Simple Thought Experiments that Falsify the Einstein's Weak Equivalence Principle

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In this article it is shown by simple thought experiments that the Einstein Weak Equivalence Principle, defined here as the equivalence of inertial and gravitational masses, is not compatible with the Special Relativity Theory, the Lorentz Coordinate Transformation, and the Maxwell Theory of Electromagnetic Field. Since the General Relativity Theory is firmly based on this principle the thought experiments and the arguments described in this article present a significant problem for the theory and thus question its fundamental correctness.

Keywords: General Relativity Theory, Weak Equivalence Principle, Lorenz Coordinate Transformation, Schwarzschild Metric, Inertial Mass, Gravitational Mass, Newton's Law of Gravitation, Gravitational Mass Dependency on Velocity.

1. Introduction

There are several definitions of various equivalence principles that might introduce confusion into the discussion. It is therefore desirable to first define which equivalence principle will be discussed in this paper. In one form the Einstein's Weak Equivalence Principle (WEP) is defined as the equivalence of gravitational and inertial masses that is absolute and independent of any motion. This should not be confused with the statement that all masses fall the same way in a gravitational field. If the gravitational mass dependence on velocity is universal, applicable to all masses, then all masses would fall the same way even if their inertial mass depended on velocity differently than the gravitational mass. The Galileo free fall experiments would still be satisfied. However, the General Relativity Theory (GRT) is based on the absolute equivalence of the inertial and gravitational masses, regardless of their relative motion, so it is this equivalence that will be investigated in this paper by way of thought experiments. In this work it will be also assumed that the Special Relativity Theory (SRT) and in particular the Lorentz Coordinate Transformation (LCT) together with the Maxwell Electromagnetic Field Theory (EMT) are valid theories that closely reflect reality.

2. EM Field Experiment

In this section EMT and LCT will be used and tested by way of conducting a simple thought experiment as follows: two nonconductive parallel plates charged by charge $+mq$ and $-mq$ respectively, with charge embedded into the plates' matrix so that it cannot move, will be placed in the laboratory coordinate system. The plates will move horizontally with a uniform velocity v as shown in Fig.1. The plates will be separated by a distance d, and their mass will be neglected in this first experiment. The plates will reflect light that will bounce between them. The impact angle α of the photons will be such that the horizontal drift speed of the photons will match the horizontal speed of the plates in order to always keep the same amount of photons confined between the plates.

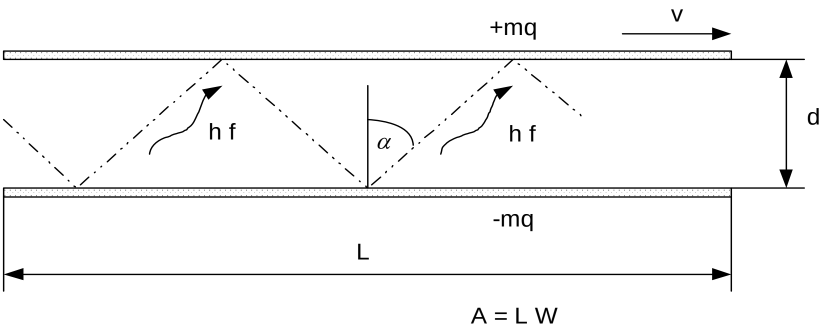

Fig.1. EMT test experimental setup: two plates built from an insulating material have charge $+mq$ and $-mq$ embedded in them, large area A, and are positioned at a small distance d apart from each other. The plate's inner surfaces are reflective and certain number of n photons is confined between the plates. The photon incidence angle with the plates is such tat the photon horizontal drift velocity matches the plate's velocity.

As is well known from EMT the charged plates are attracted to each other by the electrostatic force and since they are also moving relative to the laboratory coordinate system there is additional force component between the plates due to the Lorentz force. The resulting force acting on the plates, assuming that the peripheral field effects can be neglected, is thus described by the following equation:

$$F_q = \frac{m^2 q^2}{2\varepsilon_o A_o \sqrt{1 - v^2 / c^2}} \left(1 - \frac{v^2}{c^2} \right), \tag{1}$$

according to SRT. In order to prevent the plates from moving closer to each other and colliding, certain number of photons is injected between the plates under the angle α that exert the necessary counter pressure on the plates. The resulting photon force is exactly equal to the plates' attractive force to keep the system in equilibrium. The balancing photon force of n photons is found from the formula:

$$F_p = n\frac{\Delta p}{\Delta t} = \frac{2}{\Delta t}\frac{n\ h\ f}{c}\cos\alpha\,. \tag{2}$$

In deriving this formula it was considered that during the photon reflections between the plates the photon energy and momentum are conserved and no momentum is transferred from the photons to the plates. The time between the photon impacts on the plates and the cosine of the impact angle α are found from the following formulas:

$$\Delta t = \frac{d}{\sqrt{c^2 - v^2}}\,, \tag{3}$$

$$\cos\alpha = \sqrt{1 - v^2 / c^2}\,. \tag{4}$$

After substitution into Eq.2 the formula for the photon force on the plates becomes:

$$F_p = \frac{2n\ h\ f}{d}\left(1 - \frac{v^2}{c^2}\right). \tag{5}$$

It is reasonable to assume that the plates can always be kept apart at a constant distance d regardless of their horizontal motion. If the plates were not moving relative to the light source, but the angle of incidence of photons on the plates was changed, the frequency of the photons would have to increase according to formula $f = f_o / \cos\alpha$, where f_o is the frequency for the incidence angle $\alpha = 0$, or the frequency of the light source that moves with the plates. The final formula for the photon force can thus be written as follows:

$$F_p = \frac{2n\ h\ f_o}{d}\sqrt{1 - v^2\ /\ c^2}\ . \tag{6}$$

In order to prevent the plates from colliding the EM force and the photon force must balance $F_q = F_p$. By equating these forces it is apparent that the velocity dependent terms of the resulting equation cancel, which indicates that the balance is maintained regardless of the velocity as expected. The remaining parameters must then satisfy the following:

$$m^2 = n\frac{2h\varepsilon_o c}{q^2}\frac{A_o}{d\frac{\lambda_o}{2}} = n\alpha_o\frac{A_o}{d\frac{\lambda_o}{2}}\ , \tag{7}$$

where f_o was replaced by $f_o = c\ /\ \lambda_o$. The parameter α_o is the famous Sommerfeld fine structure constant $\alpha_o = 137.035999$. This is an interesting result suggesting that the electrons, which charged the plates, might be some sort of topological dynamic structures where the EM field is balanced by the photon inertial force. However, this is not a topic of this paper and will not be discussed here any further.

Since this thought experiment yielded the expected result using EMT, LCT, and SRT, the same considerations will be used next for the case where the EM field is replaced by the gravitational filed to test the correctness of GRT.

3. Gravitational Field Experiment

In this section the insulating plates in the EM experiment are replaced by the massive plates that will not be charged but will attract each other by the gravitational force. It is also important to note that there is no gravitational field or any static electrical or magnetic field between the plates; so that the space-time between the plates is not distorted in any way. The gravitation induced space-time distortion occurs only in the outer regions of the plates. The Earth's gravitational field is, of course, also not considered in these experiments. The standard SRT and LCT can thus be used here without any problems. The photons will move the same way as they would in any other Minkowski flat space-time along the straight lines without any bending. The repulsive photon force is calculated the same way as in the previous case. The gravitational force is calculated from the following formula:

$$F_g = \frac{2\pi\ \kappa\ M_g (v)^2}{A_o \sqrt{1 - v^2 / c^2}}, \tag{8}$$

where M_g is the gravitational mass of the plates and κ is the gravitational constant. Equating now the gravitational force with the photon force to keep the plates again in equilibrium the following result is obtained:

$$M_g (v)^2 = n \frac{hc}{2\pi\kappa} \frac{A_o}{\frac{\lambda_o}{2} d} \left(1 - \frac{v^2}{c^2}\right) = n\ M_p^2 \frac{A_o}{\frac{\lambda_o}{2} d} \left(1 - \frac{v^2}{c^2}\right), \tag{9}$$

where M_p is the Planck mass. This result is very important for judging the correctness and correspondence with reality of various theories of gravity.

4. Linearized GR Force Equation

When the standard GRT is considered and linearized for the weak fields [1] the linearization result resembles EMT equations:

$$\nabla \cdot \vec{E}_g = -4\pi\kappa\rho_g, \tag{10}$$

$$\nabla \cdot \vec{B}_g = 0\,, \tag{11}$$

$$\nabla \wedge \vec{E}_g = -\frac{1}{2c}\frac{\partial \vec{B}_g}{\partial t}\,, \tag{12}$$

$$\nabla \wedge \frac{1}{2}\vec{B}_g = \frac{1}{c}\frac{\partial \vec{E}_g}{\partial t} - \frac{4\pi\kappa}{c}\vec{j}\,. \tag{13}$$

In this approach, where the gravitomagnetic B_g and gravitoelectric E_g fields were introduced, the similar force equation as the Lorentz force equation of the EM theory is also derived:

$$\vec{F}_{gg} = M_{gg}(v)\left(\vec{E}_g + \frac{\vec{v}}{c} \wedge \vec{B}_g\right). \tag{14}$$

When these equations are used in the above described experiment, the gravitational force equation between the plates, Eq.8, becomes:

$$F_{gg} = \frac{2\pi\ \kappa\ M_{gg}(v)^2}{A_o\sqrt{1 - v^2/c^2}}\left(1 - 2\frac{v^2}{c^2}\right), \tag{15}$$

and from that the dependency of the gravitational mass on velocity is as follows [2]:

$$M_{gg}(v)^2 = n\ M_p^2\ \frac{A_o}{d\,\frac{\lambda_o}{2}}\frac{\left(1 - \frac{v^2}{c^2}\right)}{\left(1 - 2\frac{v^2}{c^2}\right)}. \tag{16}$$

It is apparent that Eq.16 will show the inertial mass type of dependency on velocity for small velocities but the dependency will not be exact and will fail for larger velocities due to the well known factor of two figuring in the denominator velocity bracket. This is a problem. Since Eq.9 was derived using SRT theory the result should perfectly match Eq.16, or Eq.16 should perfectly match the inertial mass dependency on velocity since the plates can be chosen with arbitrarily small masses to satisfy the GRT linearization assumption. The troublesome denominator bracket is the consequence of the gravitoelectromagnetic equivalent of the EM Lorentz force that has not been experimentally directly observed yet. If such a force does not exist, then the gravitational mass must depend on velocity according to Eq.9.

It is peculiar that GRT cannot provide an accurate linear equation even for the weak fields and for the gravitational mass dependency on velocity (Lorentz velocity factor) for such a simple experimental arrangement as is described in Fig.1. This fact proves that GRT is not consistent with SRT and casts a great suspicion on its correctness.

It seems unreasonable to derive the linearized vector theory from GRT or use a tensor theory to describe the gravity since the gravity has only one type of the "gravitational charge" M and only one type of the attractive force between masses. Some authors [3] suggest that the factor of two, which causes the problem in Eq.16, is the carryover from the fact that the classic graviton is a spin two particle. It is therefore reasonable to expect a vector theory for the EM fields, since there are both attractive and repulsive forces, with the photon having the spin equal to 1. However, for the gravity a simple scalar theory should suffice with the graviton having the spin equal to zero. As a consequence of the scalar theory of gravity the Lorentz factor bracket in the denominator of Eq.16 would be equal to unity. In such a theory, or in a theory without the gravitomagnetic effect, the gravitational mass and the inertial mass must have the following dependencies on velocity:

$$M_g(v) = M_o\sqrt{1 - v^2 / c^2}\,, \tag{17}$$

$$M_i(v) = \frac{M_o}{\sqrt{1 - v^2 / c^2}}\,, \tag{18}$$

where M_o is the rest mass, in order not to contradict SRT. This seems more reasonable than the dependency described by Eq.16. Of course in such a scalar theory of gravity the photons do not have any gravitational mass, only the inertial mass, which again contradicts one of the key assumptions of GRT and directly challenges all the theories and mountains of publications constantly being written about Black Holes.

5. Force Equation from the Schwarzschild Metric

To avoid potential problems and resulting errors with the linearization of GRT, the gravitational mass dependency on velocity can be also derived directly from the Schwarzschild metric, which is a solution of Einstein Field equations for centrally gravitating body. The metric is found assuming that the mass energy tensor in the space around the main body is zero. The equation for the force and from that for the gravitational mass dependence on velocity can be derived by having a small test mass with its rest mass equal to m_o fall radially in the static gravitational field of the main body. The Lagrangian describing such a motion in a coordinate system referenced to the main body is as follows:

$$L = g_{tt}\left(\frac{cdt}{d\tau}\right)^2 - g_{rr}\left(\frac{dr}{d\tau}\right)^2. \tag{19}$$

The first integrals of corresponding Euler-Lagrange equations can be easily found to be:

$$g_{tt}\frac{dt}{d\tau}=1, \tag{20}$$

$$\frac{1}{c^2}\left(\frac{dr}{d\tau}\right)^2=1-g_{tt}, \tag{21}$$

where the identity $g_{tt}g_{rr}=1$ was used together with the zero initial condition at infinity. By differentiating Eq.21 with respect to τ and substituting into the result the expression for the metric coefficient from the Schwarzschild metric: $g_{tt}=(1-R_s/r)$, with the Schwarzschild radius defined as usual: $R_s=2\kappa M_o/c^2$, the following result is obtained:

$$\frac{d}{d\tau}\left(\frac{m_o dr}{d\tau}\right)=-\frac{\kappa M_o m_o}{r^2}, \tag{22}$$

where M_o is the mass of the main gravitating body. In the next step it is necessary to reference the falling test mass to a stationary observer positioned in the proximity of the test mass and express equation Eq.22 in terms of his time t_o according to the relation:

$$d\tau=dt_o\sqrt{g_{tt}(r)}\,. \tag{23}$$

This transform affects both the observed velocity of the falling test mass as well as the observed speed of light. The observer referenced speed of light is calculated by setting the metric line element ds in the metric corresponding to Lagrangian in Eq.19 equal to zero. This leads to following equation:

$$c^2 g_{tt}=g_{rr}\left(\frac{dr_p}{dt_o}\right)^2\left(\frac{dt_o}{dt}\right)^2. \tag{24}$$

This result can be rearranged using Eq.20, Eq.23, and the identity $g_{tt} g_{rr} = 1$ to read:

$$c^2 g_{tt} = c_r^2 , \tag{25}$$

where $c_r = dr_p / dt_o$ is the sought after observer referenced radial speed of light. The value of the metric coefficient g_{tt} is obtained from Eq.21 and expressed in terms of the observer referenced speed of light and the observer referenced test mass velocity $v_r = dr / dt_o$ as:

$$g_{tt}(r) = 1 - v_r^2 / c_r^2 . \tag{26}$$

By substituting these results into Eq.22 the following relation is obtained:

$$\frac{d}{dt_o}\left(\frac{m_o v_r}{\sqrt{1 - v_r^2 / c_r^2}} \right) = -\frac{\kappa\, M_o m_o \sqrt{1 - v_r^2 / c_r^2}}{r^2} . \tag{27}$$

This equation represents the Newton's inertial and gravitational laws with the inertial mass and the gravitational mass depending on velocity as follows:

$$m_i(v) = \frac{m_o}{\sqrt{1 - v^2 / c^2}} , \tag{28}$$

$$m_g(v) = m_o \sqrt{1 - v^2 / c^2} . \tag{29}$$

The same result as in Eq.17 and Eq.18 is obtained with no indication that there is any additional force acting on the test body similar to the Lorentz force of EM theory. It may be strange that the result does not agree with the linearized gravitoelectromagnetic equations, however, the purely radial motion of the test mass may be obscuring this force.

On the other hand, there may not be any gravitomagnetic forces as discussed above.

Perhaps in the future an experiment can be devised that can determine the gravitational mass dependence on velocity with certainty and finally put to rest the questionable correctness of GRT. One possibility of a testing setup arrangement is shown below in Fig.2:

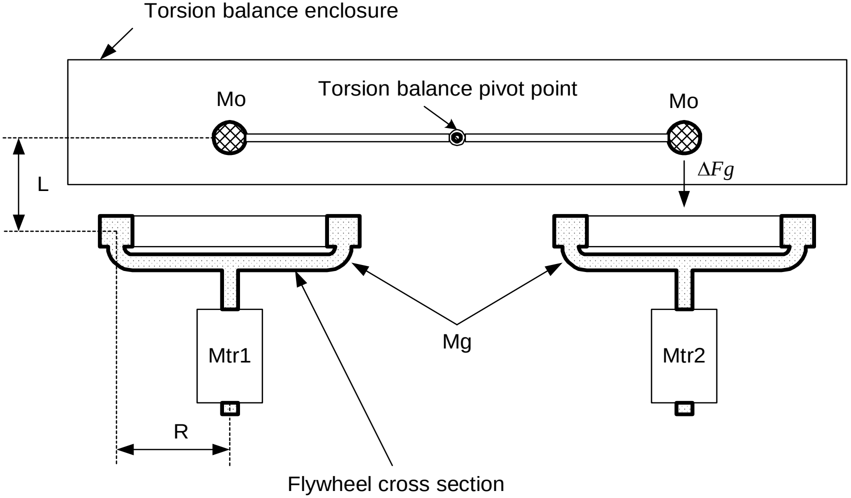

Fig.2. Gravitational mass dependence on velocity measurement setup (top view): two flywheels with masses Mg and Radius R are powered by motors Mtr1 and Mtr2. Motors are mounted horizontally on a vibration insulated table in proximity to an enclosed torsion balance. Motors, which can be integral with the flywheels such as in gas turbines [4], can be independently turned on and off as well as their RPM adjusted. L is adjusted for the optimum sensitivity and is approximately equal to. $L = R/\sqrt{2}$

Two motors that have massive flywheels with radius R and mass M_g attached to them are positioned with their turning axels in a horizontal direction on a vibration isolated table in a laboratory. Two test masses M_o are suspended vertically in a torsion balance arrangement in a shielding enclosure in such a way that the masses are in an optimum distance L from the centers of the flywheels. The torsion balance arrangement is in equilibrium when both motors are turned off. After one motor or the other motor are turned on, the deflection of the balance is read out depending on the RPM of the motors.

The expected force ΔF_{gg} acting on the torsion balance for the optimum distance L can be roughly estimated as follows:

$$\Delta F_{gg} = \pm \frac{\omega^2}{3\sqrt{3}} \frac{\kappa M_o M_g}{c^2}. \tag{30}$$

The sign depends on the theory. It is interesting to note that this result does not depend on the radius of the flywheel, just on the RPM. This may be helpful in the construction of the test setup and in developing the detail measurement technique. For the masses $M_g = 1,200 kg$, $M_o = 8,000 kg$, and the angular velocity of $10,000\ RPM$ the force will be: $\Delta F_{gg} = 1.5\, femto\ N$. This value is indeed very small, but perhaps some motor RPM modulation technique and the torsion balance resonance tuning can be employed to improve the sensitivity.

6. Conclusions

In this article it was shown, using the simple thought experiments that the gravitational mass must depend on velocity differently than the inertial mass. This falsifies the Einstein's WEP. Furthermore it was shown that the absolute equivalence of the inertial and gravitational masses, as is postulated in GRT, is not compatible with SRT and EMT theories. There have been already several similar proofs published previously [5], but these proofs are more complicated with additional tacit assumptions that usually fail to convince skeptics stubbornly adhering to the prevailing dogma. Hopefully, the approach selected in this article is simple enough and sufficiently clear with minimum hidden assumptions that this problem is now avoided. An experimental verification of the gravitational mass dependency on velocity has also been proposed and the expected magnitudes of forces that need to be detected estimated.

References

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