candidate it is inevitable the disassociation of them.

The so-called *Cosmological Constant problem* had its first seeds planted in 1916, with the ideas of Nernst [131]. He studied the non-vanishing vacuum energy density that was fulfilled with radiation-only content, which was confirmed by the Casimir effect in 1948

[125, 126, 127]. Originally, Casimir effect consists in the effect of approximation of two separated uncharged conducting plates due to the zero-point energy density of the electromagnetic field. The Casimir force is generated by the energy density difference of pairs of virtual particles and virtual-antiparticles between the plates and outside the plates. Hence, the difference of pressure outside the plates are more intensive than between the plates which generates a force on the plates approximate them. In this manner, it was the first experimental evidence of an existing non-vanishing contribution of quantum vacuum energy density.

In late 1920's, Pauli [132, 133, 134] made studies about the gravitational influence of the vacuum energy density of the radiation field, suggesting a conflict between the vacuum energy density and gravitation. If vacuum energy density is considered, then gravity must be dispensed. Moreover, based on Pauli's work, Straumann [132, 133] restated that if one can consider the static Einstein dust-dominated universe, the radius of the universe would be of the order of 31km, lesser than the Earth-Moon distance, thus confirming the conflicting Pauli's results that passed unnoticed by scientific community. Even so, in the subsequent decades, even with the dismissal of Λ by Einstein, some universe models based on Λ were still studied, for instance, the Lemaître model, as pointed out in the previous subsection. In addition, the observations of quasars in the mid-late of the 1960's suggested the reconsideration of Λ [135].

The quantum vacuum energy density as cosmological constant

In 1967, Zel'dovich [136] had a breakthrough proposing the hypothesis in which Λ is the vacuum energy. In contrast with Pauli's conclusion, the vacuum energy density is considered, and gravity must also be taken into account. By considering a perfect fluid, the vacuum energy-momentum tensor $T_{\mu\nu}^{\ vac}$ can be given in comoving coordinates as

$$T_{\mu\nu}^{\ vac} = (p_{\ vac} + \rho_{\ vac})\, U_\mu U_\nu + p_{\ vac} g_{\mu\nu} \ \ , \tag{39}$$

where $p_{\ vac} = - < \rho_{\ vac} >$ and $< \rho_{\ vac} >$ is the expectation value of quantum vacuum energy density, which is the analogue expression to the ordinary perfect fluid $T_{\mu\nu}^{\ mat}$ in eq.(31). If we take Einstein equations with Λ, we can write

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G \left(T_{\mu\nu}^{\ mat} + T_{\mu\nu}^{\ vac} \right) \ \ . \tag{40}$$

But, in vacuum, $T_{\mu\nu}^{\ mat} = 0$ and taking the covariant derivative, we find

$$< \rho_{\ vac} > = \ constant \ \ , \tag{41}$$

thus,

$$< \rho_{\ vac} > = \ \frac{\Lambda}{8\pi G} \ \ . \tag{42}$$

Therefore, according to Zel'dovich Λ can be regarded as a quantum vacuum energy density when vacuum is regarded as a perfect fluid, with $p = -\rho$. However, this situation would not reveal a mere fact with the development of phenomenological observation techniques. Effectively, the problem comes up because there is a gap of the order of 123 decimal places between the cosmological observed value of $\Lambda/8\pi G \approx 10^{-47}\ Gev^4$ and the theoretical vacuum energy density prediction $< \rho_{\ vac} > \sim 10^{76}\ Gev^4$.

From a geometrical point of view, the cosmological constant problem is shown to be a consequence of the *equivalence class of* *metric geometries* characterized by the Riemannian tensor. General relativity avoids this difficulty by postulating the Minkowski space-time as the standard flat geometry, from which we derive the concepts of particles, quantum fields and their vacuum states. On the other hand, the experimental evidences of a small but non-zero cosmological constant is not compatible with the Minkowski space-time, but it is consistent with the deSitter space-time. Either we adopt the Minkowski flat-plane standard of curvature or, in face of the observations we adopt the deSitter standard. Therefore, it appears that the emergence of the cosmological constant problem is a symptom of the lack of an independent reference standard for curvature in Riemann's geometry.

On the other hand, as a realization of Heisenberg's uncertainty principle, the theoretical value for the vacuum energy density is obtained from the individual contribution of each oscillator of mass m and wave number k_{max} cutoff in a set of harmonic oscillators(with $\hbar = c = 1$)

$$< \rho_{\,vac} >= \int_0^{k_{max}} \frac{4\pi k^2 dk}{2(2\pi)^3}\sqrt{k^2+m^2} \sim \frac{k_{max}^4}{16\pi^2}\,. \tag{43}$$

In order to avoid the ultra-violet(UV) divergence, one can impose a finite maximum value for k_{max}. And considering $\Lambda = (8\pi G)^{1/2}$, $< \rho_{\,vac} >$ results in $< \rho_{\,vac} >\sim 10^{76}\ Gev^4$, as stated before.

According to Weinberg [137], the problem was taken seriously in the 70's with the spontaneous breaking symmetry on the electroweak scales. Even if we consider the lowest energy scale of the quantum chromodynamics (QCD), which is of the order of $0.3\ Gev$, we still have a huge difference of 46 decimal places, when compared to the cosmological observational value for Λ. Such large difference cannot be eliminated by renormalization techniques in quantum field theory unless an extreme fine tuning can be applied [55, 112, 137]. But, why is the cosmological value of Λ so small and can not be regarded as zero? and why is it observed today? These are an examples of unanswered questions. Even so, the cosmological constant is one of the most important candidates to dark energy.

Therefore, the cosmological constant problem has become one of the most important problems in modern physics, because it is a problem of fundamental nature, not only because it involves the structure of the Einstein-Hilbert principle, but also because it apparently deals with the distinction between gravitation and gauge fields.

Some basic approaches about dark energy and cosmological constant

In accordance with [55], we have chosen some interpretations and proposals of solutions for the cosmological constant problem and/or the dark energy problem. Here we point out some models related to fine-tuning process, symmetry mechanisms, violation of the *equivalence principle* models and statistical approaches. Most of them are extensively explored nowadays despite of the lack of a proper explanation by first principle of a complete theory.

Fine-tuning mechanisms

Concerning *fine-tuning mechanisms*, the simplest idea is to consider the cosmological constant as a dark energy component, without separating the concepts of quantum vacuum energy density and the cosmological constant. As conceived in the Λ-*Cold Dark* *matter* (ΛCDM) model, the cosmological constant is as a source term that obeys the cosmic equation of state with pressure $p_\Lambda = w_\Lambda \rho_\Lambda$, where $w_\Lambda = -1$ and the energy density $\rho_\Lambda = \Lambda/8\pi G$. In spite of its simplicity, it is in good agreement with experimental data from WMAP and others astronomical data [76, 138].

Another attempt was, rather than supposing Λ as a constant, regard it as a function of time. The time-varying "cosmological constant" or $\Lambda(t)CDM$ predicts that the vacuum quantum energy density decays into CDM particles transferring energy to them. In fact, this model proposes the introduction of a new field produced by $\Lambda(t)$ responsible for the acceleration of the universe. The coupling between the energy-momentum tensor $T_{\mu\nu}$ and $\Lambda(t)$ is a consequence of the Bianchi identities when applied to Einstein's equations and is given by

$$T_{\mu\nu;\nu} = -\left(\frac{\Lambda(t) g_{\mu\nu}}{8\pi G}\right); \nu \tag{44}$$

where the energy density and the cosmological scale factor $a(t)$ are related by the *ansatz* $\rho_m = \rho_0 a^{-3+\epsilon}$ by taking a small deviation ϵ from the standard cosmic evolution. Calculating the conservation of the energy-momentum tensor $T_{\mu\nu;\,\nu} = 0$, one can find

$$\dot{\rho} + 3\frac{\dot{a}}{a}\rho = \dot{\rho_\Lambda} \ , \tag{45}$$

where we denote the ordinary time derivative by a dot. And hence

$$\rho_\Lambda = \bar{\rho_{\Lambda 0}} + \frac{\epsilon \rho_0}{(3-\epsilon)} a^{-3+\epsilon} \ , \tag{46}$$

where ρ_Λ is the energy density contribution from Λ, ρ_0 is the current CDM energy density and $\bar{\rho_{\Lambda 0}}$ is an integration constant

[139, 140].

In spite of some merits, the lack of some explanations from ΛCDM to, for example, predict the cusped central density on galactic sub-scales structures which are at odds with observations [141, 142, 143], has motivated other phenomenological proposals. One of them is the *X-Cold Dark matter* (XCDM) model, which is characterized by the equation of state $p_x = w_x \rho_x$ of an exotic fluid. The w_x parameter can be a constant or more generally, a function of time [144, 145, 146, 147], which has a plethora of proposals. The energy-conservation equation can be written as

$$\dot{\rho_x} = -3\frac{\dot{a}}{a}\rho_x(1 + w_x) \ . \tag{47}$$

Hence, if w_x is a constant and $w_x < 0$, one can find

$$\rho_x \sim a^{-3(1+w_x)}, \tag{48}$$

where ρ_x is the density contribution from the X-fluid. One of the justifications for $w_x < 0$ is that w_x is a sufficient smooth component making it compatible with the age of the universe as well as the rate of growth of the density perturbations in small scales, plus it gives out solutions to the problems of redshift in SNIa and gravitational lensing measurements [120].

Moreover, the values of the parameter w_x define different cosmological scenarios. For instance, in order to reproduce an expanding universe, one must set $w_x < -1/3$ which gives a large contribution to $(\ddot{a}/a)$. And when $w_x = -1/3$, it imprints no effects on $(\ddot{a}/a)$, i.e, it reproduces the same scenario as in the standard open universe without any dark energy assumption. There is still a weird scenario of a universe when $w_x < -1$ that proposes the existence of some sort of exotic fluid that violates all energy conditions and induces a huge increase of negative pressure, driving the universe to a singularity at a finite time named *Big-Rip*, where the factor scale and the curvature of the universe diverges [104, 148, 149, 150, 151]. This scenario is still an odd possibility, since it was recently constrained by observations of the Chandra x-ray observatory [61]. For a flat universe based on SNIa and CMBR data, we have $-1.11 \leq w_x \leq -0.86$[152, 153], and based on X-ray clusters and SNIa surveys $w_x = 0.95^{+0.30}_{-0.35}$ [154, 155, 156]. When $w_x = -1$, we have the ΛCDM model, which fits to recent WMAP observations [78] on CBMR. Moreover, if we define

$$\Omega_x = \frac{8\pi G\rho_x}{3H_0^2} \ . \tag{49}$$

Assuming w_x is constant and neglecting the current tiny contribution of Ω_R and Ω_k, we can rewrite Friedman equation simply as

$$H^2 = H_0^2\left[\Omega_M(1+z)^3 + \Omega_x(1+z)^{3(1+w)}\right] \ . \tag{50}$$

To analyze the evolution the universe, we can study the deceleration parameter, in terms of the redshift, given by

$$q(z) = \frac{1}{H}\frac{dH}{dz}(1+z) - 1 \ , \tag{51}$$

where H is given by eq.(50). Thus, we obtain

$$q(z) = \frac{3}{2}\left[\frac{\Omega_M(1+z)^3 + (1+w)\Omega_x(1+z)^{3(1+w)}}{\Omega_M(1+z)^3 + \Omega_x(1+z)^{3(1+w)}}\right] - 1 \ . \tag{52}$$

and plot the behavior of the deceleration parameter running the values of w terms of redshift as shown in fig.6. The values $q =$

$-0.6 \sim -0.7$ are the current values for the deceleration parameter compatible with the constraints from supernovae observations which for $q = -0.6 \sim -0.7$ is the current value for the decelera-

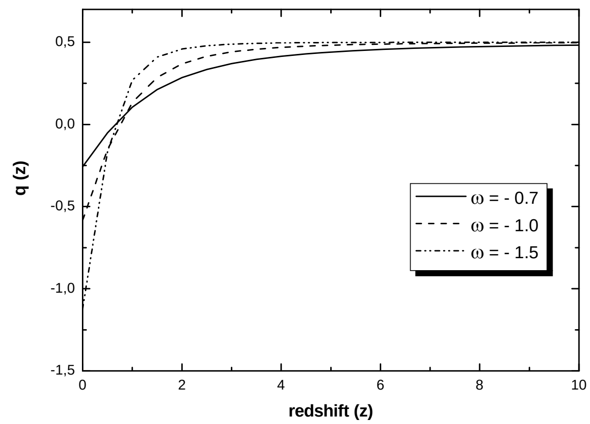

Figure 6: Deceleration parameter as a function of redshift for a fixed value of $\Omega_x = 0.7$ and some selected values of w.

tion parameter compatible with the constraints from supernovae observations.

There is another approach, where one can state that dark energy is provided due to some sort of unclustered scalar field according to the quintessence proposal in such a manner that drives the universe to speed up. The *quintessence* [157, 158] model consists in an addition of a minimal coupled scalar field $V(\varphi)$ to Einstein's equations, which yields a sought-after extreme fine-tuning to solve the hierarchy discrepancy [159, 93]. Writing the energy-tensor for a scalar field $V(\varphi)$ as

$$T_{\mu\nu} = \varphi_{,\mu}\varphi_{,\nu} - \left(\frac{1}{2}\varphi^{,\alpha}\varphi_{,\alpha} - V(\varphi)\right) g_{\mu\nu}\ , \tag{53}$$

and using the conservation law for $T_{\mu\nu}$, one can find the Klein-

Gordon equation

$$(\varphi^{;\mu}_{;\mu}) + \frac{\partial V(\varphi)}{\partial \varphi} = 0 \ . \tag{54}$$

Thus, the related field equation is

$$\ddot{\varphi} + 3H\varphi + \partial_\varphi V(\varphi) = 0 \ , \tag{55}$$

where we denote $\partial_\varphi = (\partial/\partial\varphi)$, where H is the Hubble parameter given by eq.(34) and the total energy density is defined as $\rho = \rho_m + \rho_\varphi$. The energy density of matter can be given by eq.32 while the energy density of the quintessence field φ is given by

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) \ . \tag{56}$$

The underlying idea is to create a mechanism of decaying for the energy of vacuum with a low varying expansion rate [137]. In spite of consisting of a good scheme as a phenomenological model, it lacks fundamental based grounds with *ad hoc* proposal of a quintessence potential. Nevertheless, some researches consider that the quintessence field may not only be identified as the dark component dominating the current cosmic evolution, but also as a bridge between an underlying theory and the observable structure of the universe [140].

Symmetry mechanism

To proceed further, in the *scale invariance approach* we have unimodular theories [160] of gravitation as examples of proposals. Basically they modify Einstein-Hilbert principle S_{EH} by introducing a Lagrange multiplier $\mathcal{L}$ in order to substitute the cosmological constant. This leads to a modified Einstein-Hilbert $S_{mod\,EH}$ principle and a fixed absolute space-time volume element, the so-called *modulus* [161]. The Jacobian g is regarded as a fixed constraint, and as a result we have a new integration constant Λ' in the Riemann Scalar R in such a way that

$$S_{mod\ EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R - \mathcal{L}(g-1)\right)\ . \tag{57}$$

By varying the modified action $S_{mod\ EH}$ with respect to the metric $g_{\mu\nu}$, we can obtain

$$R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} = 0\ , \tag{58}$$

and using the Bianchi identities result in

$$\partial_\mu R = 0\ . \tag{59}$$

Hence, R is a constant and can be set as $R = -4\Lambda'$ which allows us to write eq.(57) as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \Lambda' g_{\mu\nu}\ . \tag{60}$$

Although it has a reduction of 2 levels of degrees of freedom, due to the constraints $R = -4\Lambda'$ and $det(g_{\mu\nu}) \neq 1$, Λ' does not alter the dynamics of the equations and the problem endures so that the fine-tuning mechanism is still necessary. Consequently, Λ' can take any value, even not being explicitly provided in the modified Einstein-Hilbert action [162]. In spite of these shortcomings, the unimodular theories have been studied and applied to cosmology nowadays [163].

In addition, a more geometrical approach to the problem appeals again to modify Einstein-Hilbert action principle, such as in the so called $F(R)$ theories, using higher order Lagrangian, where the higher order curvature terms provide for the difference between the observed cosmological constant and the vacuum energy [164, 165]. However, once a physically justifiable Lagrangian such as the Einstein-Hilbert principle is replaced by $F(R)$, it becomes a necessity to properly justify a choice among a large variety of options. More specifically, it appears that the present astrophysical observations are not sufficient to decide on what $F(R)$ to choose[166]. On the other hand, recent studies suggests that $F(R)$ cosmology provides a accelerating behavior during attractor phase of matter-dominated era at odds with expectations, with expansion factor varying as $a(t) \propto t^{1/2}$ [167].

As we have discussed in the Dark matter section, in contrast with the dark energy repulsive effect observed in cosmological scales, dark matter is regarded as a sort of non-baryonic matter with merely attractive effect. Besides, a local effect on small scales is suggested, since it influences the growth of structures in the early universe. Thus, at first, dark matter and dark energy constitute elements with opposite gravitational characteristics and they are the main characters of the cosmological "tug-of-war" [168]. However, due to the lack of observational evidences, which can suggest that theses components are generated by different sources, there are some unification symmetry models that stick together both dark matter and dark energy. A very known model in this approach is the quartessence [169], which has as the main candidate some sort of exotic gas called Chaplygin gas [170], with the equation of state

$$p = -\frac{A}{\rho} \tag{61}$$

and, moreover, at a generalized form $p = -\frac{A}{\rho^\alpha}$, where the parameters A is restricted to $0 < A < 1$ and α to the range $-1 < \alpha \leq 1$ in a manner of reproducing the early and later times of the universe. Depending on the choice of parameters, the gas behaves sometimes attractively and sometimes repulsively, or equivalently, as similar to dark matter and dark energy which could be related to some topological change in the universe. However, it has some constraints [76, 171] based on SNIa experiments and statistics of gravitational lensing.

Another symmetry mechanism is related to supersymmetric models where, in short, the cosmological problem does not occur. Basically, a constraint imposed by supersymmetry (SUSY) on the vacuum energy prevents it from even existing. Thus, the sum of the contribution of al density states are canceled due to every supersymmetric particle has an equivalent superpartner, hence, $\Lambda = 0$. Even though Λ does not exist in supersymmetric models, it still is one of the most accepted proposals to explain dark energy. The current explanation is that when supersymmetry is broken, the dark energy, as conceived, comes up [137], and the scalar fields give the effective value of Λ in order to remedy this situation. Further information and recent works can be found in [172, 173, 174].

Violating the equivalence principle

The main example of violating the *equivalence principle* of General relativity is related to Brane-world models. Just like Superstrings or M-theory, these models bring to light the discussion of the existence of the extra-dimensions. They intend to provide a solution to the hierarchy problem, and possibly a sought-after unified physics theory of all conceived interactions.

In summary, these approaches are based on trying to decouple gravity from vacuum energy density, making it indifferent to gravitation. The general idea is that gravitation propagates in the extra-dimensions in a sort of "leakage" of gravitation into the bulk in which the brane (4-dimensional space-time) is embedded in. This hypothesis could explain how gravity is weaker than other interactions as measured by an observer on the brane, where other gauge interactions are confined. Actually, the confinement of the gauge interactions has to do with special relativity, where the standard model of particles and interactions are built on. In other words, it is a consequence of the Poincaré symmetry of the electromagnetic field, and in general, of the dualities of the Yang-Mills fields, which are consistent in four-dimensional space-time only. If we consider an odd brane-world quantum-gravity theory the gravitational particles, named gravitons, are regarded as some oscillations modes for the extra-dimensions.

One of the most known brane models is the Randall-Sundrum (RS) type II model [175]. When applied to Cosmology, the vacuum energy density in a 3-brane is smaller than the one predicted by quantum field theory, which means that the cosmological constant problem persists, even though the fundamental Tev scale energy is preserved. A similar situation occurs when we treat dark energy problem in which RS model II provides the modified Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_{pl}^2}\rho + \frac{16\pi^2}{9m_5^6}\rho^2 \ , \qquad (62)$$

where m_5 is the 5-dimensional planck scale, m_{pl} is the 4-dimensional planck scale. The correction term corresponds to the square of the energy density ρ^2 of the confined matter [176, 177, 178]. As it is well known, this result is not compatible with recent observational data [77, 179] since the additional term on Friedmann's equation, i.e, the energy density ρ^2, provides a deceleration scenario of the universe, besides affecting the nucleosynthesis of large structures. To remedy this situation, other attempts have been studied, such as particular classes of bulk and brane scalar potentials [180] that lead to a fine-tuning mechanism.

Another proposal is the Dvali-Gabadadze-Porrati or DGP [181] model where the 5-dimensional bulk is flat and the brane is fixed, that is, the embedding of the brane into the bulk is rigid with a noncompact, infinite-volume extra dimension. It also presents some difficulties related to strong interactions and massive gravitons and it does not duly adjust to the accelerated expansion scenario, even when studied on its general, the Dvali-Turner model [41, 55] which still requires, as it does the RS model, an extreme fine-tuning to make it compatible with the observational data.

A promising brane-world approach stated in [182] proposes a covariant (model independent) formulation of the brane-world theory based on the perturbational theory of local embedded submanifolds rather than particular junction conditions as commonly used in RS model and variants, hence the extrinsic curvature is considered as an independent field of spin-2 as compared with the metric. The main motivation of this approach has its roots in the classic problem in differential geometry, originated in the early days of the Riemannian geometry, whose solution was suggested by L. Schlaefli [183] in 1873, by comparing two geometries, so that one is gauged by the other. The general solution for the problem was given by J. Nash [184] in 1956. Nash showed how any Riemannian geometry can be generated by metric perturbations against a bulk space (which he assumed to be Euclidean, but it was soon extended to a pseudo Riemannian bulk by R. Greene [185]). As it happens, any embedded metric geometry can be generated by a continuous sequence of small metric perturbations of a given geometry with metric of the immersed manifold, i.e.,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta y\, \bar{k}_{\mu\nu} + (\delta y)^2\, \bar{g}^{\rho\sigma} \bar{k}_{\mu\rho} \bar{k}_{\nu\sigma} \cdots \quad .$$

When applied to cosmology, the brane-world modified Friedman equation is obtained

$$\dot{a}^2 + k = \frac{8\pi G}{3} \rho a^2 + \frac{\Lambda}{3} a^2 + \frac{b^2}{a^2} \quad , \tag{63}$$

where the $b(t)$ correction term with respect to the standard Friedman equation is given by the component $k_{11}(t)$ of the extrinsic curvature. When compared with the x-fluid state equation $b(t)$ has the form

$$b(t) = b_0 (\frac{a}{a_0})^{\frac{1}{2}(1-3\omega_x)} \quad , \tag{64}$$

where a_0 and b_0 are integration constants, respectively representing the current expansion parameter and the current warp of the universe. From the theoretical point of view, it would be a satisfactory solution for the dark energy problem if the $b(t)$ function was a unique solution, but, in fact, it depends on a choice of a family of solutions for the extrinsic curvature induced by the homogeneity of the Codazzi equation. Thus, to be free from these pathologies a proper mechanism or an additional dynamical equation for extrinsic curvature should be implemented. In spite of Brane-world models get some attention on recent years due to several options for dark energy, their mechanisms are still not completely understood or justified.

Statistical approach

The *statistical approaches* focuses basically on an explanation for the value of the cosmological constant and, in a general manner, the physical constants. The main debate concentrates on why the physical constants have the value which are measured today.

An example of statistical approach approach is the *Anthropic Principle* [137, 186], mainly based on Bayesian statistics. This principle plays a important role when applied to Superstrings theory with an implementation of the Calabi-Yau manifolds. These manifolds were used to explain the extra 6-dimensions of the theory built in 10 dimensions. For each one of those compactions, there exists a wide landscape of possible universes, where vacuum energy is anthropically allowed. Each one of these universes have different values of Λ and different physics [187]. This situation is solved with the strong anthropic principle that states the universe we live in is the only one adequate to man existence, rather, the law and physical constants have the value they have to provide intelligent life. In a weak version of the anthropic principle, Weinberg [137] says that the intelligent life is the way it is only with the minimum value oscillations of Λ and $< \rho_{vac} >$. On the other hand, in the strong version, the *a priori* probability is more "problematic" because the *ensemble* gathers the set of cosmological models fixed with different values of the fundamental physic constants violating the logical principle called *Occam's razor* that states "*entia non sunt multiplicanda praeter necessitatem*" (Entities should not be multiplied beyond necessity). Although interesting, such approach sets a too distant target for theoretical physics as an experimentally based discipline since we can only make experiments in one universe. Another criticisms and discussions of anthropic principle can be found in [188, 189, 190].

In addition, in a manner of avoiding the anthropic principle, by analogy to theoretical biology *the cosmological natural selection* [191] tries to give an explanation for the choices of parameters and fine-tuning process in a landscape theory without appealing to the anthropic principle. The basic idea is that the universe was designated to the black-hole production stating an existing population of correlated universes. Such populations must attend to a very specific type able to evolve. For instance, the physical parameters are fixed in each universe but they can vary in different universes. If one of these universes can produce black-holes, it is called *active universe*, i.e, in this universe *child-universes* are produced in the event horizon of each black-hole. Each child-universe carries part of the characteristics (the values of the physical parameters) from the *parent-universe*. The natural selection occurs precisely at the biggest possibility of an universe can dominate among an universe population, i.e, the biggest values of the physical parameters can be achieved in a manner of maximizing the black-hole production, hence the child-universe birthrate and the rate of a life-permitting universes. Following this rationalization, one can conclude that our universe with life is the result of an evolutive chain of birth and death of preceding universes. The main problems of this proposal are that there are not explicit reasons of why the choice of populations of specific characteristics evolve exactly by *natural selection* and also if our universe is really the first universe or not. If the choice is *randomly*, there is no *progeny*, hence there is no natural selection [192]. In fact, the natural selection does not make an improvement upon the weak anthropic principle.

On the other hand, in the Horava-Witten's [193] superstring model, the Calabi-Yau manifolds are not used. This model is built in a 10-dimensional space-time, which is reducible to Anti-deSitter ADS_5 space-time by using ADS/CFT correspondence, as proposed by Maldacena [194] in 1998. By taking the Anti-deSitter space-time in five dimensions, Maldacena concluded that every theory built in the M_4 Minkowski space-time corresponds to a theory in the ADS_5 space-time in which one can relate to the Yang-Mills theory of a gravitational theory. Hence, the ADS/CFT correspondence is extended to supersymmetry only in 10 dimensions. Thus, Horava-Witten superstring model has gained more attention lately because it does not appeal to the anthropic principle in a manner to deal with the cosmological problem.

Moreover, when adding the holographic principle [55, 195, 196, 197, 198] to the M theory has brought up some interesting questions. The holographic principle states that all information contended to a physical system in a region of space is defined by its surface and can be represented as a hologram in a ADS_5 boundary in Horava-Witten's model. The hologram rules the boundary regions in this same space in which contains at most one degree of freedom per Planck area. This area is defined as a small square with side L_p; that is the Planck length $L_p(10^{-33}\ cm)$ [199]. Thus, the number of degrees of freedom that describes such region is finite and much smaller than the one expected on quantum field theory [55] constrained by the hologram mechanism. This fact could explain the smallness of the cosmological vacuum energy density since the energy density decreases with the area [200], considering that the universe is large when compared to Planck scales. Such as the anthropic principle, the hologram principle provides several discussions about its validity and range, which only further observations can shed light on these issues. For further information about the hologram principle and dark energy see [200, 201].

Nevertheless, we note that there is a lack of a fundamental theory which could give a satisfactory explanation to these fundamental problems, and that also conciliates theory and phenomenological data. To get to this final step, extra information about the universe become more and more vital to unriddle the issues surrounding the foundations of physics.

Some recent and upcoming projects on Dark matter and Dark energy

Despite of facing striking problems, the interest on topics such as dark matter and dark energy, which are undoubtable related to the inner foundations of physics, has increased. We live in a very interesting moment of non-stop process of improvement of measurement devices, like the space probes and the ground-based telescopes and terrestrial laboratories around the world. One point to note is that the consortium between several research centers and institutions all over the world are the cornerstone to achieve success in all scientific projects. Here, we restricted ourselves to short comments in a manner of giving some examples of current and upcoming projects on modern cosmology and astrophysics related to dark matter and dark energy.

Dark matter experiments

Due to the importance of the contribution of dark matter for the total energy density of the universe, several experiments have been proposed and carried out by many scientific groups around the world in search of relic dark matter candidates.

The current experiments on dark matter focus mainly on *Wimps*. This apparent preference on *Wimps* is due to the characteristic of clumping of cold dark matter and, possibly, the formation of a bulk around the galaxy. In this sense, the DArk MAtter or DAMA collaboration is one of the projects focused on Wimps detection, the solar axions and perhaps Simps detection as proposed in [204]. The project is a result of a initial collaboration between Italy and China, and *a posteriori* other groups from India, UK, Russia, Ukraine and Spain [203]. The method is based on the measurement of the annual modulation signature which accounts for the supposed variation of signal of dark matter due to the positions and velocities of the Earth and Sun with respect to the galactic plane [70]; the material used is NaI crystal scintillator detectors. The initial results of measurements were controversial due to the negative results from other experiments, as, for instance, the US project called the Cryogenic Dark Matter Search (CDMS) [205, 206]. Current efforts are being made in a manner of reconciling DAMA experiments with other projects in order to gain maximum level of confidence on measurements. A more detailed information about the project and recent results can be found in [69, 203, 207, 208].

To proceed further, The Cern Axion Solar Telescope (CAST) [209, 210] a collaboration between Germany, Greece, Italy, UK, USA and Russia, is intended to detect axions that escape from the solar core. The CAST apparatus is basically composed of a Large Hadron Collider (LHC) prototype magnet of order of $9T$. It resides in the interior of two parallel pipes of length $L = 9,26m$, and cross-sectional area $A = 2 \times 14,5cm^2$. With the current upgrade of LHC it has been expected information about, for example, dark matter, production of mini-black holes, the existence or not of extradimensions and the Higgs boson, and possibly the appearance of new paradigms in physics. Due to the high level of sensitivity, as stated in [211], CAST experiment can provide a probe for the existence of extra-dimensions. The first phase results reached the limit bound of axion mass m_a of order of 0.20eV. The on-going second phase intends to reach a region mass of order of 1eV, which can provide the observation of some new effect [212]. Moreover, the Tokyo axion helioscope is a Japanese experiment which is based on “helioscope method” which intends to reach the same limit mass bound as the CAST experiment (1Gev) on its current third phase. The experiment has been applied extensively on the study of solar axions since 1997, and the actual *status quo* is the on-going third phase. The apparatus is composed basically of a 2.3-m long 4T superconducting magnet, a gas container (for hydrogen or helium), PIN-photodiode X-ray detectors, and a telescope. For further information see [213, 214].

Another interesting experiment is the Cryogenic Rare Event Search with Superconducting Thermometers (CRESST) [215, 216]. It is based on the analysis of the elastic scattering of the relic particles. The possible small energy of the recoil can be detected by sensitive cryogenic detectors. Just like DAMA experiment, the CRESST apparatus is located at the Gran Sasso National Laboratory, about 1.4 km below ground. Both experiments have such caution to avoid an interference of any kind. The project has been updated to its second phase started in 2007 and it has expanded cryogenic detectors with scintillating crystals up to 33 detector modules [215].

Several other experiments on dark matter have been proposed and explored, SOLAX [217] and COSME [218] to name a few. All of these are in search of the relic particles, but none have conclusive facts yet. Nevertheless, the efforts are in progress and more sensitive devices have been constructed for detecting such particles. Recently, physicists of the international collaboration DZero experiment at the U.S. Department of Energy's Fermi National Accelerator Laboratory have discovered "doubly strange" [219] particle called the Omega-sub-b (Ω_b^-) which is constituted with two strange quarks and a bottom quark. However, a profound analysis must be applied to study and understand this new discover. A more complete list of dark matter experiments can be found in [220].

Projects on dark energy

Just like dark matter experiments, the dark energy surveys have been extensively explored in the recent years, mainly on space probes and the ground-based telescopes. One very known example of a space probe is the WMAP experiment [77] launched in 2001, intended to measure the CMBR anisotropies. The CMBR collected data plays an essential role in modeling and analyzing models of theories about the universe. The project is a partnership between Princeton University and NASA's Goddard Space Flight Center. As stated in former section, the current fifth year has shown us an improvement of the measurement of the power spectrum and the composition of the universe. In fact, the WMAP has been considered an essential tool for the current cosmology and astrophysics.

In contribution to calibrate with WMAP collected data, we can mention the CBI and BOOMERANG experiments. The Cosmic Background Imager (CBI) is a radio telescope designed to study the CBMR fluctuations on arcminute scales at frequencies between 26 and 36 GHz. It is located in the Chilean Andes, at the Chajnantor Observatory. This project is a collaboration between the California Institute of Technology, the Canadian Institute for Theoretical Astrophysics, the University of Chicago, the National Radio Astronomy Observatory, the Max-Planck-Institute für Radioastronomie (Bonn), Oxford University, the University of Manchester, the Universidad de Chile, and the Universidad de Concepción [221, 222]. Moreover, the BOOMERANG balloon, just like WMAP and CBI, is also designed to measure anisotropies in the CMB. It consists of an array of detectors cooled to 0.28

Kelvin mounted at the focus of a 1.3 meter telescope. The instrument flows on a gondola beneath a NASA/NSBF high altitude balloon [223].

In addition, the PLANCK space probe [73] programmed from the European Space Agency is designed to measure the anisotropies of the CBMR in a manner to provide more information, besides sharpening recent data about the universe. It also tests and makes constraints on cosmological models and theories. The space probe has basically a 1.5 meter off-axis telescope which will be capable of covering the entire sky with a precision of approximately 2 parts per million. It was launched on October 2008, the Operations are on 2009-2010, and the scientific product delivery, is dated to mid 2012.

Besides the space probes and balloons, there are important contributions from the grounded-based telescopes. They play a fundamental role in providing reliable data surveys comparing to other surveys, helping us to trail with higher level of confidence the collected data. As it happens, we point out the spectroscopic project named Sloan Digital Sky Survey (SDSS) [179], which plays a fundamental role in providing data for dark energy issues. It is essentially a 2.5 meter spectroscopic telescope located on Apache point. Its first and second-finished phases of operations were active during 2000-2005 and 2005-2008, respectively. In addition, the successor of the SDSS-II, the SDSS-III is the current research program and it is planned to start operating in june 2008 until 2014 with four new surveys using SDSS facilities.

In five years of operations, the SDSS program collected approximately 200 million cosmic objects, such as galaxies, quasars and stars. As the result of a consortium between 25 institutions, the SDSS-II carried out 3 fronts of surveys: the Sloan Legacy, which was applied to make a general survey of the cosmos; the Sloan Extension for Galactic Understanding and Exploration (SEGUE experiment), which was driven specifically to study the Milk-way, and the Sloan Supernovae project, which was turned to study and detect supernovae in the universe. This project will be improved with the new surveys driven by the SDSS-III program: the Baryon Oscillation Spectroscopic Survey (BOSS) will be intended to measure the cosmic distance scale and mapping luminous distant galaxies and quasars. The SEGUE-2 experiment is a second phase of the SEGUE experiment started by the SDSS-II program and it will be intended to make an improvement of studying of the Milky way structure, kinematics, and chemical evolution. The study of inner structure of the Milky way will be contemplated by the third program called the APO Galactic Evolution Experiment (APOGEE) based on high-resolution infrared spectroscopy. The last survey is the Multi-object APO Radial Velocity Exoplanet Large-area Survey (MARVELS) intend to locate and study distant bright stars and possible giant extrasolar planets. In this manner, the SDSS program is intended to cover a surprisingly amount of data and range of several scales of the universe.

Another project is the upcoming Dark Energy Survey project(DES), expected to start operating in September 2009, during 5 observational seasons till 2014. It is an international collaboration between 10 institutions (at present time), and is managed by Fermilab, The University of Illinois' National Center for Supercomputing Applications (NCSA) and The National Optical Astronomy Observatory (NOAO) [224]. The DES project is designed to study dark energy and its influence on the universe. It will use a device for photometric surveys constituted of a 62 CCDs camera (approximately 500 megapixels) coupled to the 4-meter Blanco telescope on Cerro Tololo Inter-American Observatory (CTIO) in Chile. With 3 square-degree-field it is capable of covering an eightvo of the sky in 4 bandpasses and it will detect approximately 300 million cosmic objects improving at least, for example, the SLSS surveys by a factor 2. It is a powerful experiment on the dark energy issues, and it will give us a large amount of collecting data in the next couple of years.

Truth of matter, all the former projects will indirectly provide ground to the ambitious Large Synoptic Survey Telescope (LSST). This project will consist of a ground-based 8.4-meter and 10 square-degree-field telescope, schedule to operate in 2014, and it is a conjoined effort between nineteen other organizations atop Cerro Pachon in Chile. Due to its advanced hardware devices, it will be capable of covering the sky every three nights. This project certainly will imprint a massive impact on the observational astrophysics and Cosmology ever seen with its great power of resolution and amount of data, providing over 3 Gigapixels per image to be processed and study. For further information see [225].

No other moment in mankind's history we have had so much information about the universe, about its content and evolution, even if our understanding about it is unsatisfactory due to the lack of consistent and general explanations of the phenomena. Clearly, all these host of recent and upcoming experiments will help us to shed light on the *dark* problems and issues of physics. To better summarize the importance of these projects on current issues we quote Gary Hinshaw of NASA's Goddard Space Flight Center in Greenbelt [77]

"*Ours is the first generation in human history to make such detailed and far-reaching measurements of our universe*".

Conclusions

Modern Cosmology has been an important source of data that provides a deeper comprehension of the gravitational structure and evolution of the universe. Not only this, but it seems to call for new gravitational theories far beyond Einstein's approach. Even though we are long way from a concrete fully-developed theory, dark matter and dark energy play a major role on this quest, representing fundamental constraints to these new gravitational models.

Based on recent data in Astrophysics, mainly on the WMAP experiment and SDSS surveys, we notice that dark energy is a disturbed element on the geometry of the universe which is best described by the Friedmann-Lemaître-Robertson-Walker model. The same characteristic seems to appear on primordial dark matter issues, which are related to the formation of large structures in the universe. It is also important to point out that the Cosmological Constant problem is keenly related to the Dark Energy problem, which means that not only we have to understand dark matter and dark energy better, but also its relation with the Cosmological Constant, which is still itself the only viable explanation to conciliate theory and phenomenological data. More recently, the problem has escalated to a central issue in the context of the

ΛCDM cosmological scheme. Although the problem is rooted in General Relativity, it shows up in other branches, such as non-supersymmetric theories, like brane-world gravity. Moreover, the cosmological constant problem can be taken as a problem of fundamental nature because it involves the Einstein-Hilbert principle, and also because it involves the different nature and energy scales of the four fundamental interactions. Hence, we need a theory to deal with these intrinsic features of data phenomenology, and not just a simple mechanism, such as a fine-tuning, but indeed, a reasonable description based on the first principles of a more general theory.

As it is laid out right now, from a theoretical point of view, the Cosmological Constant problem is a fundamental problem, such as the hierarchy problem, and its solution must come from a complete theory independent of particular models. As we want to point out to the reader, besides a *quantitative* difference between gravity and gauge interactions, gravity *seems* to behave *qualitatively* different than other gauge interactions, as we see on the efforts of unifying all fundamental interactions, as well as dark component problems and observational data. Clearly, the current examples of gravitational effects imprinted by dark matter and dark energy require a deeper insight on the structure of the universe and on what gravity really means. In truth, these are problems of fundamental nature that exercise a serious effect on the foundations of gravitation conceived since the studies of Galileo, Newton and Einstein.

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