Further Evidence of Photon-Graviton Recycling in White Dwarf Luminosities

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Assuming that the universe is not expanding and instead operating under conditions of general equilibrium, the author recently proposed that photons and gravitons are steadily being interconverted at fractional rates proportional to the Hubble constant, H_0. On the cosmic scale, the decay of photons was suggested to give rise to the cosmological redshift, while the decay of gravitons was linked to gravitation. Within a single body, such as a planet or star, the model rate of photon energy production is $dE/dt = -UH_0$, where U is the body's internal gravitational potential energy. Previously, observed rates of planetary heat emission and possible planetary expansion were shown to be consistent with this process. Low luminosity stars at the end of their cycles are also potential candidates for the effect to be observed. Here we consider a group of 21 white dwarfs for which the mass and radius have been most reliably determined. It is found that the luminosities of the 16 hotter objects in the sample are consistent with the model. The five cool white dwarfs in the sample ($T_{\mathrm{eff}} \leq 12{,}000$ K) have lower than expected luminosities. The theoretical and observational picture concerning these particular stars is complicated, however, and inclusion of more recent findings eliminates much of the discrepancy.

Keywords: White dwarfs, graviton decay, tired light, static universe, Le Sage gravity, planetary heating, expanding earth hypothesis

1. Introduction

Ever since Hubble's discovery that the spectra of galaxies are progressively redshifted with distance, the nature of these redshifts has been open to debate. While the universal expansion interpretation is generally accepted today, many so-called 'tired light' models, such as the one first proposed by Zwicky (1929), have periodically been suggested. In these models, the energy of a photon slowly dissipates as it transits space. Many of the tired light models which have been proposed, such as those positing a direct interaction with atoms or electrons in space, have known difficulties. At the same time, the general concept of tired light has never actually been disproved. For example, the discovery of time dilation in the light curves of Type Ia supernovae was deemed conclusive evidence for expansion and against tired light (Leibundgut *et al*., 1996). As recently noted, however, time dilation appears to be associated with diverse kinds of redshifts and the supernova evidence merely places the restriction on a putative tired light mechanism that it too exhibit time dilation (Edwards, 2006, 2007). Applying this restriction would even confer an advantage to tired light models with respect to the Tolman galaxy surface brightness test, since the surface brightness of galaxies would be diminished by an extra factor of $(1 + z)^{-1}$. In addition, a variety of other tests have tended to favour non-expanding or 'static' models (for discussions, see Lopez-Corredoira, 2003; Edwards, 2006, 2007).

Working under the premise that the universe is not expanding and instead existing in a state of general equilibrium, the author recently proposed that gravitons too are subject to an analogous decay process over time. Specifically, it was proposed that graviton energy and photon energy are everywhere being interconverted at fractional rates proportional to the Hubble constant, H_0 (Edwards, 2006). These recycling processes were considered to be viable if gravitons are modelled not as the traditional spin-2 gauge bosons, but rather as a form of virtual photon. The conversion of gravitons to photons and the reverse process of photon to graviton decay in this case could each be expressed as $(dE/dt)/E = -H_0$, where E is the initial photon or graviton energy. Due to the regeneration of gravitons from photons, gravitational forces between bodies, as well as the gravitational constant G, would not diminish over time. In this respect the model thus differs fundamentally from earlier suggestions by Dirac (1937) and others that G decays at a fractional rate proportional to H_0, for which observational evidence has been mostly negative (Uzan, 2003).

Evidence for the proposed graviton conversion was suggested to lie in the observed heat emissions from planets and in the possible expansion of the Earth (Edwards, 2006). In the model, the quantity of energy tied up in the gravitons exchanged between two bodies is assumed to be equal in magnitude to the gravitational potential energy (U) of the gravitational system. The expression for the conversion of graviton energy in a system to photon energy, E, is then

$$\frac{dE}{dt} = -UH_0 . \tag{1}$$

As mentioned, the quantity U is not changed by this graviton decay *per se*, since gravitons are simultaneously being reconstituted in gravitational systems. As was shown, the heat produced in this decay can possibly explain the excess heat emissions of the Earth, the Moon and the planets. Typically, the emissions of these bodies amount to 5-10% of the total energy available through the postulated gravitational decay (see Table 1). The additional energy potentially available for expansion in these planets is thus about 90-95% of the gravitational energy lost. As was shown, this amount of energy is sufficient to have expanded the Earth from an initial radius about 60% of its present size. Several lines of evidence suggest a slow expansion of the Earth at a rate of .5-.7 mm/yr since the time of Earth's formation (Wesson, 1978, 1980; Weijermars, 1986; for recent discussions, see Scalera and Jacob, 2003).

On the universal scale, it was also shown that the decay of long-range gravitons to radio photons could be the agent for gravitation (Edwards, 2007). In this case it would be the gravitational potential energy that a mass has with respect to the most distant stars in the visible universe which is slowly decaying to photons. The photons arising from such decay constitute a cosmic background of radio photons and it is the reverse process − absorption of these photons back into the graviton lattices of masses − which is suggested to cause gravitational attraction. The mechanism operates in a manner analogous to the wave versions of Le Sage's theory of gravitation (for discussions of the latter, see Edwards, 2002).

To test the model further, it would be desirable to compare the luminosities of diverse kinds of astrophysical objects with their model predictions. Main sequence stars have luminosities far too high for the postulated effect to be observable. In the case of the Sun, for example, the quantity $-UH_0$ amounts to 5×10^{30} erg s^{-1}, almost three orders of magnitude lower than its actual luminosity. However, for stars that are either too small to have initiated sustained fusion (*e.g.*, small brown dwarfs) or have finished their time on the main sequence and now conduct little fusion (*e.g.*, white dwarfs, neutron stars), the luminosities resulting from the model equations are much more likely to be measurable. Recently, a binary pair of brown dwarfs was discovered and characterized (Stassun *et al.*, 2006). Unfortunately, the rather high luminosities of this pair are indicative of recent formation and exclude them as useful test objects. In this paper, we will instead focus on white dwarf luminosities, for which a large body of data now exists.

2. Overview of Current Theory of White Dwarf Evolution

Present theories of white dwarf cooling and evolution are complex and are continuously being updated in the light of new observations (for reviews, see d'Antona and Mazzitelli, 1990; Koester, 2002; Hansen and Liebert, 2003; Hansen, 2004). The following discussion represents just a brief overview of the current theoretical situation, emphasizing those aspects of relevance to the model.

White dwarfs (WDs) are the end stage of stellar evolution in progenitor stars with masses less than 6-8 $M_\odot$. Thus, some 90% of all stars end up as white dwarfs. The difference in mass between the progenitor and the white dwarf is lost to the interstellar medium during the explosive process giving rise to the white dwarf. The white dwarfs begin their lives with very high luminosities and are considered to gradually cool off and fade away over a period of 5-10 billion years. With their very high densities (~ 10^6 g/cm^{-3}), the electrons in white dwarfs are degenerate and this in turn determines many of the physical and structural characteristics of white dwarfs. Degeneracy limits the masses of white dwarfs to approximately 1.4 $M_\odot$. The WD atmospheres in the majority of cases are either predominantly H or He. The presence of these lighter elements in the atmosphere is thought to result from diffusion in the early stages of WD evolution of heavier elements, primarily C and O, towards the core. The degeneracy of electrons in white dwarfs provides the theoretical basis for the much studied WD 'mass-radius relationship', which may be expressed simplistically as $R \propto M^{-1/3}$. As will be discussed, this relationship potentially renders white dwarfs enigmatic as test objects for the model.

The main equation which governs WD cooling is given by

$$L = L_{th} + L_g + L_{nuc} + L_\nu , \tag{2}$$

where L is the bolometric luminosity and the other terms are defined as below. The main contributor to L is the loss of thermal energy (L_{th}) acquired by the white dwarf during its gravitational collapse from the precursor star. It is generally assumed that the white dwarf has a degenerate, nearly isothermal core, which gradually cools as it releases its residual heat through the nondegenerate outer layers. During the first phases of WD evolution, neutrino cooling (L_ν) drains off most of this energy. After ~ 30 million years, neutrino cooling declines and becomes negligible once log $(L/L_\odot)$ falls to ~ −1.5. However, for massive white dwarfs ($1.0\ M_\odot < M < 1.4\ M_\odot$), neutrino emission arising from fusion in the outer shell can cause significant neutrino emission once more. Another input to L is nuclear energy (L_{nuc}) from proton-proton and CNO burning. H burning, in particular, is thought to provide 50% or more of luminosity for the star down to log $(L/L_\odot)$ ~ −2.5 to −3. At later stages of evolution, gravitational energy release (L_{g}) becomes significant, when lower luminosities permit nondegenerate external layers to settle at near constant radius. As noted earlier, the core temperature is thought not to be significantly affected by changes in density during the cooling process, since any gravitational energy released is immediately absorbed by degenerate electrons being forced into higher energy levels. As the white dwarf cools further, a final source of energy is latent heat of crystallization, considered a contribution to L_{th}. The latent heat can theoretically increase by 5-30% the period during which a star has luminosities in the range log $(L/L_{\odot}) \sim 10^{-3}$-$10^{-3.5}$.

For white dwarfs with $T_{\mathrm{eff}} > 12{,}000$ K, radiative cooling through the surface non-degenerate layer is the dominant mode of cooling. Below this temperature, a convection layer forms which increases in thickness as the white dwarf cools. For an envelope made of helium, the convection layer forms at higher temperatures and extends deeper into the star. Interactions between the convective layer and the surface layer complicate the physics of the cooling process (see, *e.g.*, Hansen, 2004) and will be discussed again later.

Since white dwarfs are end stages of stellar evolution, all the inputs to L in Equation 2 eventually diminish to insignificant levels over time. In recent surveys, large numbers of cool white dwarfs have been found, some with T_{eff} less than 5,000 K (Hansen & Liebert, 2003; Kilic *et al.*, 2006). These low temperatures are considered to reflect the ages of the white dwarfs and thus, in the conventional interpretation, potentially provide constraints on the ages of the galactic disk and the universe (Leggett *et al.*, 1998; Hansen, 2004).

3. The Model Process Applied to White Dwarfs

From the previous section, it can be seen that white dwarfs have several features which may complicate their use as test objects for the graviton recycling model. Due to electron degeneracy pressure, the radii of white dwarfs are insensitive to the core temperatures and, at least in theory, depend only on the mass. As a consequence, any extra source of internal heating is thought simply to raise the degenerate electrons to higher energy states. This energy sink could mask the novel heating effect being proposed in the model. At the same time, *if* white dwarf radii are in fact approximately constant, then any internally derived heat would tend to be reflected in external heat emission rather than radial expansion. These two considerations could thus potentially offset each other to some extent.

From the standpoint of the model, the period of interest is when the luminosity falls below log $(L/L_\odot) \sim -1.5$. At these stages of WD evolution, the main inputs to L are from L_{th} and L_{nuc}. The model prediction, if correct, would imply that one or both of these inputs are in some cases currently being overestimated.

According to the model, the gravitational potential energy of a white dwarf, even at constant radius, is being converted to photons and heat according to Equation 1. For a polytropic star, the quantity U is given by

$$U = -\frac{3}{5-n}\frac{GM^2}{R}, \tag{3}$$

where M and R are the mass and radius respectively and n is the polytropic index. This study will focus on white dwarfs with masses much less than the Chandrasekhar limit of $\sim 1.4\ M_\odot$. Such polytropes are supported by the pressure of a non-relativistic gas and thus correspond to the case $n = 1.5$. For these stars the quantity U is then given to a fair approximation as

$$U \equiv -\frac{GM^2}{R}. \tag{4}$$

Supposing that *all* of the heat generated in the model process is radiated into space, the expression for the model WD luminosity, L_{m}, is then

$$L_m = \frac{GM^2}{R} H_0. \tag{5}$$

In addition to the polytrope index, the other major source of uncertainty in our model is in the choice of value for H_0. Historically, a wide range of values for H_0 have been found, from 50-100 km s^{-1} Mpc^{-1}. For consistency with our earlier work, an intermediate value for H_0 of 66 km s^{-1} $Mpc^{-1} = 2.2 \times 10^{-18}$ s^{-1} will be used. This value is also very close to the statistically derived one of 67 km s^{-1} Mpc^{-1} found by Gott *et al.* (2001) using a large number of previous estimates of H_0.

From Equation 5, it is apparent that the predicted luminosity of white dwarfs in our model is determined mainly by the WD mass and radius. Many catalogues of white dwarfs are now available in which mass and radius are reported. As noted by Provencal *et al.* (1998), however, the values listed for mass and radius in most cases are not the result of independent determinations of both mass and radius, but rather are based in some way on the WD mass-radius relationship. In their efforts to study the mass-radius relationship itself, those authors therefore restricted their analysis to a group of 21 white dwarfs for which mass and radius had been independently determined. They noted that the mass-radius relationship did not seem to hold especially well among their sample of objects, since white dwarfs with similar masses were observed to have a wide range of radii. They suggested accordingly that certain members of the sample had denser cores than predicted by theory.

Mathews *et al.* (2006) more recently considered whether the apparent deviations from the mass-radius relationship amongst this same group of white dwarfs could be due to the presence of strange-quark matter in some of the white dwarf cores. Their study included an updated estimate for the radius of Procyon B, following Provencal *et al.* (2002), and corrected values for the radii of L481-60 and G154-B5B, which appeared as discrepant values in Table 5 in Provencal *et al.* (1998).

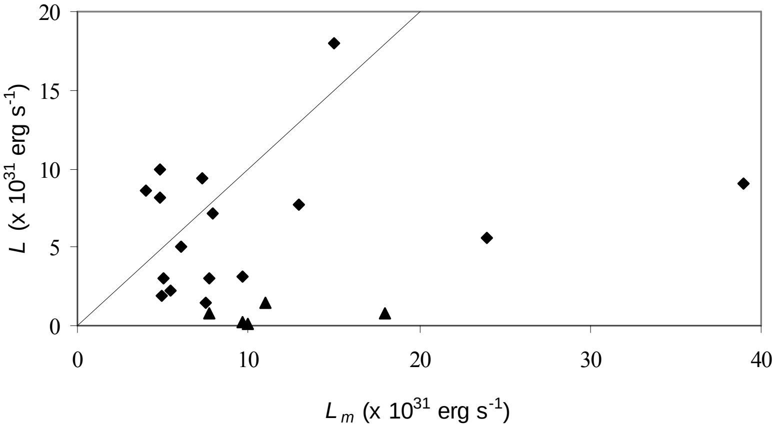

Fig. 1. The observed white dwarf luminosity, L, is plotted against the predicted luminosity, L_m. The diamonds are the hotter objects in the sample (T_{eff} > 12,000 K). The triangles are the cool white dwarfs ($T_{\text{eff}} \leq$ 12,000 K). The solid line is the 1:1 correspondence.

The present study will use the data for the group of 21 white dwarfs analyzed by Provencal *et al.* (1998), as updated by Mathews *et al.*, to test the model.

4. Results and Discussion

The relevant data from Mathews *et al.* for the 21 objects are given in Table 2. Application of the model equation to this group of white dwarfs yields the results shown in Table 3. The model luminosity, L_{m}, is the expected luminosity if all the energy generated from the model process were to be radiated away (*i.e.*, no expansion is occurring). The total luminosity, L, for each star is obtained from $4\pi R^2\sigma T_{\text{eff}}^4$, where σ is the Stefan-Boltzmann constant (= 5.67 × 10^{-5}

ergs cm^{-2} (deg-K)$^{-4}$ s^{-1}). These results are plotted in Figure 1. The error ranges for estimates of mass, radius and effective temperature shown in Table 2 are not carried over to the results shown in Table 3 and Figure 1, since these sources of error are minor compared to the uncertainties in H_0 and the polytrope index n.

From Table 3 and Figure 1, it is seen that for the 16 hotter DA white dwarfs in the sample ($T_{\rm eff} \geq 12{,}000$ K), the predicted luminosities are well within an order of magnitude of the observed luminosity, with a mean ratio $L/L_{\rm m} = 0.84$. This includes all of the field white dwarfs in the sample, except for G226-29. Several stars – CD-38 10980, L711-10, Wolf 1346, Feige 22 and G238-44 – have observed luminosities greater than their model predictions, with the maximum values being about double the predicted value. However, the excess luminosity for these stars is readily explicable using the traditional modes of white dwarf cooling, particularly $L_{\rm th}$ and $L_{\rm nuc}$.

It was noted above that white dwarfs could be enigmatic objects from the standpoint of the graviton decay model. On one hand, any extra internal heat generated would in theory simply raise the energy states of degenerate electrons. At the same time, the theoretical constancy in WD radii would allow all excess heat to be radiated away rather than be used to drive expansion. The heating effect could thus be more fully displayed than might be the case, for instance, in planets. In regards to these potentially offsetting uncertainties, we can only note that the observed luminosities of the 16 hotter white dwarfs in the sample appear to be consistent with the model process.

4.1 The Cool White Dwarfs

For the five cool white dwarfs ($T_{\rm eff} \leq 12{,}000$ K) in the sample – G226-29, L268-92, Procyon B, L481-60, and G156-64 – the observed luminosity is only 1-10% of the model luminosity. We discuss these stars in turn.

L268-92 has a luminosity that is 14% of the model prediction. While low, this ratio is consistent with the planetary ratios (Table 1). On the other hand, in their spectropolarimetric study of magnetism in H-rich white dwarfs, Kawka *et al.* (2007) analyzed the stellar spectra to provide effective temperature and mass estimates. Compared to the values listed in Table 2, L268-92 was found to have a much higher T_{eff} of 14,660 K and a lower mass of 0.62 $M_{\odot}$. No radius value was given, but if the radius given in Table 2 is used, the ratio of observed luminosity to our model prediction for L268-92 would be revised upwards to .41. This would place L268-92 with the 16 hotter DA white dwarfs of the sample.

L481-60 has a luminosity about one tenth of the model luminosity, a ratio also not different from the planetary values. In this instance, it may perhaps be relevant that photometric observations of L481-60 are contaminated by a bright nearby star. For this reason, it was excluded from a sample of 152 cool white dwarfs compiled by Bergeron *et al.* (2001) in their photometric and spectroscopic WD study.

Concerning G226-29, a ZZ Ceti star, a controversy has existed surrounding its effective temperature, with reported values from 12,100 K to 13,600 K. G226-29 is one of the field white dwarfs in the sample, for which the masses were determined using spectroscopic surface gravity estimates. In their study, Provencal *et al.* chose a value of 12,000 K, which yields a mass of .77 $M_{\odot}$. By contrast, they noted that the higher T_{eff} of 13,600 K would imply a much lower mass of 0.47 $M_{\odot}$. Subsequent studies focusing on G226-29 have not fully clarified this issue. For example, Allard *et al.* (2004) studied G226-29 using *FUSE* and found an optimal fit of their spectroscopic data with $\log g = 7.9$, significantly lower than the earlier reported values of $\log g = 8.2$-8.3. This lower value for surface gravity would imply a much smaller mass for G226-29 and/or a larger radius. In either case the model luminosity prediction would be significantly lowered and there would thus be better agreement with the observed luminosity.

The two white dwarfs in our sample which fall well short of their model predictions are G156-64 and Procyon B, having L/L_m ratios of .011 and .016 respectively. G156-64 was one of the stars occurring as common proper motion pairs in the Provencal *et al*. study. Since the error ranges in mass determination for the CPM pairs were significantly greater than for the visual binaries and field stars in the sample (see Table 2), these could cause a portion of the discrepancy. Provencal *et al*. also noted that the gravimetric mass of G156-64 is indicative of a typical WD, whereas the surface gravity pointed to a mass larger than that of Sirius B. Previously, it had been shown that, for white dwarfs with T_{eff} < 12,000 K, large amounts of spectroscopically invisible He may be brought to the surface convectively and give rise to pressure effects which mimic increased surface gravity. Thus, cool DA white dwarfs with large surface gravities can also be interpreted as stars with dense helium envelopes and normal masses. Provencal *et al.* therefore concluded that a thick layer of helium was likely mimicking increased surface gravity in this case. Cool white dwarfs with helium atmospheres exhibit many other unusual characteristics affecting determinations of effective temperature, mass and radius (Hansen, 2004). Thus, the status of G156-64 within the model framework could improve with further study of these kinds of stars.

At one point, Procyon B was considered to deviate strongly from the WD mass-radius relationship and to possibly have an iron core. A subsequent analysis by Provencal *et al*. (2002) determined that Procyon B was in fact a rare DQZ star and, correspondingly, that its T_{eff} was smaller and radius larger. This allowed Procyon B to be placed on the WD mass-radius curve. The analysis by Provencal *et al*. later received support from Dufour *et al*. (2005), who used it as a template in their broader study of DQ stars. Observations of Procyon B have always been difficult due to its close proximity to Procyon A. Nonetheless, among all the stars in our sample, the luminosity of Procyon B is most difficult to reconcile with its model prediction.

In recent years a number of surveys of cool white dwarfs have been done (*eg.*, Kilic *et al.*, 2006). In some cases, surface temperatures of 4,000 K or lower have been reported. However, unlike the present sample, the WD masses and radii in these surveys are not determined independently. Instead, the WD mass-radius relationship is called upon to furnish these data. In addition, the aforementioned difficulties in spectroscopic determinations of mass and radius at $T_{\mathrm{eff}} < 12{,}000$ K apply in these studies. This is an unfortunate situation, as it is precisely for the coolest white dwarfs that the model predictions differ most sharply from traditional models. In the conventional view, the surface temperatures of the coolest white dwarfs reflect their considerable ages and thus give possible constraints on the age of the galactic disk and the universe. Conversely, in the present model, the low temperatures of these objects merely reflect their small masses and/or large radii.

5. Conclusions

In this study we have further tested the model of photon-graviton recycling using a group of 21 white dwarfs for which radius and mass have been independently measured. Using a polytrope index $n = 1.5$ and $H_0 = 66$ km s^{-1} Mpc^{-1}, we find general agreement with our model for the 16 hotter DA white dwarfs of the sample. While the approximate correspondence obtained for this small sample of objects might be viewed as mere coincidence, this seems less likely when combined with the similar pattern found for the planets (Table 1). These two populations of objects, planets and white dwarfs, have internal gravitational potential energies separated by over 10 orders of magnitude. A double coincidence of this nature would therefore seem highly unlikely.

For the 5 cooler white dwarfs in the sample, the observed luminosity is only 1-15% of the model expectation. As discussed, however, the general difficulties in making accurate determinations of mass and radius for cool white dwarfs, in addition to specific anomalies affecting several of the five stars, place this group of white dwarfs in an uncertain category from the model perspective.

In conclusion, the luminosities of the hotter white dwarfs in the sample can be interpreted as lending additional support to the graviton recycling model, which was based on the premise of a static universe in overall equilibrium. Further opportunities to test the model may be expected in the near future with observations of low mass brown dwarfs, wherein deuterium burning and other residual fusion processes may be negligible.

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	$-U$ (erg)	**$-UH$ (erg/s)**	**Excess Heat Flux (erg/s)**	**Ratio**
Earth	2.49×10^{39}	5.49×10^{21}	3.2×10^{20}	.058
Jupiter	2.63×10^{43}	5.79×10^{25}	3.35×10^{24}	.0579
Saturn	3.60×10^{42}	7.92×10^{24}	7.9×10^{23}	.10
Neptune	2.19×10^{41}	4.82×10^{23}	3.28×10^{22}	.0680
Uranus	1.59×10^{41}	3.50×10^{23}	3.3×10^{21}	.0094

Table 1. Planetary heat emissions and model predictions. The internal gravitational potential energy of each body is in the first column. The heat generated by conversion through the model process is in the second column, where $H = 2.2 \times 10^{-18}$ sec^{-1}. The third column shows the observed excess heat emission of each body. The ratio of the observed heat emission to the model prediction is in the last column. The anomalously low heat emission of Uranus may be due the large tilt in its axis of rotation, which makes accurate measurements difficult.

Object	T_{eff}	$M/M_\odot$	$R/R_\odot$
Sirius B	24700 ± 300	1.0034 ± 0.026	0.00840 ± 0.00025
G226-29	12000 ± 300	0.750 ± 0.030	0.01040 ± 0.0003
G93-48	18300 ± 300	0.750 ± 0.060	0.01410 ± 0.0020
CD-38 10980	24000 ± 200	0.740 ± 0.040	0.01245 ± 0.0004
L268-92	11800 ± 1000	0.700 ± 0.120	0.01490 ± 0.0010
Procyon B	7740 ± 50	0.602 ± 0.015	0.01234 ± 0.00032
Wolf 485 A	14100 ± 400	0.590 ± 0.040	0.01500 ± 0.0010
L711-10	19900 ± 400	0.540 ± 0.040	0.01320 ± 0.0010
L481-60	11300 ± 300	0.530 ± 0.050	0.01200 ± 0.0040
40 Eri B	16700 ± 300	0.501 ± 0.011	0.01360 ± 0.0002
G154-B5B	14000 ± 400	0.460 ± 0.080	0.01300 ± 0.0020
Wolf 1346	20000 ± 300	0.440 ± 0.010	0.01342 ± 0.0006
Feige 22	19100 ± 400	0.410 ± 0.030	0.01367 ± 0.0020
GD 140	21700 ± 300	0.790 ± 0.020	0.00854 ± 0.0005
G156-64	7160 ± 200	0.590 ± 0.060	0.01100 ± 0.0010
EG 21	16200 ± 300	0.580 ± 0.050	0.01150 ± 0.0004
EG 50	21000 ± 300	0.500 ± 0.020	0.01040 ± 0.0006
G181-B5B	13600 ± 500	0.500 ± 0.050	0.01100 ± 0.0010
GD 279	13500 ± 200	0.440 ± 0.020	0.01290 ± 0.0008
WD2007-303	15200 ± 700	0.440 ± 0.050	0.01280 ± 0.0010
G238-44	20200 ± 400	0.420 ± 0.010	0.01200 ± 0.0010

Table 2. White dwarf properties. The second column gives the effective temperature, the third column the ratio of the WD mass to the solar mass and the last column the ratio of the WD radius to the solar radius.

Object	L ($\times 10^{31}$ erg s^{-1})	L_m ($\times 10^{31}$ erg s^{-1})	L/L_m
Sirius B	9.0	39	.23
G226-29	0.77	18	.043
G93-48	7.7	13	.59
CD-38 10980	18	15	1.2
L268-92	1.5	11	.14
Procyon B	0.19	9.7	.016
Wolf 485 A	3.0	7.7	.39
L711-10	9.4	7.3	1.3
L481-60	0.81	7.7	.11
40 Eri B	5.0	6.1	.82
G154-B5B	2.2	5.4	.41
Wolf 1346	9.9	4.8	2.1
Feige 22	8.6	4.1	2.1
GD 140	5.6	24	.23
G156-64	0.11	10	.011
EG 21	3.1	9.7	.32
EG 50	7.2	7.9	.91
G181-B5B	1.4	7.5	.19
GD 279	1.9	4.9	.38
WD2007-303	3.0	5.0	.60
G238-44	8.2	4.9	1.7

Table 3. Model relationships. The second column gives the observed luminosity, the third column the model prediction and the last column the ratio of the observed luminosity to the model luminosity.