

# Modeling The Expansion Of The Universe By A Steady Flow Of Space-Time

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Assuming that the Universe is spatially infinite, we depict a simple model where gravitation does not decelerate the expansion, which occurs at constant speed for any two distant galaxies. This Steady Flow model fits the SNe Ia observations without a repulsive dark energy. We review the main problems of Big Bang cosmology and see how they can be solved. Particularly, the singularity and the problem of the cosmic origin could be avoided in our framework. The time evolution of the scale factor ( $R \propto t$ ) leaves a longer period for early structure and galaxy formation. However, the expansion time coincides with the Universe age of the standard model. Other features of Big Bang, such as the evolution of temperature with the scale factor and the primordial nucleosynthesis, remain unchanged.

*Keywords:* Cosmology, Type Ia supernovae, Early Universe, Primordial nucleosynthesis

# 1. Introduction

Centuries ago, Newton postulated the idea of an infinite Universe in order to avoid the stars falling into a hypothetical centre of mass of a finite space. For similar reasons Einstein introduced a cosmological constant ( $\Lambda$ ) to allow for a static Universe. When Hubble discovered his fundamental law describing the universal expansion, the prerequisites of infiniteness and  $\Lambda$  were not needed anymore. In spite of that,  $\Lambda$  still appears in the fundamental equations describing the standard cosmological model.

The universal expansion seems to imply a moment in the past when the entire observable Universe was reduced to a single point. The classical Big Bang model postulates the creation of space, time, matter and energy from such a singularity. During decades the expansion of the Universe was assumed to be decelerating due to gravitation. However, recent measurements of type Ia supernovae (SNe Ia) show that the expansion of the Universe is faster in our epoch than classical models expected (e.g. Riess et al. 1998, 2001). In order to fit these observations within the standard cosmology, a repulsive dark energy, derived from the concept of cosmological constant, has been postulated. Such energy was first identified with the vacuum energy, but both energies are ca. 122 orders of magnitude apart, so that the nature of dark energy remains a mystery. The Inflationary model (Guth & Lightman 1997) was constructed, using a scalar field whose vacuum energy essentially plays the role of time-varying  $\Lambda$ , in order to solve old issues of the Big Bang such as the horizon, flatness and initial fine-tuning problems (Hu et al. 1994; Ellis 2000). Inflation proposes that an extremely fast exponential acceleration of the expansion took place for a very small fraction of a second after the Big Bang.

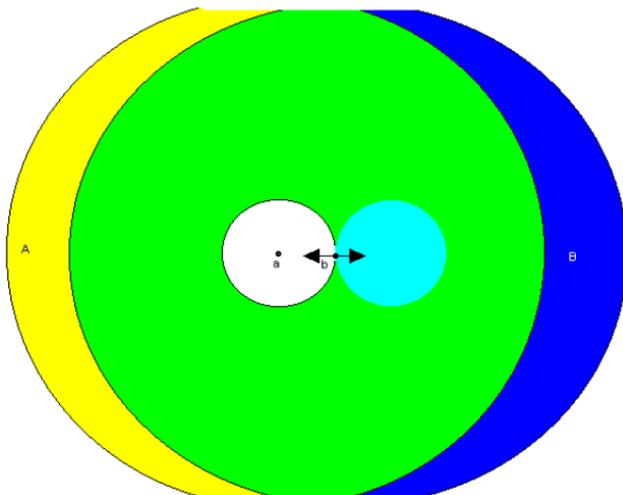
Very shortly after, the expansion decayed to a 'normal' Hubble flow. In summary, we have a quite complex picture for the dynamics of the Universe: the Inflation period, with a very fast acceleration, followed by a long lasting deceleration era (with 2 different expansion regimes depending on radiation or matter dominance) and, since redshift  $z \approx 1$ , a new acceleration era, much slower.

The standard model allows the Universe to be either spatially infinite or finite (Coles & Lucchin 2002). The apparent acceleration seems to favour an infinite one, unless the dominance of dark energy vs. gravity changes again in the future. Let us assume that we are in a homogeneous, isotropic and spatially *infinite* Universe. Consider an isolated 'particle' (such as a galaxy) in it. The gravitation of the rest of the Universe is pulling it from all the directions in such a way that its effects are practically cancelled due to symmetry reasons. A particle should feel no net force from matter homogeneous and isotropically distributed around it in an infinite space. Moreover, according to Mach's Principle the *inertial* properties of any particle are due to the background provided by the rest of the objects in the Universe. Therefore, the probe galaxy (**b** in figure 1) will not behave as if it was feeling the gravitation of a sphere centred in the observer's galaxy (white circle around **a** in fig. 1), and its relative recession velocity should not decrease. In fact, for each of these spheres, one can define another one of the same mass and radius (pale blue in fig.1), but in the opposite direction to the observer's galaxy, that would cancel the force of the first sphere. Even if each galaxy only felt the gravitation from a portion of the Universe, its own observable Universe (big circles **A** and **B** in fig.1), the net result would be 0 (disregarding local companions or clusters) due to the homogeneity, isotropy and *spherical* symmetry of the cited portion.

Obviously gravitation acts locally clustering stars, galaxies, and the rest of celestial matter. However, in this paper we will study some

physical consequences of the postulate that gravitation does not play a relevant role in the overall expansion of the Universe (provided that it is spatially infinite, homogeneous and isotropic) because their long range effects cancel each other from every two opposite directions in space.

To further justify this assumption from another point of view, let us remind that, even in the standard cosmological model, space itself is considered as an expanding media that carries out the galaxies within, instead of considering galaxies receding from each other through a static space. In other words, the expansion of the space causes the recession of galaxies. Then, why should the expansion of the space (which posses no mass) be decelerated by gravity? Gravitation is an interaction between physical particles with



**Fig. 1.** Scheme illustrating the practical cancellation of gravitation over a probe galaxy **b** in our model. The force pulling **b** towards the distant observer's galaxy **a** is neutralised by an opposite force from the mass within the pale blue sphere, since the portion of the universe that interacts gravitationally with **a** (yellow sphere **A**) is different to that for **b** (deep blue sphere **B**) except in case of a finite universe of scale coincident with **A** and **B**.

mass/energy and there is no evidence of space being constituted by particles or having any mass. According with General Relativity, gravitation implies a curvature of space around massive objects, but does not imply contraction or expansion of space itself. If one admits that there is no compelling reason why gravity should decelerate the expansion of an entity free of mass such as the space, a steady Hubble flow at constant rate appears as more natural.

Notice that, though this postulate could be seen as artificial, the introduction of a cosmological constant or a dark energy is not less artificial. Thus, it would be not epistemologically legitimate to set aside this possibility without a previous analysis of it. Anyway, the aim of this paper is to study the consequences of the postulate rather than to justify it *a priori*. Therefore, we will briefly study how our hypothesis influences some of the most salient features of cosmological interest and we will see that it can explain a number of observations in a simpler way than the standard model.

## 2. The supernovae evidence Section title

Although measurements of WMAP are compatible with the standard cold dark matter model including a repulsive  $\Lambda$  (Komatsu et al. 2008), so far the only *direct* evidence favouring an apparent slight acceleration of the Universe in recent times comes from distant supernovae observations (Blanchard et al. 2003).

Our analysis begins with SNe Ia data summarised by Tonry et al. (2003) in a residual Hubble diagram with respect to an empty Universe (see Fig.8 therein). A careful inspection of that figure evidences the following features:

- Average error bars are larger for data at redshift  $z > 0.1$ .

- In general, error bars appear underestimated. Several of them should be larger because no single smooth curve can be drawn along all of them.
- Overall data fit to a Universe of null density as well as to the standard model including dark energy, the Concordance model. While the differences between both models are not  $> 0.2$  magnitudes, deviations  $\geq 0.4$  magnitudes from any of them are frequent among the plotted data points.

Another plot of similar data, shown in figure 2 (Wright 2003), shows 2 data sets: Wang et al. (2003) in orange and Tonry et al. (2003) in black. It can be observed again that the scattering of data

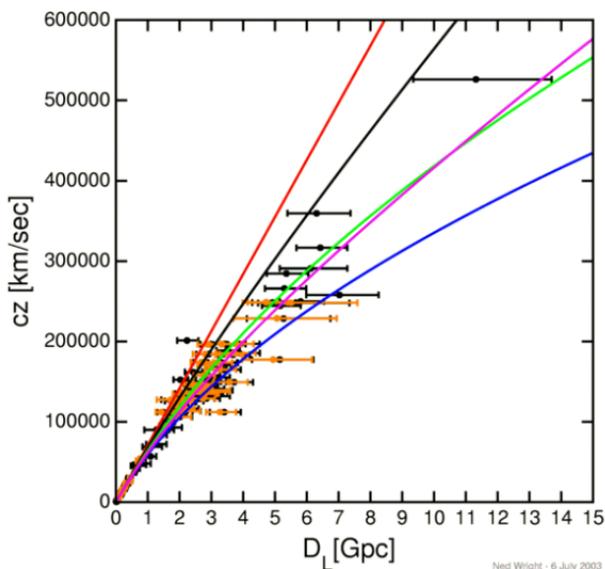


Fig. 2- Recession velocities vs. luminosity distance for SNe Ia. The curves show a closed Universe ( $\Omega = 2$ ) in red, the critical density Universe ( $\Omega = 1$ ) in black, the empty Universe ( $\Omega = 0$ ) in green, the steady state model in blue, and the Concordance model with  $\Omega_M = 0.27$  and  $\Omega_\Lambda = 0.73$  in purple. A Hubble parameter  $H_0 = 71$  km/sec/Mpc has been used to scale the distances in the plot.

and the error bars are much larger than the differences between the Concordance model (purple curve) and the empty Universe (green curve). In fact, the scattering of data only allows to empirically rule out the closed Universe (red curve) and the steady state model (blue curve). Blanchard et al. (2003) studied the case for an Einstein De Sitter model (black curve) with a low Hubble parameter ( $H_0 < 50 \text{ km/sec/Mpc}$ ), although these authors acknowledged severe problems. We present in here the case of a Universe which expansion occurs *as if* it was empty. A similar, although different, general relativistic model leading to a scale expansion exponential with time has been already described (Masreliez 2004).

Notice that measurements of peak brightness of these remote supernovae are extremely difficult and require several corrections. Moreover, there are systematic differences in the corrections made for the same objects by different groups of observers (Leibundgut 2000). Considering this, the self-consistency of the data is remarkable. However, the decelerated expansion at  $z > 1$  is still based in too few observations to be considered conclusively proved beyond doubt.

Evidently the Universe has a non-zero density, so that the curve for an empty Universe shown by Tonry et al. and by Wright was considered as a mere approximation or a reference. However we analyse in this paper the possibility that this curve could be a good description of the actual expansion kinematics since space could grow *as if* it was empty.

We suggest that the space among distant galaxies, unbounded by gravitation, increases with time in a very simple way: the Hubble law with constant recession velocities. In short, space flows with time following a steady Hubble flow. That's why we dub this model the Steady Flow model. Therefore, assuming a steady expansion at constant recession velocity for each pair of distant galaxies we have:

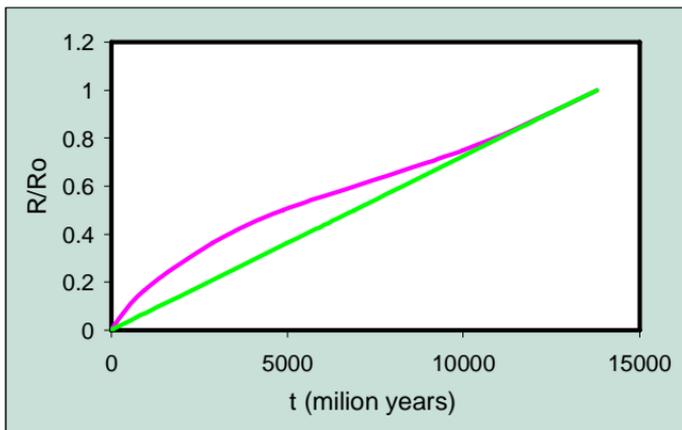


Fig. 3- Evolution of the normalised scale factor  $R/R_0 (=1/1+z)$  with expansion time. The curves show our Steady Flow model (which behaves as an empty Universe) in green and the Concordance model in purple.

$$\frac{t_0}{t} = \frac{R_0}{R} \quad (1)$$

$R$  is the cosmic scale factor and  $t$  is the comoving proper time since the beginning of this linear expansion.

The simple equation (1), along the Hubble law, describes the expected behaviour for an empty Universe where gravitation does not decelerate the expansion, since gravity would have no effect at all in the absence of mass. But we propose that it also describes the behaviour of an infinite, homogeneous and isotropic Universe with  $\rho > 0$ , such as our real Universe may be. Notice that this simple model avoids the use of free parameters or free functions to fit the observations. From (1) and the Hubble law we get immediately  $H=1/t$  and, for a currently accepted  $H_0$  value of  $71 \text{ km/sec/Mpc}$  (Wright 2003),  $t_0$  should be  $13.8 \cdot 10^9$  years. Remarkably, this time coincides with recent determinations of the Universe age obtained from the more complex Concordance model (Turner 2007).

Our model (green curves in figs. 2 and 3) predicts -without need of introducing *ad hoc* a repulsive dark energy to accelerate universal expansion- that supernovae with  $0.1 < z < 1$  are farther away, in terms of luminosity distance, than previously expected (black curve in fig. 2). Therefore they appear fainter, as actually observed (Riess et al. 1998, 2001; Wang et al. 2003; Tonry et al. 2003). The reason is that the recession velocities of any two distant galaxies do not decrease (nor increase) with time. This constancy suggests some kind of cosmological homogeneity not only in space, but also in time. Our hypothesis can be tested through further SNe Ia data at different redshift, and specifically predicts that supernovae of  $z > 1$  should not show any consistent deceleration in the past expansion of the Universe.

Somehow, this steady flow of space with time is not surprising because space and time, as the theory of Relativity disclosed, are deeply entangled. But, how could and infinite space develop from a point-like singularity in a finite time? This open question leads us to briefly discuss several Big Bang controversies in the next section.

### **3. The Big Bang problems and the proposed explanations**

Despite its success, the classical Big Bang cosmology has several problems such as (Coles & Lucchin 2002):

- The horizon problem: The Universe is observed to be highly homogeneous and isotropic, but how did it become so when all regions of the observable Universe were not in mutual causal contact at early times after the Big Bang?
- The flatness problem: The Universe seems to be nearly flat today, but this implies that it must have had a normalised

density  $\Omega$  very nearly equal to one at early times. Unless the Universe is exactly flat, this requires fine-tuning.

- The structure problem: What formed the perturbations that lead to the structure we see around us? Why is structure the same everywhere, even though different parts of the Universe were not causally connected early in the Big Bang model?

These problems were solved by Inflation theory (Guth & Lightman 1997), at the cost of adding some amendments to the classical Big Bang cosmology. Shortcomings of Inflation remain, however. These include the required fine-tuning of the coupling constant in order to obtain the correct density profile in the present Universe, the vacuum energy problem and the unnaturally flat potentials needed to solve the initial value problems (Moffat 2002). Non-adiabatic expansion in order to bypass a decrease of entropy is another controversial issue of the inflationary model (Hu et al. 1994). Finally, the low-order quadrupole of the temperature anisotropy power spectrum from WMAP has lower amplitudes than expected from Inflation (Efstathiou 2003).

The so-called cosmological constant problem still remains, i.e., why the vacuum energy density expected from particle physics, roughly  $10^{76}$  GeV, is 122 orders of magnitude larger than the value derived from the SNe Ia observations? In other words, why  $\Lambda$  is so amazingly small without being 0? The flatness and cosmological constant problems have been also addressed by theories of nonconformally coupled massless fields within General Relativity (Guendelman 1988). The strengths of gravity and dark energy (which should change by many orders of magnitude with time) are very similar particularly at the present epoch. But the radiation energy density and the  $\Lambda$  energy density should be set to an accuracy of better than one part in  $10^{120}$  at the Planck time in order to ensure that

the densities in matter and  $\Lambda$  become comparable at present times. This is called the coincidence problem (Sahni & Starobinsky 2000; Padmanabhan 2003; Peebles & Ratra 2003).

Within our model, the horizon problem is solved in a way different to the inflationary scenario, since the Universe would always have been homogeneous, at least at the observable scale. If the Universe is spatially infinite, it should have been always so since, no matter how many times one can imagine contracting the space whilst going backwards in time, an infinite space would never collapse into a finite dimensionless singularity. The contrary could be envisaged for a finite Universe or a finite portion of the Universe, such as our observable Universe, but it appears unfeasible, unphysical for an overall infinite Universe. In any case, the original singularity results from a very strong extrapolation backwards of the universal expansion, which remains not justified since the theory of Relativity has its own limitations, particularly when one approaches the realm of the very hot and very dense, where quantum effects can not be disregarded. Furthermore, it has been suggested that in a complete theory of quantum gravity, the cosmological singularity would not exist even for a finite Universe (Coles & Lucchin 2002). Taking this into account, there is neither reason nor evidence to assume that *if* the cosmological principle of homogeneity is nowadays correct, it would have been otherwise in the past. On the contrary, the evidence from WAMP (Komatsu et al. 2008) confirms a very smooth, homogeneous and isotropic scenario at an early stage of the present expansion epoch. In short, if the Universe is now infinite and homogeneous it should have been always so, even if, obviously, it was denser and hotter in the past.

Therefore, we hypothesise the Universe at the beginning of its expansion as a homogeneous and extremely dense fluid at Planck temperature, filling an already infinite space. The universal expansion

could be a consequence of the repulsive (quantum) interactions and collisions among the very high-energy particles on that fluid. Before that, one can barely speculate, for instance, that the only particles in the Universe were bosons, which do not feel such repulsive interactions. Later on, when the average distance between fermions grew to the point to make the effects of these interactions negligible, the recession velocity between any two particles distant enough reached a constant value, according to the inertia law.

Let us consider a last speculation on this fascinating issue. If the expansion before the  $R \propto t$  regime was, for instance, exponential, e.g.  $R \propto e^{Ht}$  (as proposed by standard Inflation), or perhaps could be described by a Hubble law with  $dH/dt \geq 0$ , the scale factor should never reach zero going backwards in time. So, the Universe would also be infinite in age. Whatever the case, the present model considers a beginning of the expansion, but not a beginning of time. The eternity of the Universe should be a feature of our model since reconciling an infinite space with a finite time seems unfeasible. In addition, an eternal Universe would have plenty of time to reach a homogeneous state (at least at the scale of our observable Universe) if eventually it was not so in a remote past, thus facilitating the solution of problems such as the cosmological horizon. Furthermore, without a beginning of time the singularity and the problem of the cosmic origin would be avoided -there are also ways of avoiding the singularity in the framework of General Relativity (Coles & Lucchin 2002)-. In this scenario there would be no 'creation' and, concomitantly, no violation of the principle of conservation of the energy.

We want to emphasise at this point that the present conjecture is not questioning General Relativity as the most accurate theory of gravity we presently have. Our model just postulates that gravity is not limiting the space expansion rate. Gravitation is crucial, among many other fundamental questions, in the development of the cosmic

structure, but it would not control the overall expansion of the Universe according to this model.

The flatness of a Universe as described by the present Steady Flow model seems not surprising. The practical cancellation of gravitation at very large scales implies that the overall space-time must be flat or very nearly flat. Except in the neighbourhood of local concentrations of matter, such as galaxies, light does not deviate from rectilinear propagation, and the metric of the space, as far as we can measure it, appears to be Euclidean.

In our scenario, the cosmological constant, dark energy and quintessence are superfluous. Thus, the cosmological constant/vacuum energy problem above mentioned and the surprising coincidence of matter and dark energy dominance in present-day Universe (Caldwell et al. 1998) are also avoided.

On the other hand, within the Steady Flow model some of the main features in the history of the Universe, such as the cosmic recombination (decoupling) or the primordial synthesis of light elements remain the same as in Big Bang model. However, the times elapsed since these events are shorter in our model than in the Concordance model (see fig. 3), as we will see in the following section.

## **4. The cosmic background radiation temperature evolution and the primordial nucleosynthesis**

To discuss the evolution of temperature of the cosmic background radiation (CBR) as function of time we take the classical result of blackbody radiation thermodynamics:

$$\frac{S}{V} = \frac{4}{3} a T^3 \quad (2)$$

$S$  is the entropy,  $V$  is the volume and  $a$  is the blackbody constant. Then, since  $V \propto R^3$ , in an adiabatic expansion one has that the product  $RT$  is constant. Therefore, the present model agrees with the standard one in the way that temperature of the CBR decreases as the cosmic scale increases, i.e.:

$$\frac{T}{T_0} = \frac{R_0}{R} \quad (3)$$

$T_0$  denotes today's temperature.

It is commonly accepted that recombination of nuclei and electrons to form atoms was allowed when the Universe temperature dropped to  $T_c \approx 3000\text{K}$  (Kolb & Turner 1990), rendering the Universe transparent to electromagnetic waves. From this temperature we have:

$$\frac{T_c}{T_0} = \frac{3000}{2.73} = \frac{R_0}{R_c} \quad (4)$$

So that this ratio is approximately 1100, and since  $R \propto t$  we get immediately  $t_c = t_0/1100 \approx 1.3 \cdot 10^7$  years, a recombination time longer than the standard figure ( $3.8 \cdot 10^5$  years) to account for the appearance of the structure seeds observed by WMAP. Let us remind, by the way, that non-inflationary mechanisms for the origin of CBR inhomogeneities have also been proposed (Vilenkin & Shellard 1994, Khoury et al. 2001).

The above calculation tells us something else: for any distant object, the corresponding elapsed time since the beginning of the linear expansion should be longer than in the standard model, as depicted in fig. 3. For instance, a galaxy of redshift 5 is observed as it was when the scale factor was 6 times smaller than today ( $R/R_0 \approx 0.17$  in fig. 3), i. e. less than  $10^9$  years after Big Bang, according to the standard cosmology, but after about  $2.3 \cdot 10^9$  years of steady Hubble flow within our model. This longer time for the formation and

evolution of galaxies could help to understand why high redshift galaxies appear to be more fully formed and mature than previously expected (e.g. Cimatti et al. 2004).

The Steady Flow model also leads to the interesting consequence that the behaviour of  $R$  as a function of time is the same in the radiation era and in the matter era, namely  $R \propto t$ . This result unifies the dynamical behaviour of the expanding Universe and differs from the standard model, where  $R \propto t^{1/2}$  in an early radiation-dominated era, and  $R \propto t^{2/3}$  in the matter-dominated era, before the 'recent' dominance of dark energy. This is consistent with our approach because these dependencies in the standard model follow from considering the equations of state for radiation and matter as the respective sources of gravitation slowing the expansion.

Notice however that the present model does not change the standard prediction concerning the relative abundance of primordial H and He in the Universe. Indeed, these values depend essentially of the relative abundance of protons and neutrons at a temperature of the order of the annihilation energy of the electrons and positrons ( $T_a \approx 10^9$  K). The relative abundance of neutrons and protons at that temperature, given by (Clark 1997):

$$\frac{N_n}{N_p} \approx \exp\left(-\frac{(m_n - m_p)c^2}{kT_a}\right), \quad (5)$$

remains unchanged in our model, as consequently does the ratio of primordial H/He abundances, the reason being that the nuclear reactions yielding helium occurred until practically all the neutrons were combined. A too fast expansion would have halted this process before completion due to baryon dilution. But, according to our model, this could not happen since the expansion in that epoch was even slower than in the standard theory. Notice that this successful

result of Big Bang is based on the hypothesis that the Universe was also homogeneous and isotropic at those very early stages (Coles & Lucchin 2002), thus giving further support to the steady homogeneity of the Universe, a feature of our Steady Flow model, discussed in section 3.

## 5. Conclusions

In a homogeneous, isotropic and spatially *infinite* Universe the net gravitational force on any galaxy should be (almost) null for symmetry reasons. Thus, gravitation does not decelerate the universal expansion, which follows a steady Hubble flow with constant recession velocities. The results of SNe Ia are consistent with this simple model for a commonly accepted value of  $H_0$  and without any free fitting parameters. Neither cosmological constant nor a repulsive dark energy are required in our model. The horizon problem vanishes since the Universe should have been always spatially infinite and homogeneous. The flatness problem is avoided because of the lack of any spatial curvature (except at local scale) due to the overall cancellation of the gravitational field. The time evolution of the scale factor,  $R \propto t$ , is specific of our model and is the same independently of either radiation or matter dominance in the Universe. Subsequently, we obtain longer times for the development of structure seeds observed in CBR and for the formation of the first galaxies. On the other hand, the steady expansion time coincides with the age of the Universe obtained from the Concordance model, and some other features of Big Bang cosmology, such as the evolution of temperature with the scale factor or the primordial nucleosynthesis of He, remain unchanged.

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