Cosmological Origin of Gravitational Constant

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The base units contributing to gravitational constant are supposed to be related to mass and size of universe and to the speed of light. We test this idea in respect to its conformity with estimations concerning the present parameters of universe. Considering Hubble's law, the size-dependence of gravitatonal constant determines its variability. In turn, this involves the variability of Planck units containing G. Provided that mass of universe and Planck mass coincident at Planck epoch, we arrive at new parameters of universe in quantum state, described by the newly derived Planck units of mass, size, time and temperature.

Keywords: universe; gravitational constant; Planck units; Planck epoch; Planck constant

1. Introduction

The fundamental questions concerning physical constants are still unsolved. What is the relationship between initial conditions of universe, laws of physics and physical constants? Do constants follow from (hypothetic) fundamental laws, or determine these laws? Are all of constants really constant? As long as these questions are not properly recognized, the restrictions put on the constant's values by the present state of universe cannot be treated as ultimate.

For the last few decades, the main attention in that connection has been focused on the Sommerfeld fine-structure constant α. If it increases in time as some of recent observations seem to suggest [1] then at least one of its components must be variable, so that it affects the whole ratio. The main 'suspected' became the speed of light, supposed to be time dependent:

$$c \propto 1/t \tag{1}$$

Another case of that kind concerns the gravitational constant. The conjecture of its variability has been advanced by P.A.M. Dirac in 1937, in the framework of his 'large numbers hypothesis' [2]. Comparing two ratios: 1) radius of visible universe / radius of quantum particle, and 2) gravitational force / electric force that, with the (supposed) accuracy to the order of magnitude, amount 10^{40} and 10^{-40} respectively, Dirac deduced that the value of gravitational constant decreases in proportion to the age of universe:

$$G \propto 1/t \tag{2}$$

The conjecture submitted in this paper (hereafter called in short 'the conjecture') also consists in relating the gravitational constant to some of fundamental parameters describing the universe. However, the exact content of the conjecture essentially differs from Dirac's hypothesis. Namely, we assume that gravitational constant is a physically meaningful quantity of variable value, related in a definite way to the size and mass of universe and to the speed of light.

2. Physical premises for introducing the conjecture

All of the units describing dimensional constants refer to the comprehensible physical quantities such as mass, length, velocity, charge or action. Meanwhile, according to the current state of the art, the only sensible physical explanation of G unit amounts to the need of completing the Newton's law of gravitation, so as to obtain the right dimension of force.

Let us take a closer look at G from that point of view. Its usually applied unit is

$$Nm^2 / kg^2 \tag{3}$$

It can be also expressed by

$$m^3 / kgs^2 \tag{4}$$

Both forms are mathematically equivalent, yet preferable is the latter since it reflects the quantum-mechanical connotation of gravitational constant, thought to have fundamental significance. Namely, G relates to Planck length, Planck mass and Planck time as

$$G = \frac{\ell_P^3}{m_P \times t_P^2} \tag{5}$$

This relationship seems to contradict the claim that physical meaning of gravitational constant's dimension is unclear or unrecognized. However, since G contributes to all of the above Planck units:

$$\ell_P = \left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}};\ m_P = \left(\frac{\hbar c}{G}\right)^{\frac{1}{2}};\ t_P = \left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} \tag{6}$$

so, in fact, obtaining G from Planck units consists in drawing out that what has been previously inserted to them; namely, rewritting (5) gives

$$\left(\frac{\hbar G}{c^3}\right)^{\frac{3}{2}} \times \left(\frac{\hbar c}{G}\right)^{-\frac{1}{2}} \times \left(\frac{\hbar G}{c^5}\right)^{-1} = \hbar^0 c^0 G = G \tag{7}$$

Nevertheless, Eq. 5 brings in two essential informations. The first one is that, mathematically, this equation does not determine the value of G. Let the value of gravitational constant be different from the usual one. What are the consequences of such assumption to Planck units? Let's start from Planck mass. By definition, it is the mass for which the Schwarzschild radius r_S equals the Compton wavelength λ divided by π. Substituting Planck mass to Compton wavelength gives

$$\frac{\lambda}{\pi} = \frac{2\hbar}{m_P c} = 2\hbar\left(\frac{\hbar c^3}{G}\right)^{-\frac{1}{2}} = 2\left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}} = 2\ell_P \tag{8}$$

which conforms to the value obtained by substituting Planck mass to Schwarzschild radius:

$$r_S = \frac{2Gm_P}{c^2} = 2G\left(\frac{\hbar c}{G}\right)^{\frac{1}{2}} c^{-2} = 2\left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}} = 2\ell_P \tag{9}$$

If, therefore, we assume gravitational constant equal to ξG, with ξ the factor of proportionality of a free value, then we get the Planck mass:

$$\xi^{-\frac{1}{2}} m_P = \left(\frac{\hbar c}{\xi G}\right)^{\frac{1}{2}} \tag{10}$$

In consequence, the equality between Compton wavelength and Schwarzschild radius would take place for Planck length of the value $\xi^{1/2}\ell_P$, and Planck time equal to $\xi^{1/2}\ell_P/c$.

The second information obtained from Eq. 5 is the following. Since $\ell_P/t_P = c$ then the relationship between gravitational constant and Planck units can be also written in the form:

$$G = \frac{\ell_P c^2}{m_P} \tag{11}$$

We conjecture that

$$\frac{\ell_P}{m_P} = \frac{R_u}{M_u} \tag{12}$$

where M_u is the (constant) mass of universe, and R_u is its (present) size. Thus, the gravitational constant can be expressed as

$$G = \frac{R_u c^2}{M_u} \tag{13}$$

Considering that R_u is , in the general case, variable accordingly to the Hubble's law, we may define gravitational constant as

$$\xi G \stackrel{def}{=} \frac{\xi R_u c^2}{M_u} \tag{14}$$

where ξ is the dimensionless factor, such that $0 < \xi \leq 1$, provided that $\xi = 1$ refers to the present epoch.

3. The gravitational constant and the magnitude of universe

An ideal test of the conjecture should consist in substituting the right values describing the present size and mass of universe to the new formula of gravitational constant (Eqs. 13, 14), so as to obtain the proper value of G :

$$G = \left(6,67428 \pm 0,00067\right) \times 10^{-11}\, m^3 \big/ kgs^2 \qquad (15)$$

It is clear, however, that we cannot expect the exact outcome on this way since all avaliable data concerning the mass and size of the whole (or visible) universe are merely approximations. Nevertheless, the recently applied observational methods are sufficiently advanced to obtain the credible results with the accuracy to the order of magnitude.

The lower bound for the the size of universe, obtained from the WMAP data, is $24\, Gpc$. This gives in SI units $7,2 \times 10^{26} \approx 10^{27}\, m$ [3]. There are convincing arguments in favour of supposition that the real size does not considerably exceed this limit.

The estimations of the mass of universe range from $3 \times 10^{50}\, kg$ to $1,6 \times 10^{60}\, kg$ (ignoring the assumption of infinite mass) [4]. Such a big discrepancy is, to a large extent, determined by diversity of the applied cosmological assumptions. However, the most reliable valuations referred to the whole universe range from $10^{53}\, kg$ to $10^{55}\, kg$. They conform to the valuations of mass of visible universe that amount approximately $3 \times 10^{52}\, kg$, with the age of universe estimated by various methods (including Hubble‘s law, WMAP and astrophysical data) for about $13,7$ billion years. Considering the above mentioned value of size and the estimation of density

($\rho \approx 10^{-27}\ kg/m^3$), it seems reasonable to regard in calculations the mass value $M_u \approx 10^{54}\,kg$.

Considering $c^2 \approx 10^{17}\ m/s$, we may calculate for $\xi = 1$

$$G = \frac{R_u c^2}{M_u} \approx \frac{10^{27} \times 10^{17}}{10^{54}} \approx 10^{-10}\left(m^3/kgs^2\right) \tag{16}$$

Taking into account that the right approximation to the order of magnitude of gravitational constant is $G \approx 10^{-10}$, we may conclude that the conjecture and the recently obtained estimates concerning mass and size of universe, validate each other with the fair accuracy.

4. Coincidence of Planck mass and the mass of universe at Planck epoch

If gravitational constant relates to the (increasing) size of universe then all Planck units containing G also become variable. Let us write the four (originally proposed by M. Planck [5]) base natural units, in the form including factor ξ. We use the superscript $\tilde{}$ to differentiate the general (new) form of Planck unit from the usual form that, according to the conjecture, applies to the present instant only:

$$\tilde{m}_P = \left(\frac{\hbar c}{\xi G}\right)^{\frac{1}{2}} \tag{17}$$

$$\tilde{\ell}_P = \left(\frac{\hbar \xi G}{c^3}\right)^{\frac{1}{2}} \tag{18}$$

$$\tilde{t}_P = \left(\frac{\hbar \xi G}{c^5} \right)^{\frac{1}{2}} \tag{19}$$

$$\tilde{T}_P = \left(\frac{\hbar c^5}{\xi G k^2} \right)^{\frac{1}{2}} \tag{20}$$

Considering $0 < \xi \leq 1$ it follows

$$\infty = \lim_{\xi \to 0} \widetilde{m}_P(\xi) = \left(\frac{\hbar c}{\xi G} \right)^{\frac{1}{2}} \tag{21}$$

$$0 = \lim_{\xi \to 0} \tilde{\ell}_P(\xi) = \left(\frac{\hbar \xi G}{c^3} \right)^{\frac{1}{2}} \tag{22}$$

$$0 = \lim_{\xi \to 0} \tilde{t}_P(\xi) = \left(\frac{\hbar \xi G}{c^5} \right)^{\frac{1}{2}} \tag{23}$$

$$\infty = \lim_{\xi \to 0} \tilde{T}_P(\xi) = \left(\frac{\hbar c^5}{\xi G k^2} \right)^{\frac{1}{2}} \tag{24}$$

Certainly it is not correct to mix quantum terms with infinitesimal values. Let us notice, however, that, if Planck units refer to the nature, the Planck mass shouldn‘t exceed the mass of universe. Considering that, we may assume that at Planck epoch the Planck mass coincidences with the mass of universe:

$$m_{P_0} \equiv M_u \tag{25}$$

Hence

$$M_u = \left(\frac{\hbar c}{\xi G}\right)^{\frac{1}{2}} \tag{26}$$

It follows

$$\xi = \frac{\hbar c}{G M_u^2} \tag{27}$$

The Planck length at Planck epoch is therefore

$$\ell_{P_0} = \xi^{\frac{1}{2}} \left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}} = \left(\frac{\hbar c}{G}\right)^{\frac{1}{2}} M_u^{-1} \left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}} = \frac{\hbar}{M_u c} \tag{28}$$

The analogous derivations for Planck time and Planck temperature at Planck epoch (t_{P_0}, T_{P_0}) are:

$$t_{P_0} = \xi^{\frac{1}{2}} \left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} = \left(\frac{\hbar c}{G}\right)^{\frac{1}{2}} M_u^{-1} \left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} = \frac{\hbar}{M_u c^2} \tag{29}$$

$$T_{P_0} = \xi^{-\frac{1}{2}} \left(\frac{\hbar c^5}{G k^2}\right)^{\frac{1}{2}} = \frac{M_u c^2}{k} \tag{30}$$

By substituting numerical values: $M_u \approx 10^{54}$, $\hbar \approx 10^{-34}$, $c \approx 10^{8}$, $c^2 \approx 10^{17}$, $k \approx 10^{-23}$ to Eqs. 28, 29, 30, we obtain respectively (see the counting rule expressed in [6]):

$$\ell_{P_0} \approx \frac{10^{-34}}{10^{54} \times 10^{8}} \approx 10^{-97}\,(m) \tag{31}$$

$$t_{P_0} \approx \frac{10^{-34}}{10^{54} \times 10^{17}} \approx 10^{-105}\,(s) \tag{32}$$

$$T_{P_0} \approx \frac{10^{54} \times 10^{17}}{10^{-23}} \approx 10^{94} \left(K\right) \tag{33}$$

The density at Planck epoch would be then

$$\rho_{P_0} = \frac{M_u}{\ell_{P_0}{}^3} \approx \frac{10^{54}}{10^{-291}} \approx 10^{345} \left(kg/m^3\right) \tag{34}$$

Let us call the newly derived parameters the 'basic Planck units'. Hence, the Planck epoch that we postulate here is defined by the basic Planck units of length, time, temperature and density.

The numerical value of the factor ξ at Planck epoch (ξ_0) can be obtained either directly from $\xi_0 = \hbar c / G M_u{}^2$ (Eq. 27) or, considering $m_P = \left(\hbar c/G\right)^{1/2}$, from

$$\xi_0 = \left(m_P / M_u\right)^2 \tag{35}$$

It follows

$$\xi_0 \approx \left(\frac{10^{-8}}{10^{54}}\right)^2 \approx 10^{-124} \tag{36}$$

Since we assume that ξ is linearly related to R_u so we should expect that

$$\frac{\ell_{P_0}}{\xi_0} = R_u \tag{37}$$

Substituting the numbers obtained for ℓ_{P_0} and for ξ_0, gives

$$R_u \approx \frac{10^{-97}}{10^{-124}} \approx 10^{27} \left(m\right) \tag{38}$$

which confirms the conjecture. We may write then

$$G_0 = \frac{\ell_{P_0} c^2}{m_{P_0}} = \frac{\xi_0 R_u c^2}{M_u} \tag{39}$$

where G_0 stands for gravitational constant at Planck epoch. Numerically, this gives

$$G_0 \approx \frac{10^{-97} \times 10^{17}}{10^{54}} \approx 10^{-134} \left(m^3 / kgs^2 \right) \tag{40}$$

Thus

$$\frac{G}{G_0} \approx 10^{124} \tag{41}$$

From the variability of gravitational constant it follows the variable (in time) value of the gravitational force. However, if the distances between atracting masses are equal as measured in Planck units relevant to given epochs, then gravitational force does not change, i.e.

$$\frac{G_1 m_1 m_2}{\ell_{P_1}^2} = \frac{G_2 m_1 m_2}{\ell_{P_2}^2} = F_{const.} \tag{42}$$

Rewriting the Planck length (Eq. 18) gives

$$\frac{G_1}{\ell_{P_1}^2} = \frac{G_2}{\ell_{P_2}^2} = \frac{c^3}{\hbar} \tag{43}$$

Let us consider the Planck constant from the point of view of the obtained results. It relates to Planck units as

$$\hbar = m_P c \ell_P \tag{44}$$

Rewriting Planck mass and Planck length, we obtain

$$\hbar = \left(\frac{\hbar c}{\xi G}\right)^{\frac{1}{2}} c \left(\frac{\hbar \xi G}{c^3}\right)^{\frac{1}{2}} \tag{45}$$

which means that Eq. 44 reduces to the identity $\hbar \equiv \hbar$, and therefore holds for any value of ξ. This can be written as

$$\hbar = \widetilde{m}_P c \tilde{\ell}_P \tag{46}$$

For m_{P_0} and ℓ_{P_0} (the Planck epoch) it takes the form

$$\hbar = M_u c \ell_{P_0} \tag{47}$$

Numerically, this gives

$$\hbar \approx 10^{54} \times 10^{8} \times 10^{-97} \approx 10^{-34} \left(J \cdot s\right) \tag{48}$$

According to the assumption $m_{P_0} \equiv M_u$, the Planck energy at Planck epoch is

$$E_{P_0} \equiv E_u = M_u c^2 \tag{49}$$

From $E_{P_0} = \hbar \omega_{P_0}$, where $\omega_{P_0} = t_{P_0}^{-1}$, we get

$$E_u = \hbar / t_{P_0} \tag{50}$$

which is the energy equation for the universe at Planck epoch.

Note that the universe at Planck epoch, as well as the Planck particle, satisfies the Schwarzschild equation for the black hole: $r_S = 2G_0 m_{P_0} / c^2$. Namely

$$r_{S(P_0)} = \ell_{P_0} \approx \frac{10^{-134} \times 10^{54}}{10^{-17}} \approx 10^{-97} \left(m\right) \tag{51}$$

In all probability, the present universe also fulfils the Schwarzschild condition:

$$r_{S(u)} = R_u \approx \frac{10^{-11} \times 10^{54}}{10^{17}} \approx 10^{27}\,(m) \tag{52}$$

We may, therefore, calculate the entropy of the universe at Planck epoch, and in present epoch, by applying the Bekenstein-Hawking formula [7], [8] for the entropy of black hole:

$$S = \frac{Akc^3}{4G\hbar} \tag{53}$$

where A is the surface area of the event horizon and k is the Boltzmann constant. For the spherically symmetrical black hole we get $A = \left(m^2 8\pi G^2\right)\big/c^2$, and therefore (53) takes the form:

$$S = m^2\, 2\pi \frac{kcG}{\hbar} \tag{54}$$

Thus, for the Planck epoch we get

$$S_{(P_0)} = M_u^2\, 2\pi \frac{kcG_0}{\hbar} \tag{55}$$

which gives

$$S_{(P_0)} \approx \frac{10^{108} \times 10^{-23} \times 10^{8} \times 10^{-134}}{10^{-34}} \approx 10^{-7}\left(\frac{Jm^2}{Ks^2}\right) \tag{56}$$

This value is equal to the value of entropy for the Planck particle-black hole of the mass m_P. Namely

$$S_{(P)} = m_P^2\, 2\pi \frac{kcG}{\hbar} \approx \frac{10^{-16} \times 10^{-23} \times 10^{8} \times 10^{-11}}{10^{-34}} \approx 10^{-7}\left(\frac{Jm^2}{Ks^2}\right) \tag{57}$$

Instead, for the present universe we have

$$S_{(present)} = M_u^2 2\pi \frac{kcG}{\hbar} \tag{58}$$

which gives

$$S_{(present)} \approx \frac{10^{108} \times 10^{-23} \times 10^{8} \times 10^{-11}}{10^{-34}} \approx 10^{117} \left(\frac{Jm^2}{Ks^2} \right) \tag{59}$$

Hence, the increase of the entropy of universe conforms the factor ξ_0^{-1} and amounts

$$\frac{S_{(present)}}{S_{(P_0)}} = \frac{G}{G_0} \approx 10^{124} \tag{60}$$

Conclusion

The revealed dependence of gravitational constant from the size of universe involves the redefinition of Planck units that eventually appears inconstant. The base parameters of universe (including mass) coincident with basic Planck units at Planck epoch, which means that initial state of universe can be considered in terms of a single quantum. This induces to interprete Planck time rather as a purely technical parameter related to the Heisenberg uncertainty principle than as a hypothetic period ranging from the ‘zero‘ instant to the ‘end point‘ of Planck epoch.

The conjecture seems to entail profound consequences to physics and cosmology. It brings solution to the fundamental cosmological questions concerning the very early universe: the problem low entropy, the horizon problem and the flatness problem. The thermodynamic arrow of time is well defined by the initial value of entropy (10^{-7}) and its present value (10^{117}). The possibility of casual contact within the whole volume of universe at Planck epoch follows from $ct_{P_0} = \ell_{P_0}$. In turn, the ratio between the density at Planck epoch (dramatically greater in comparison with the hitherto estimations) and the relevant spatial curvature (that is constant as related to Planck particle /Eq. 43/) explains fairly well the apparent flatness of the present universe.

Besides, the conjecture confirms the time invariability of Planck constant and the speed of light. This, in turn, speaks against the hypothesis of the varying fine structure constant as related to inconstant c. Last but not least, the conjecture may also help in solving the problem of quantum gravity.

The variable value of gravitational constant involves specific predictions referring to the creation of different cosmic objects such as stars, galaxes and black holes in the deep past. This makes the idea of inconstant G experimentally testable. The same refers to the varying ratio between gravity and the electric forces.

Since, according to GRT, gravitation is recognized as a phenomenon basically connected with spacetime, then it shouldn't be very surprising in general that evolving spacetime determines the (variable) properties of gravitation. Nevertheless, the postulated dependence of gravitational constant from the size of universe demands physical justification. Only then, in our opinion, the conjecture would be able to evolve into a matured theory.

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