Gravitational and Kinetic Scaling of Physical Units

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One of Einstein's great innovations in relativity theory was to break with the idea that there is an exchange of kinetic and potential energy as an object changes its position in a gravitational field. Instead, he assumed that the unit of energy varies in a systematic manner that can be deduced from Newton's Universal Law of Gravitation and his own famous mass/energy equivalence relation. In the present work the way in which the units of other physical quantities vary with position in a gravitational field is derived on the basis of experimental observations such as the gravitational red shift and the angle of displacement of star images during solar eclipses. A computational method introduced by Schiff in 1960 to predict the latter value that does not involve general relativity plays a key role in this discussion. In particular, it has been possible to extend this approach successfully for the first time to the calculation of the precession angle of the perihelion of planetary orbits around the Sun, obtaining quantitative agreement with Einstein's original results. The present work on the gravitational scaling of physical units complements earlier work dealing with the analogous

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"kinetic" scaling of the units of energy, time, length and gravitational mass and other quantities derived therefrom.

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I. Introduction

In recent work [1] it has been shown that the units of physical quantities differ in a well-defined manner from one inertial system to another. A key example is the unit of time. It is known from experiment [2], and was predicted by Einstein's special theory of relativity (STR [3]), that atomic clocks run $\gamma(u) = \left(\frac{1-u^2}{c^2}\right)^{-0.5}$ times slower when they are in motion with speed u relative to the observer than when they are stationary in the latter's rest frame. It therefore follows that the unit of time is directly proportional to $\gamma(u)$ in the inertial system in which the clock is at rest. The values for elapsed times such as radiative periods and non-radiative lifetimes measured by different observers are therefore found to be inversely proportional to the unit of time in their respective inertial systems. Once the ratios of the units of a small number of key physical quantities in different inertial systems are known, specifically for energy, time, length and gravitational mass, it becomes possible to determine the corresponding ratios of units of all related quantities on the basis of their definitions in terms of the latter four units.

The above relationships have been characterized by the term "kinetic scaling." In the following discussion an analogous set of relationships (gravitational scaling) will be derived for observers located at different gravitational potentials. The determination as to how the various units change with distance from a gravitational source will be based primarily on the early work of Einstein [4], but also on a classic paper by Schiff [5] that computes the angle of displacement of star images during solar eclipses.

II. Variation of the Unit of Energy

The classical view of how the total energy of an object varies with altitude is that there is a gravitational potential energy. When the object falls it is subject to a constant acceleration, which according to Galileo is exactly the same for all masses, but its energy does not change due to the fact that the accompanying increase in kinetic energy is exactly balanced by a decrease in its potential energy. Einstein had shown with his STR [2] that the energy of an object at rest is equal to the product of its inertial mass and the square of the speed of light ($E=m_I c^2$). He deviated further from the traditional view by assuming that the increase in kinetic energy as the object falls from point P to point O is due to the fact that *the unit of energy decreases* with its distance dr from a gravitational source. He concluded that the relationship between the respective total energies at these two points is given by the formula:

$$E_O = E_P\left(\frac{1+gdr}{c^2}\right), \tag{1}$$

where $dr>0$ is the (differential) vertical distance between the two points and g is the local acceleration due to gravity as computed by Newton's Universal Law of Gravitation.

When an object falls a finite distance from infinity toward a gravitational source, the corresponding change in the unit of energy is obtained by integration [5] of the quantity in parentheses on the right-hand side of eq. (1). It is helpful therefore to make the following definition:

$$A_O = 1 + \frac{GM_S}{c^2 r_O}, \tag{2}$$

in which G is the universal gravitational constant (6.67 x 10^{-11} N m^2/kg^2), M_S is the gravitational mass of the source and r_O is the distance from point O to the source (active mass). The unit of energy is A_O times *smaller* at O than it is at an infinite distance from the gravitational source. The quantity A_O plays an analogous role in the gravitational scaling procedure as α_O does for kinetic scaling [1]: clocks at rest on the Earth by definition run α_O times faster than their counterparts in the rest frame of inertial system O.

As a consequence of the above definition, it follows that the ratio of the energy values measured for the same object by two observers located at different distances from the gravitational source, O at r_O and P at r_P, is given by

$$E(O) = \left(\frac{A_O}{A_P}\right) E(P). \tag{3}$$

If O is located at a higher altitude than P, i.e., if $r_O > r_P$, his energy value will be smaller than that of P because $A_O < A_P$ according to eq. (2). In the following discussion the ratio $\frac{A_O}{A_P}$ will be referred to as S. It is then analogous to the clock-rate ratio $R = \alpha_M/\alpha_O$ employed in kinetic scaling [1]. Both the ratios R and S are used directly in the Global Positioning System (GPS) technology in order to “pre-correct” the rates of atomic clocks on satellites so that they run at exactly the same rate as identical counterparts located on the Earth’s surface [6, 7], for example. It will be shown that the ratios of the respective measured values by observers O and P for all physical quantities are given by integral powers of S, just as in kinetic scaling the analogous ratios are always integral powers of R. An important goal thus becomes the determination of the *powers* of R and S that correspond to each physical quantity such as energy, time and distance.

The conservation of energy principle allows one to obtain a relationship between A_O in eq. (2) and the key quantity in STR, $\gamma(u) = \left(\frac{1-u^2}{c^2}\right)^{-0.5}$, as discussed elsewhere [8]. As an object falls from P to O, its speed, and therefore its energy in terms of the local units, increases. If the speed of the object is u_P at P and u_O at O ($u_O > u_P$), its energy *increases* as the ratio $\frac{\gamma(u_O)}{\gamma(u_P)}$ during the fall according to STR if gravitational effects are ignored. On the other hand, according to eq. (3) its energy is *lower* when it reaches O by a factor of $\frac{A_O}{A_P}$ if kinetic effects are not taken into account. Since energy is actually conserved throughout the fall, it therefore follows that

$$\frac{\gamma(u_O)}{\gamma(u_P)} = \frac{A_O}{A_P}, \tag{4}$$

i.e., $\gamma(u_O)$ increases in direct proportion to A_O as the object falls in the gravitational field. Schiff employs a similar argument [5] to obtain the equivalent of eq. (1) based on a power-series expansion of $\gamma(u)$ and another relation from Newton's classical gravitational theory. One may look upon the proportionality in eq. (4) as the definition of the A_O and A_P factors, which in turn may be slightly different than that given in eq. (2).

III. Gravitational Scaling of Time, Length and Mass

In his 1907 paper [4] Einstein used an equivalence principle to prove that clocks run faster as their distance from a gravitational source increases. The relationship between elapsed times measured by two observers at different locations in the gravitational field is

$$T(O) = \frac{A_P}{A_O} T(P). \tag{5}$$

Measurements of the rates of standard atomic clocks have verified Einstein's prediction quantitatively [9].

Comparison of eq. (5) with eq. (3) reveals something quite interesting, however. The factor in parentheses in eq. (5) for time is the reciprocal of that for the ratio of energy values. The power of $S = \frac{A_O}{A_P}$ is +1 for energy by definition, but it is –1 for time. This is in stark contrast to the situation found in the case of kinetic scaling of the same two quantities. In that case the power of $R = \frac{\alpha_M}{\alpha_O}$ is the same for both [1], namely +1. This means, for example, that when the speed of a metastable object increases, both its energy and its lifetime become greater. On the other hand, when the same object is taken to a higher altitude, its energy increases but its lifetime *decreases* by the same factor. It is especially interesting that Einstein was able to correctly predict the variation of radiative periods with distance from a gravitational source by using arguments that are based directly on STR [4], that is, by means of his equivalence principle. Yet energy and lifetime vary in opposite directions when the position of the object in a gravitational field is changed, but in the same direction when their speed relative to the observer is altered. When the metastable object is in free fall from P to O, its energy stays constant so that eq. (4) holds, but its lifetime increases by the square of $S = \frac{A_O}{A_P} > 1$. The rates of clocks decrease because of the increased speed as a consequence of the fall, but they also decrease by the same proportion by virtue of the change in altitude, so that the overall effect is magnified by S^2.

The above scaling relationships also have an interesting application with regard to Planck's radiation law [10]. As already discussed in the previous study [1], the ratio of the energy and frequency of emitted radiation varies with the state of motion of the source relative to the observer. This is most easily seen from the fact that the unit for Planck's constant h is J s. As the speed of the radiative source increases, *both* the energy and time units increase as well. According to the relativity principle, the value of h is the same in every inertial system, but only when expressed in terms of the "local" units. Since both the energy and time units increase in direct proportion to R, it follows that the energy/frequency ratio *of a moving object* measured by the stationary observer varies as R^2. For him the energy of the radiation increases, while the frequency decreases by virtue of the transverse Doppler effect. Since the units of energy and time are inversely proportional in gravitational scaling, however, no such variation in the energy/frequency ratio occurs when only the position of the source in the gravitational field is changed relative to the observer. Indeed, this fact was paramount in Einstein's original conclusion that clock rates increase with distance from the gravitational source [4].

Einstein first predicted [11] a value for the angle of displacement of star images during solar eclipses that was only half as large as ultimately was observed. The reason for the discrepancy was that he only took into account changes in measurements of time with gravitational potential in his 1911 paper [12]. His arguments based on the equivalence principle four years earlier [4] indicate that distances measured along the radial direction also depend on the position of the observer in the gravitational field, however. Schiff's approach [5] demonstrates that if *both* the units of time and radial distance are scaled for the observer on Earth, the same value for the displacement angle is obtained as Einstein found in 1916 on the basis of his general theory of relativity [13].

It is important to note, however, that the unit of distance *perpendicular* to the gravitational field is assumed to be *independent* of the location of the observer in Schiff's calculations. By contrast, the unit of length increases in kinetic scaling [1] in the same proportion (and in all directions) as the unit of time. As a consequence, the speed of light and all *relative* velocities are found to be independent of the state of motion of the observer, in accordance with the second postulate of STR [3]. The conventional derivation of the Fitzgerald-Lorentz contraction effect leads to a quite different (and erroneous [1, 7]) conclusion about the variation of distances between accelerated objects.

The question remains as to how the unit of length scales with position in a gravitational field. Both Einstein [11] and Schiff [5] assumed that lengths measured along a direction *perpendicular* to the field are independent of the distance from the gravitational source, while those measured in a radial direction are proportional to S. In other words, a stationary observer would find that an object "grows" in the radial direction as it is moved to higher altitude, whereas no change in directions perpendicular to the field would be noted. As a consequence, the unit of distance would seemingly have to vary with orientation in the gravitational field.

A key piece of evidence in this connection is that Schiff's calculations for the displacement angle of star images [5] assume that the observer on Earth scales both time and radial distance. It is important to recall the criterion he used for determining the displacement angle, however. It is the same as Einstein used in his original work [11, 13]: Huygens' principle. What is actually done is to scale the *radial component of the speed* of light differently than that in the tangential direction relative to the Sun. Huygens' principle states [5] that the angle of displacement is proportional to *the rate of change* of the speed of light with respect to its distance of approach to the Sun. The "local" value of the speed of light is always assumed to be c in Schiff's method and its direction is constant, however. In other words, *no acceleration of the local speed of light* is assumed, only a change in its speed as measured by the observer on Earth or other gravitational potential.

In his discussion of the above results Schiff [5] remarks that "the curvature is such that the ray is concave to the Sun." Einstein had come to the same conclusion in his original work [11, 13, 14]. In neither case is the trajectory of the light actually calculated, however. Instead, the curvature of the light trajectory is deduced from the fact that the speed of light as observed on Earth "increases with increasing y [5]," i.e., as the distance of the light ray from the Sun increases.

Since Schiff assumed that the light ray travels in a perfectly straight line for the local observer, with constant speed c, it is nonetheless straightforward to actually compute the trajectory as seen by the observer on Earth. The result is shown in Fig. 1.

First of all, it is clear that the scaling of the time coordinate has no effect on the trajectory as observed on Earth. The light simply travels more slowly for him than for the local observer, but the ratio of the velocity components in the radial and transverse directions must be the same for both on this basis. This observation by itself demonstrates that there is no direct connection between the shape of the trajectory of the light and Huygens' principle, contrary to what is usually assumed.

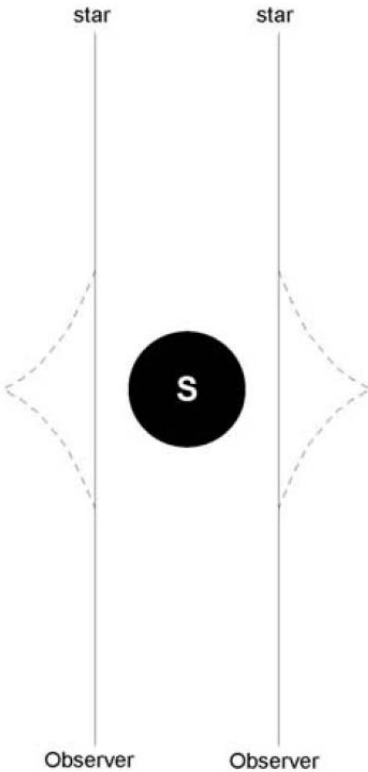

Fig. 1 Diagram illustrating the "pseudo-trajectory" inferred from the velocity vector computed for the observer on Earth in Schiff's procedure (see ref. [5]). Note that on the initial approach the light appears to veer away from the Sun (convex trajectory) because the Schiff [5] gravitational scaling reduces the magnitude of the radial component relative to the local straight-line path. This result demonstrates that the direction of the latter velocity is ignored in Schiff's method, which nonetheless obtains perfect agreement with Einstein's value for the angle of displacement of star images during solar eclipses because of its reliance on Huygens' principle to define this angle (see Fig. 2). General relativity employs the same definition for the displacement angle.

Even when the scaling of the radial component of the velocity is taken into account, however, the effect on the light's trajectory is different from what is normally expected. The transverse component is scaled by a factor of $S = \dfrac{A_O}{A_P}$ in Schiff's method [5, 15], which means that it is smaller for the observer on Earth (at infinity) than it is for his counterpart near the Sun (where O stands for the observer on Earth and P for the local observer). The corresponding radial component is scaled by a factor of S^2, so that for the observer at infinity this component decreases more than the transverse one. This means that when the light is still approaching the Sun from infinity, the direction of the light trajectory is shifted away from the Sun for O (see Fig. 1). It is as if a brake were applied to the light in the radial direction, which causes it to veer away and therefore to follow a trajectory that is *convex* to the Sun (see Fig. 1). The computed curvature gradually increases as the light draws nearer to the Sun until a maximum is reached at the closest point of approach. From then on the light moves away from the Sun, and that means the effect of the radial velocity scaling is to make it appear as if the light is now bending toward the Sun. The result is a Δ-shaped trajectory (Fig. 1) that is quite different from the concave path for the light generally assumed. Nonetheless, the computed value for the displacement angle for star images [5, 15] is exactly the same as Einstein obtained in his original work on general relativity [13, 14].

A key point that is easily missed in discussing Schiff's method, however, is that at each stage of the calculation the starting point is a local straight-line trajectory for the light. Only the speed of light measured on Earth is affected by the gravitational scaling, not its direction. To obtain the correct result for the displacement angle it is necessary to always continue the scaling procedure from a point on the local trajectory [15]. On this basis the only way to have internal consistency in this approach is to assume that *both* observers measure *the same* trajectory for the light and, therefore, that the trajectory deduced from Schiff's scaling arguments shown in Fig. 1 *is not actually observed*. The fact that his method leads to the correct value for the displacement angle can be explained quite easily by assuming that the radial distance scaling in his treatment *only applies to the component of velocity v_r in that direction.* In other words,

$$v_r(O) = \left(\frac{A_O}{A_P}\right)^2 v_r(P). \tag{6}$$

The corresponding scaling relationship for the tangential component v_t is

$$v_t(O) = \left(\frac{A_O}{A_P}\right) v_t(P). \tag{7}$$

The additional factor of $S = \frac{A_O}{A_P}$ in eq. (6) does *not* imply that the component of the distance vector between two points that lies in the direction radial to the gravitational field of the Sun in not the same for both observers. There is a common $\frac{A_O}{A_P}$ factor in eqs. (6, 7) due to the scaling of time, as indicated in eq. (5).

In this connection it is helpful to define a separate quantity L_r* to be distinguished from the actual radial component of the distance L_r. They scale differently for O and P, namely,

$$L_r(O) = L_r(P), \tag{8}$$

$$L_r{}^*(O) = \left(\frac{A_O}{A_P}\right) L_r{}^*(P). \tag{9}$$

There is no need to distinguish between corresponding transverse distance components (L_t), i.e. they satisfy the equivalent of eq. (8): $L_t(O) = L_t(P)$ [note that this means that the scaling of distance is isotropic, the same as for kinetic scaling]. In practice in Schiff's computational procedure [5], eq. (9) is used by the observer at infinity (O) *only* to determine the radial component of the velocity of light from the perspective of the local observer P [15]. In order to simulate this procedure in a computer program it is necessary to determine both the velocities $\boldsymbol{v}(O)$ and $\boldsymbol{v}(P)$. The direction followed by the light is parallel to $\boldsymbol{v}(P)$ for both observers, consistent with the above arguments [15]. The path taken is a straight line in each case *and is exactly the same for both observers*. The *speed* with which the light follows this trajectory is different for the two observers, however. It is determined by the absolute values of $\boldsymbol{v}(O)$and $\boldsymbol{v}(P)$, respectively. In effect, the scaling of the radial component of the speed of light only affects the elapsed time required by the light to follow a particular trajectory as measured on a given observer's clock.

The role that Huygens' principle plays in the above procedure is illustrated in Fig. 2. Two light rays separated by a distance dy each travel along straight lines from a star to the observer on Earth. Because their speed is different, however, they do not reach the Earth simultaneously. As is clear from the diagram, this has the effect of rotating the wave front of light [16]. A simple calculation of the angle of rotation shows that it has the same value as that determined by Huygens' principle. The reason that the star images *appear* to be displaced is not because the light travels in a curved trajectory, but rather because the direction from which the light is coming is determined by the naked eye (or corresponding photographic device) to be normal to the above wave front. More details regarding the above calculations and their interpretation are given elsewhere [15].

Experience with calculations of planetary trajectories indicates the scaling of gravitational mass m_G is especially simple by contrast. One obtains consistent results from Newton's classical theory,

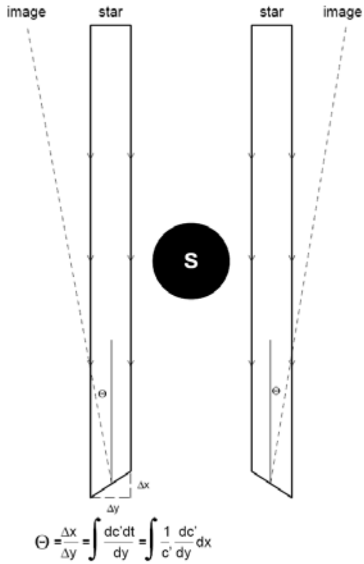

Fig. 2. Schematic diagram showing light rays emitted by stars to follow straight-line trajectories as they pass near the Sun. Gravitational scaling causes the speed c' of the light rays to increase with distance from the Sun, with the effect that the corresponding Huygens' wave front gradually rotates away from it. As discussed in the text, the normal to a given wave front points out the direction from which the light appears to have come, causing the star images to be displaced by an angle θ during solar eclipses.

for example, by assuming that the masses of the Sun and planets are independent of both their state of motion relative to the observer and their location in a gravitational field. Hence,

$$m_G(O) = m_G(P). \tag{10}$$

The same assumption has been employed successfully in calculations of the precession angle of Mercury's perihelion [8].

As noted in previous work [1, 17], the unit of inertial mass m_I of an object does change with its state of motion, however. The same is true for gravitational scaling, but the corresponding power of S is not the same as for R in kinetic scaling. The unit of inertial mass is $\mathrm{N}\frac{\mathrm{s}^2}{\mathrm{m}}$, which is to be distinguished from kg, the unit of gravitational mass m_G. Accordingly, m_I scales as R but as S^{-1} because of the way that energy, time and length vary with gravitational potential [see eqs. (3, 5, 8)]. A summary of the factors involved in kinetic and gravitational scaling is given in Table 1. Their application to the computation of trajectories of objects in gravitational fields is the subject of the next section.

IV. Scaling of the Acceleration Due to Gravity

The main thesis of Schiff's 1960 paper [5] is that the trajectories of objects moving under the influence of a gravitational field can be accurately computed using the methods of classical mechanics when account is taken of the way the units of physical quantities change with distance from the gravitational source such as the Sun. He speculated that the same techniques he employed to compute the angle of displacement of star images during solar eclipses could be successfully applied to the determination of the orbital trajectories of planets such as Mercury and Venus. In order to accomplish this objective he claimed that it was necessary to have "an equation of motion for a particle of finite rest mass" and to take account of the fact that the planets move at speeds less than c. He also felt that higher orders of the $\frac{A_O}{A_P}$ factor [see eq. (2)] would have to be included in such calculations.

In a companion paper [8], Schiff's approach has been modified successfully to obtain the angle of precession of the perihelion of the orbits of Mercury and other planets. Rather than relying on an equation of motion for the planets, it is simply assumed that Newton's Universal Law of Gravitation holds for the *local observer*, that is, for someone (M) at the same distance from the Sun and co-moving with

Table 1. Variation of the units of various physical quantities with gravitational potential. The object of the measurement is at point P in the gravitational field with clock-rate parameter α_M (see text), whereas the observer carrying out the measurement is at point O with clock-rate parameter α_O. The ratio of the respective units of a given quantity is conveniently given as the integral power n of the ratio $S = \frac{A_O}{A_P}$ (see eq. 2 for definition). The corresponding ratio in kinetic scaling is indicated in the last column by means of the integral power q of the ratio $R = \frac{\alpha_M}{\alpha_O}$ (see ref. 1) in each case.

Physical Quantity	**Standard Unit (mks system)**	**Power of ratio *S*** ***N***	**Power of ratio *R*** ***q***
Energy	J=Nm	1	1
Time	s	-1	1
Length	m	0	1
Gravitational mass	kg	0	0
Velocity	$\frac{m}{s}$	1	0
Differential length [a] radial to field (L_r*)	m*	1	1
Velocity radial to field[a]	$\frac{m^*}{s^2}$	2	0
Inertial mass	$N\frac{s^2}{m}$	-1	1
Acceleration	$\frac{m}{s^2}$	2	-1
Accel. due to gravity g[a]	$\frac{m^*}{s^2}, \frac{kg^*}{m^{*2}}$	-3	-2
Speed of light c	$\frac{m}{s}$	1	0
Force	N	1	0
Angular momentum	Nms	0	2
Momentum	Ns	0	1

[a] Scaling only used in Schiff's procedure [5] with A_o=1 (observer at infinity)

the planet. Accordingly, the local acceleration to gravity is given by the classical expression:

$$a_M = g = \frac{GM_S}{r^2}, \tag{11}$$

with M_S equal to the gravitational mass of the Sun (1.99x10^{30} kg); r is the distance of the planet from the Sun and G is the universal gravitational constant (6.67 x 10^{-11} m^3/kg s^2). In order to carry out the calculations from the perspective of an observer at infinity (O), it is necessary to know how g scales with both the relative speed of the observer to the planet and their respective distances from the Sun. For this purpose it is important to follow Schiff's suggestion [5] of introducing a third observer P who is located at the same gravitational potential as M but is co-moving with O and also with the Sun.

For the kinetic scaling [1] it is assumed that g in eq. (11) varies as R^{-2}. This follows directly from the fact that the distance r scales as R, whereas the gravitational mass M_s does not vary with R (R^0, Table 1). In the present case, $R = \frac{\alpha_M}{\alpha_P} = \frac{\alpha_M}{\alpha_O} = \gamma(u)$, where u is the speed of the planet relative to observer P expressed in his units. It is important to see that this choice for the power of R in the kinetic scaling of g is consistent with Schiff's assumption [5] that the "local" speed of light is always equal to c. For light, the value of $\gamma(u{=}c)$ is infinite, so the result of applying the R^{-2} scaling in eq. (11) is that the acceleration due to gravity as measured by P is exactly zero. This means of course that the velocity of light is indeed constant for him, as required. There is another subtle point to be considered, however. The ratio $\frac{\alpha_M}{\alpha_O}$ is generally only equal to $\gamma(u)$ if M was previously accelerated from the rest frame of O [18], as has been assumed above. The justification for this choice in the present application is clearly that the planet was ejected from the body of the Sun at some point in time, as seems quite plausible. The successful calculation of the orbits of the planets on this basis gives clear support to the latter cosmological assumption.

The *gravitational* scaling of g remains to be discussed. If it is assumed that g scales in the same manner as radial velocity divided by elapsed time (and therefore as S^3, see Table 1), the result obtained for the advancement angle is quite different than that inferred from both experiment and general relativity [13]. Consideration of Newton's classical definition of g in eq. (11) suggests another possibility, however. If one assumes instead that the distance from the Sun in this expression scales in the same manner as L_r* in eq. (9), then a contribution of S^{-2} to the scaling factor of g is indicated. Moreover, an additional factor of S^{-1} is required if one assumes that the mass in eq. (11) scales as the inertial mass for this purpose, so that the actual gravitational scaling factor for g is S^{-3}, as indicated in Table 1. Calculation of the planetary orbits employing a standard numerical time-step procedure [8] obtains results that are in very good agreement with those observed experimentally when the scaling factors of Table 1 are employed to account for the distinctions in the units of time, length and mass of various observers in different states of motion and locations in a gravitational field. In particular, the same dependence of the precession angle of the perihelion of orbiting planets on both the average distance from the gravitational source and its mass M_S is found as in Einstein's general theory of relativity [13, 14]. More details of these calculations may be found in the original reference [8].

V. Variation of the Properties of Light with Kinetic and Gravitational Scaling

Einstein consistently used the properties of light as a guide in developing the special and general theories of relativity [3, 13], particularly its speed in free space. After determining the manner in which the units of various physical quantities vary with changes in the state of motion of the observer and his location in a gravitational field, it is of particular interest to consider what influence these conclusions have on the general subject of light. A summary of these results is given in Table 2. It is assumed that the light source is in the rest frame of inertial system M, with clock-rate parameter α_M, and is located at point P in the gravitational field, whereas the observer is located at point O in an inertial system with clock rate α_O.

The speed of light observed by O is proportional to the ratio $S = \dfrac{A_O}{A_P}$. In accordance with Einstein's second postulate of STR [3], when the source and the observer are at the same gravitational potential (A_O=A_P), the value of the speed of light is always equal to c, regardless of the magnitude of their relative speed. The different rates of clocks in the respective rest frames of the source and the observer that are caused exclusively by the time dilation effect have no influence on the measured value, consistent with the results of the Michelson-Morley and Kennedy-Thorndike experiments [19, 20]. This is the justification for concluding in Table 1 that lengths scale as R, the same ratio as for elapsed times. The dependence of the speed of light on location in a gravitational field given above and in Table 1 is the same as deduced by Einstein in his 1911 paper [11]. It is greater than c when the source is located at a higher potential than the observer O, and it is smaller when it is located at a lower potential as in the classic example of light passing close to the Sun.

Table 2. Dependence of various properties of light on the positions of the object and observer in a gravitational field and on the corresponding clock-rate parameters α_O and α_M (see Table 1 and text for definitions). The corresponding dependence for *in situ* measurements (A_O=A_P and α_M=α_O) is given in the third column, whereas in the fourth, the corresponding results are given for the case of light in free fall in a gravitational field from point *P* to *P'* for the same (stationary) observer ($A_{P'}$>A_P). The powers of *S* and *R* are given in parentheses below each quantity (see Table 1 for definitions).

Quantity	**General Value (not in free Fall)**	***In Situ* Value**	**Free Fall Value (emitted from P)**
Speed of Light (1, 0)	$\left(\frac{A_O}{A_P}\right)c$	c	$\left(\frac{A_O}{A_{P'}}\right)c$
Wavelength (0, 1)	$\left(\frac{\alpha_M}{\alpha_O}\right)\lambda$	λ	$\left(\frac{\alpha_M}{\alpha_O}\right)\left(\frac{A_P}{A_{P'}}\right)\lambda$
Frequency (1, −1)	$\left(\frac{\alpha_O}{\alpha_M}\right)\left(\frac{A_O}{A_P}\right)\nu$	ν	$\left(\frac{\alpha_O}{\alpha_M}\right)\left(\frac{A_O}{A_P}\right)\nu$
Energy (1, 1) (“*E=hv*”)	$\left(\frac{\alpha_M}{\alpha_O}\right)\left(\frac{A_O}{A_P}\right)E$	E	$\left(\frac{\alpha_M}{\alpha_O}\right)\left(\frac{A_O}{A_P}\right)E$
Momentum (0, 1) (“$p=\frac{h}{\lambda}$”)	$\left(\frac{\alpha_M}{\alpha_O}\right)p$	p	$\left(\frac{\alpha_M}{\alpha_O}\right)\left(\frac{A_{P'}}{A_P}\right)p$
Planck’s Constant (0, 2)	$\left(\frac{\alpha_M}{\alpha_O}\right)^2 h$	h	$\left(\frac{\alpha_M}{\alpha_O}\right)^2 h$
Accel. due to gravity (−3, −2)	$\left(\frac{\alpha_O}{\alpha_M}\right)^2\left(\frac{A_P}{A_O}\right)^3 g$	g	g=0 for light

The wavelength of light is proportional to $R=\frac{\alpha_M}{\alpha_O}$, i.e., it is dependent on both the state of motion of the source and the observer [21]. The *in situ* value for a given source is the same for all observers, however, consistent with the relativity principle and experiment [20]. The transverse Doppler effect [22] allows a verification for the dependence on R. There is an additional factor of

$\left(1 \pm \frac{u}{c}\right)$ if the light source is moving directly away/toward the observer O due to the motion of the latter into the waves (first-order Doppler effect). The wavelength of light is independent of the location of both the source and the observer in a gravitational field. The Doppler factor mentioned above is not actually a relativistic effect, but simply reflects the fact that the light source is moving either out of or into the wave pattern.

The variation of the light frequency must be such as to satisfy the rule that in free space it is always equal to the quotient of the light speed and the corresponding wavelength. It is, therefore, proportional to both $R^{-1} = \frac{\alpha_O}{\alpha_M}$ and $S = \frac{A_O}{A_P}$ (see Table 2). There is also a first-order Doppler factor in this case if the light source is moving toward or away from O. It is the reciprocal of that required for wavelengths, consistent with the light-speed constancy requirement.

In order to deduce the variation of the energy and momentum of the photons it is necessary to know how Planck's constant [10] varies with the state of motion of the source [1] and with its relative location in the gravitational field. Since energy is a scalar quantity, its value must be independent of the direction in which the light travels, whereas the light frequency is subject to the Doppler effect. This means that the value of Planck's constant must vary with the orientation of the light velocity to the observer as well. A simple means of circumventing this complication is to restrict one's attention to the case of the transverse Doppler effect. This eliminates the $\left(1 \pm \frac{u}{c}\right)$ factor from the calculations. Further, only light moving perpendicular to the gravitational field needs to be considered to establish the value of the required energy/frequency ratio, from which it can be concluded that its value is proportional to R^2, as already discussed elsewhere [1]. This relationship can be tested experimentally by means of the photoelectric effect. The variation of the energy of the photons is then comparatively simple. It is proportional to both *R* and *S*, independent of the orientation of the light velocity to both the observer and the gravitational field.

The magnitude of the corresponding momentum of the photons can be obtained by using the same value for Planck's constant in the relation "$p = \frac{\mathrm{h}}{\lambda}$". The quotation marks are used to indicate that the value to be inserted for each quantity is that actually measured by the observer in a given situation. On this basis, "*p*" is found to be proportional to $R = \frac{\alpha_M}{\alpha_O}$ and is completely independent of the location of both the light source and the observer relative to the gravitational source (momentum scales in the same manner as length; see Table 1). Note that there is no Doppler effect for either momentum or energy, i.e., no $\left(1 \pm \frac{u}{c}\right)$ factor is required in either case.

The corresponding values for the above quantities that are observed *in situ* are shown in the third column of Table 2. These values are obtained from the previous (general results) by setting $A_O = A_P$ and recalling that the observations are being made by observer *M* in each case (i.e., $\alpha_M = \alpha_O$). In accord with the relativity principle, they are all independent of both the state of motion of the observer and his location in a gravitational field.

On the basis of the results discussed above, it is possible to make definitive predictions regarding the variation of the properties of light when the source is lowered (as opposed to dropping in free fall) in the gravitational field (without altering its velocity). Let us assume that the source is initially at point *P* in the gravitational field and is then lowered to point P' [$A_{P'} > A_P$, see eq. (2)]. The speed of light decreases by a factor of $\frac{A_P}{A_{P'}}$ for *O* after the source has changed its position. This result is obtained by simply replacing A_P by $A_{P'}$ in the general expression for this quantity in Table 2 (second column). The light frequency decreases in the same proportion, whereas the wavelength remains constant throughout. The value of Planck's constant does not change either, and thus the energy decreases in direct proportion to the frequency while the momentum stays the same. The variation of the photon energy is independent of the orientation of the light velocity to the gravitational field.

All the above results can be obtained from those given first by simply replacing A_P by $A_{P'}$ (see Table 2). The situation is different if the light is emitted from point *P* and is later observed by *O* at point P' in the gravitational field, however. The energy is unchanged during the free fall of the light, whereas the speed of light decreases by a factor of $\left(\frac{A_P}{A_{P'}}\right)$. The value of Planck's constant is determined by the relative speed of the *light source* only, so the frequency also does not change as the light descends (i.e., the value in the fourth column is independent of $A_{P'}$). In order to satisfy the local "$c=\lambda v$" condition, it is necessary for the wavelength of light to gradually decrease during its fall by a factor of $\frac{A_P}{A_{P'}}$. This, in turn, means that the momentum of the light increases by the reciprocal of this factor (see fourth column in Table 2).

It may seem strange that the momentum of the light increases while its speed decreases, but this is required by the local "*E=pc*" condition. A similar situation has been noted when light passes through dispersive media [23]. The energy of the photons does not change as the light moves from one medium to another. Newton claimed that the corresponding momentum must increase in direct proportion to the index of refraction n, but concluded incorrectly that the speed of light must also increase. After experiment showed that the speed of light actually decreases in the medium of higher n, it was concluded that Newton had also been in error with regard to his prediction of an increase in photon momentum. There is strong theoretical evidence that Newton was, in fact, correct in his conclusion about momentum, however, because it is also known that the wavelength λ of light decreases in direct proportion to n and quantum mechanics has shown that $p = \frac{h}{\lambda}$ holds quite generally. The phenomenon of light refraction is therefore consistent with what is indicated for light in gravitational free fall, namely that the momentum and the speed of light must change in opposite directions since the photon energy remains constant [16].

Thus far in the discussion, three cases have been considered: a) the light is emitted from a source fixed at a higher gravitational potential than the observer, b) the light source is moved to a different position in the field prior to emission, and c) the light itself is in free fall from a source fixed at a higher gravitational potential than the observer. A fourth case is considered in Table 3, namely when the source itself is

Table 3. Dependence of various properties of light on the positions of the source and observer in a gravitational field and their relative states of motion on the kinetic scaling factor $R = \frac{\alpha_M}{\alpha_O}$ and the gravitational scaling factor $S = \frac{A_O}{A_P}$ (see Table 1 and text for definitions). The first two rows contain results for the case when the light source (or other object) begins free fall at point P (S>1) in the gravitational field. The succeeding two rows give the corresponding results when the light source is in free fall and has reached point P' in the field [$\gamma = \frac{A_{P'}}{A_P} > 1$, see Table 2 and eq. (4)].

Observer	Lightspeed/Velocity	Energy	Frequency	Momentum	Wavelength	Inertial Mass	Planck's Constant
In situ P	c	E	ν	p	λ	m_I	h
O	$S\,c$	$RS\,E$	$\frac{S\nu}{R}$	Rp	$R\lambda$	$\frac{Rm_I}{S}$	$R^2 h$
In situ P'	c	E	ν	p	λ	m_I	h
O	$\frac{Sc}{\gamma}$	$RS\,E$	$\frac{S\nu}{R\gamma^2}$	γRp	$\gamma R\lambda$	$\frac{\gamma^2 Rm_I}{S}$	$\gamma^2 R^2 h$

in free fall as it emits light. It is assumed that the frequency and wavelength of the emitted light have the values ν and λ in the observer's rest frame when measured *in situ* at his location in the gravitational field. The corresponding values for the speed of light, its energy, momentum and inertial mass are c, E, p and m_I, respectively, as in Table 2. The source has been accelerated so that its clock-rate parameter is α_M, and the corresponding kinetic scaling factor is, therefore, $R = \frac{\alpha_M}{\alpha_O}$. It is also located at a higher gravitational potential, with the gravitational scaling factor $S = \frac{A_O}{A_P}$ (see Table 2). The *in situ* values do not change as a result of the light source's movement, in accord with the relativity principle.

The second row of Table 3 gives the corresponding values for these quantities as measured by the stationary observer (O) before the source starts to fall. The energy of the light for O is now $RS\,E$, whereas the frequency is $\frac{S\nu}{R}$ because the value of Planck's constant for light generated from the moving source is R^2h, as discussed in Section III. The speed of light is Sc because of the difference in gravitational potential of the source; the value of the wavelength is $R\lambda$. The momentum is thus "$\frac{h}{\lambda} = \frac{E}{c}$" $= \frac{R^2h}{R\lambda} = \frac{RSE}{Sc} = Rp$. The inertial mass is obtained as "$\frac{E}{c^2} = \frac{p}{c}$" $= \frac{Rm_I}{S}$.

When the light source falls, it is necessary to apply eq. (4) to the scaling factors. Because of energy conservation, as discussed in Section II, the kinetic scaling factor (R) increases by the same amount (γ) as the gravitational scaling factor (S) decreases. The instantaneous values of the *in situ* light properties are not affected by the free fall of the source, as already noted. The corresponding values for O are obtained by making the substitutions of $\frac{S}{\gamma}$ for S and γR for R relative to his initial measurements. The observed frequency becomes $\frac{S\nu}{\gamma^2 R}$, for example, whereas the wavelength becomes $\gamma R\lambda$.

The quantum mechanical relationships, "$E = h\nu$" and "$p = \frac{h}{\lambda}$", are satisfied in all four cases in Table 3. In addition, the mass/energy equivalence relation holds throughout as well "E" = "$m_I c^2$" = "pc". As a result, all property values are determined from the speed of light and its corresponding energy and momentum as determined by the observer. This state of affairs is not restricted to light, however. The same relationships hold for all particles of matter, although the Planck energy/frequency relation is of little practical significance for particles such as the electron because of their finite mass and correspondingly high rest energy. The energy of "massive" particles is no longer equal to the product of momentum and the speed of light, but "$E = m_I c^2$" continues to be valid for them. Their velocities between fixed points in a given inertial system also scale in the same way as the speed of light. These observations are consistent with Einstein's original work on STR [3], in which he showed that the relativistic velocity transformation is equally valid for massive particles as it is for photons.

VI. Conclusion

The units of physical quantities depend on the location in a gravitational field and also the state of motion of the observer. In the present work attention has centered upon the precise specification of the scaling of the units with changing gravitational potential, and comparison has been made with the corresponding "kinetic" scaling that occurs when the state of motion changes. In each case, Einstein has led the way. He deduced that clocks run slower on a fast-moving rocket than they do in the rest frame of the Earth, and that they run faster when they are taken to higher altitude. The unit of time therefore increases in the first case and decreases in the second. The unit of energy increases in both cases, however.

Determining the manner in which the unit of length varies in these two cases has been complicated by theoretical arguments based on the Lorentz transformation, specifically by the derivation of the Fitzgerald-Lorentz contraction effect of STR [3]. It has been pointed out recently [1, 7], however, that the unit of length must vary in direct proportion to that of time in order to satisfy Einstein's second postulate. This means *length expansion*, and by the same proportion in all directions. An important observation is that the units of energy, length and time always remain in the same ratio when the state of motion of the observer changes. By contrast, gravitational mass is completely unaffected by acceleration. The situation is different for inertial mass, however. The unit of the latter quantity is derived in an unambiguous manner from its relation to the units of energy, time and distance. In other words, the unit of *inertial mass* is not kg in a strict sense, but rather $\mathrm{N}\dfrac{\mathrm{s}^2}{\mathrm{m}}$.

A key quantity in expressing the variation of units in gravitational fields is $A_O = 1 + \dfrac{GM_S}{c^2 r_O}$, as defined in eq. (2). This quantity was used successfully by Schiff [5] to compute the angle of displacement of the images of stars during solar eclipses. The unit of energy is $S = \dfrac{A_O}{A_P}$ times greater/smaller at point P than for the observer at point O who is at a closer/farther distance from the gravitational source. The gravitational scaling of units can be specified quite succinctly in terms of integral powers of S, as shown in Table 1. Energy scales as S, time and inertial mass as S^{-1}, length, momentum and gravitational mass as S^0, for example, and the manner of scaling for other quantities derived from these fundamental quantities is easily accomplished on this basis.

Schiff has shown, based on Einstein's original work [11], that the scaling of velocities requires an additional measure *when computing trajectories from the standpoint of an observer at infinity*, namely the component radial to the gravitational field must have an extra factor of S than the perpendicular components in this special case. If one ignores this distinction, the value of the above deflection angle is computed to be only half as large as that observed and predicted from the general theory of relativity [13]. Analysis of Schiff's results for this deflection angle has shown that the above scaling procedure should *only be used to compute the ratio of the relative speeds of a given object (light or a planet) to an observer at infinity and a counterpart located at the same gravitational potential as the object*. In his method there are two velocities and two position vectors for each light ray, one set for the "local" and one for the primary observer on Earth (infinity). The position vectors must be the same at all times, however, by virtue of the fact that the unit of length is the same (Table 1) for both observers (it is assumed that they are not in relative motion in Schiff's procedure [5]). The corresponding light trajectory is therefore a straight line. Only the elapsed time is different from these two perspectives. The "bending" of light by the Sun is actually seen to be a misunderstanding of the criterion used for the angle of displacement, namely Huygens' principle. In reality, what the calculations show, whether those of Einstein or Schiff, is that the *wave front* of the light coming from the star *is rotated away from the Sun*, creating an optical illusion which merely tricks the eye into believing that the stars have been displaced during the eclipse. The kinetic scaling of the acceleration due to gravity ensures that the light is in fact *not bent* as it passes by the Sun, as discussed in a companion article [15]. The direction of the light's velocity (along this straight-line trajectory) is the same as for the local velocity for all observers, regardless of their position in a gravitational field or their state of relative motion. But the light speed itself for *the observer at infinity* is equal to *the magnitude* of the corresponding velocity vector in his units. In effect, the additional scaling of the radial component of the velocity produces a further decrease in the light speed over and above that caused by the normal scaling of time with S^{-1} (Table 1). When all is said and done, however, the speed with which an object moves with respect to a given observer in Schiff's method can simply be determined by knowing the relative rate of his atomic clocks to those located at infinity. The slower one's clocks run, the faster the object appears to move from his perspective. Thus velocity scales as S in all directions.

A very interesting aspect of Schiff's method is that it allows one to compute the displacement angle of star images without the full apparatus of general relativity. He has also pointed out that the gravitational red shift is easily accounted for by scaling the unit of time (i.e. as S^{-1} in Table 1). He therefore concluded that the superiority of general relativity could only be demonstrated by means of its prediction of the precession of the perihelion of planets orbiting the Sun. He could only speculate about whether a similar scaling procedure as used above could be successful in this application as well, and, in fact, apparently never succeeded in accomplishing this objective. In recent work [8], it has been shown that quantitative agreement with general relativity is obtained in this application as well by computing the planetary trajectories with a conventional time-step procedure and taking account of the gravitational scaling of units for all quantities, including the acceleration due to gravity *g*. The same dependence of the precession angle for the perihelion of the orbits upon the solar mass and the length and eccentricity of the planetary radius is obtained in this relatively simple procedure as Einstein found in his original work employing the general theory of relativity. The implication is thus that once one takes proper account of both the kinetic and the gravitational scaling of physical units, it becomes possible to remove all errors in the computation of trajectories of objects moving in gravitational fields. The scaling procedures thus perfect Newton's original Law of Universal Gravitation without resorting to the notably more complicated field theoretic approach introduced by Einstein.

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