New Time Dependent Gravity Displays Dark Matter and Dark Energy Effects

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It is shown that a time dependent gravitational field that is getting weaker with time will produce the effects measured for both the tangential velocity in the arms of spiral galaxies and for the high z supernovas. These results show that the effects that have led to the hypothesis of Dark Matter and Dark Energy may come from the same basic physical phenomena, namely that gravity is getting weaker as a function of time, and not from the existence of exotic matter.

Keywords: distances and red shifts, dark matter, dark energy, theory

Introduction

Much has been written, hypothesized, and calculated on the subject of Dark Matter and Dark Energy. However, none consider a time dependent gravitational field. A gravitational field that gets weaker with time will display galaxy dynamics responding to a much stronger field before sending light from space towards the Earth.
that can only be received many light years later. The theoretical basis for such a time dependent gravitational field has already been presented [1][2][3]. Three elements of this theory apply to the potential explanation of Dark Matter and Dark Energy. These elements include:

1. The theory is a five dimensional gauge theory with Weyl geometry [4]. This means that the fields within the theory are gauge fields. However, the theory is not another Kalusa-Klein type of theory in that the fifth dimension describes a real physical property, mass density, and, therefore, is not hidden or obscured by some mathematical technique. The five dimensionality of the gauge theory requires that the gravitational field be time dependent.

2. Quantum Mechanics is required by restricting the Weyl scale factor within the gauge theory to have only a value of unity. This was noted by Schrödinger [5] before he published his wave equations and later it was shown by London [6] that this restriction required Schrödinger’s wave equations. This quantization requires that the gauge potentials be non-singular [1].

3. The fundamental Weyl geometry requires that the Poisson brackets and the unit of action be dependent upon the gauge function [1]. This variable unit of action leads to a relation determining the red shift of light coming to Earth from distant stars [12].

These three aspects of the new theory suffice to offer a different view of the data from which the hypothesis of dark matter and dark energy have evolved.
Dark Matter

Data wherein the tangential velocities of stars in the arms of a spiral galaxy differed from Newtonian predictions were first reported nearly seventy years ago [7]. A fundamental theory supporting these data has not heretofore been given, though empirical theories have been presented. The best of these theories is the Modified Newtonian Dynamics (MOND) [8][9][10]. The theory presented here had its beginning in 1974 and only recently has it been applied to the dynamics of spiral galaxies.

Newtonian uniform circular motion equates the gravitational acceleration to the centripetal acceleration so that

\[
\frac{GMm}{r^2} = \frac{mv^2}{r}.
\]  

(1)

A time-dependent, non-singular gravitational field, such as the Dynamic Theory predicts, alters Equation (1) to

\[
\frac{GMm(1 - H_0 \tau)}{r^2} \left( 1 - \frac{\lambda}{r} \right) e^{-\frac{\lambda}{r}} = \frac{mv^2}{r},
\]  

(2)

where \( H_0 \) is Hubble’s constant and

\[
\lambda = \frac{GM}{c^2}
\]  

(3)
as determined by planetary orbits. For this time dependent gravitational field the gravitational acceleration acting on an arm of a galaxy feels is due to the gravitational field of the mass \( M \) at a previous time. This previous time is given by the time that it takes for the field to travel from the site of the gravitational field to the point on the arm under consideration. This means that when all the mass is considered to be at the center of the galaxy the time that enters into
Equation (2) is \[ \tau = \frac{-r}{c} \] so that, when \( r >> \lambda \) the velocity of the arm of the galaxy would be given by

\[ v = \sqrt{\frac{GM(1-H_0\tau)}{r}} = \sqrt{GM \left( \frac{1}{r} + \frac{H_0}{c} \right)}. \] (4)

Equation (4) shows a very different character than the expression for the velocity for the time independent gravitational field. This expression shows that the velocity of the galaxy arms should not be expected to drop off as the time independent Newtonian gravitational field does.

We may look at the 5-dimensional approach of the Dynamic Theory by looking at the Lagrangian

\[ L = \frac{1}{2}mc^2 \left( \dot{\tau} \right)^2 + \frac{1}{2}mr^2 + \frac{1}{2}m \left( r\dot{\theta} \right)^2 + GMm \left(1 - H_0\tau\right)\frac{e^{-\frac{\lambda}{r}}}{r} \] (5)

where the universe time, \( \tau \), is treated as another variable and \( t \) is the local time. The universe time, \( \tau \), becomes a geometrical coordinate that makes the problem local-time independent in five dimensions.

The time Lagrange equation may then be written as

\[ \frac{d}{ds} \left[ \frac{\partial L}{\partial \dot{\tau}} \right] - \frac{\partial L}{\partial \tau} = 0 = \frac{d}{dt} \left[ m\dot{\tau} \right] + H_0\lambda m\frac{e^{-\frac{\lambda}{r}}}{r}. \] (6)

For a spherically symmetric field the radial equation is

\[ \frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{r}} \right] - \frac{\partial L}{\partial r} = 0 = \frac{d}{dt} \left[ m\dot{r} \right] - mr\dot{\theta}^2 + GMm \left(1 - H_0\tau\right) \left(1 - \frac{\lambda}{r}\right)\frac{e^{-\frac{\lambda}{r}}}{r^2}. \] (7)

The third Lagrange equation becomes
\[
\frac{d}{dt} \left[ \frac{\partial E}{\partial \dot{\theta}} \right] - \frac{\partial E}{\partial \theta} = 0 = \frac{d}{dt} \left[ m r^2 \dot{\theta} \right].
\] (8)

For the problem of spiral galaxy behaviour we may assume the \( \lambda << r \) and write the equations of motion as

\[
\ddot{\varpi} = \frac{-H_o \lambda}{r},
\] (9)

\[
\ddot{r} - r \dot{\theta}^2 = -\left(1 - H_o \tau\right) \frac{GM}{r^2} \left(1 - \frac{\lambda}{r}\right)
\] (10)

and

\[
\ddot{\theta} + \frac{2}{r} \dot{r} \dot{\theta} = 0.
\] (11)

If we now look at uniform circular motion we find that Equation (9) becomes

\[
\ddot{\varpi} = \frac{-H_o \lambda}{r} = \text{constant} \Rightarrow \frac{d \ddot{\varpi}}{dt} = \frac{-H_o \lambda}{r}
\] (12)

so that this may be integrated to get

\[
\ddot{\varpi} = \ddot{\varpi}_o - \frac{H_o \lambda}{r} (t - t_o)
\] (13)

which may be integrated again to get

\[
\tau = \tau_o - \frac{H_o \lambda}{2r} t^2 + \left(\ddot{\varpi}_o + \frac{H_o \lambda}{r} t_o\right) t.
\] (14)

Also for the assumed uniform circular motion Equation (10) may be written as
\[ v^2 = (1 - H_o \tau) \frac{GM}{r} \left(1 - \frac{\lambda}{r}\right) \]  \hspace{2cm} (15)

where \( v \) is the tangential velocity of the uniform circular motion. Putting Equation (14) into Equation (15) obtains

\[ v^2 = \frac{GM}{r} \left[1 - H_o \tau_o + \frac{H_o^2 \lambda}{2r} t^2 - \left(H_o \dot{\tau}_o + \frac{H_o^2 \lambda}{r} t_o\right) t\right]\left(1 - \frac{\lambda}{r}\right). \hspace{2cm} (16) \]

We must keep in mind there are two times to be considered. First there is the time it takes for the gravitational change to travel from the center of the galaxy to the point of measurement in the galaxy arm. The second time is for the light signal to travel from the galaxy to the Earth.

Let us set \( \tau_o = 0 \) and \( t_o = 0 \) at the point in time when the light left the star on its way toward Earth. Now our Equation (16) becomes

\[ v^2 \approx \frac{GM}{r} \left[1 + \frac{H_o^2 \lambda}{2r} t^2 - H_o \dot{\tau}_o t\right]\left(1 - \frac{\lambda}{r}\right). \hspace{2cm} (17) \]

Time runs from the time the gravitational signal left the center of the galaxy at

\[ t = \frac{-r}{c}, \hspace{2cm} (18) \]

where \( r \) is the distance from the center of the galaxy.

Using Equation (18) in Equation (17) we find

\[ v^2 \approx \frac{GM}{r} \left[1 + \frac{H_o \dot{\tau}_o r}{c} \left(1 - \frac{H_o^2 \lambda}{2c^2}\right)\right] \approx \frac{GM}{r} \left[1 + \frac{H_o \dot{\tau}_o r}{c}\right]. \hspace{2cm} (19) \]

Now we need to establish a value for \( \dot{\tau}_o \). Look at the energy at time \( t=0 \) and \( \tau=0 \) with \( r \gg \lambda \), or
This may be rewritten as
\[ \frac{2E_o}{mc^2} = (\dot{\tau}_o)^2 + \frac{v^2}{c^2} - \frac{\lambda}{r_o}. \] (21)

Since the tangential velocities are non-relativistic this requires that
\[ \dot{\tau}_o = \sqrt{\frac{2E_o}{mc^2} + \frac{\lambda}{r_o}}. \] (22)

Equation (22) shows that the initial conditions establish the point at which the tangential velocities begin to differ from those predicted by Newtonian gravity. In the absence of a means of evaluating the initial conditions we may turn to experimental results. First, suppose we write the acceleration in the arms of the galaxy as
\[ a = a_N \left\{ 1 - \frac{H_o^2 \lambda}{2r} t^2 - H_o \dot{\tau}_o t \right\}. \] (23)

We now use the data that shows the acceleration begins to deviate from Newtonian when the acceleration drops to a value of $1.2 \times 10^{-10}$ m/sec$^2$ so that
\[ a_N = \frac{GM}{r_c^2} \approx 1.2 \times 10^{-10} \Rightarrow r_c \approx \sqrt{\frac{GM}{1.2 \times 10^{-10}}} . \] (24)

Then requiring
\[
\dot{\tau}_o = \frac{c}{r_c H_o}
\]  

sets a value of \(\dot{\tau}_o\) in keeping with the data. Equation (23) becomes

\[
a \cong a_N \left\{ 1 + \frac{r}{r_c} \right\}
\]

where we see the short range Newtonian acceleration and the long range acceleration predicted by MOND.

It should be noted that the approximate linearity of the tangential velocity with respect to time of Equations (17) and (19) displays an independence of the time it takes for light to travel from the galaxy to Earth. This apparent independence of time masks the fact that the gravitational strength of the galaxy, relative to the current epoch, depends upon the time of light travel to Earth.

**Dark Energy**

Data displaying evidence that provided the beginning of the hypothesized dark energy was first presented in 1998 [11]. To date no fundamental theory has had success in explaining these data.

The universe expansion factor is taken from general relativity and is

\[
\frac{\ddot{a}}{a} = -\frac{4}{3} \pi G \left( \rho + 3 \frac{p}{c^2} \right).
\]  

The mean density and pressure are currently taken to include dark energy and are taken to obey the local conservation of energy relation

\[
\dot{\rho} = -3 \frac{\dot{a}}{a} \left( \rho + \frac{p}{c^2} \right).
\]
The first integral of Equations (27) and (28) is the Friedman equation
\[ \dot{a}^2 = \frac{8}{3} \pi G \rho a^2 + \text{constant}. \]  

But consider what happens if one wishes to compare this with the cosmology produced by the non-singular, time dependent, gravitational gauge potential. Then Equation (27) becomes
\[ m_g \frac{d^2x}{dt^2} = \frac{4\pi}{3} x^3 \rho(t) G m_g \left( \frac{1 - \frac{\lambda}{x}}{x} \right) \left( 1 - H_o \tau \right) e^{-\frac{\lambda}{x}}, \]  
\[ = \frac{4\pi}{3} G \rho(t)x \left( 1 - \frac{\lambda}{x} \right) \left( 1 - H_o \tau \right) e^{-\frac{\lambda}{x}}, \]  
where \( \tau \) is the universe time.

Now let us replace \( x \) with the co-moving coordinate \( x = R(t)r \) where \( R(t) \) is the scale factor of the universe and \( r \) is the co-moving distance coordinate as is done in the standard model. When we also normalize the density to its value at the present epoch, \( \rho_o \), by \( \rho(t) = \rho_o R^{-3}(t) \) we obtain
\[ \frac{d^2R}{dt^2} = \frac{4\pi G \rho R}{3} \left( 1 - \frac{\lambda}{R} \right) \left( 1 - H_o \tau \right) e^{-\frac{\lambda}{R}}. \]  
(31)

If we multiply Equation (31) by \( dR/dt \) and integrate with respect to time we find
\[ \int \dot{R} \frac{d^2R}{dt^2} dt = \frac{4\pi G \rho}{3} \int \left( 1 - H_o \tau \right) R \dot{R} dt \]
\[ \frac{\dot{R}^2}{2} - \frac{\dot{R}_o^2}{2} = \frac{4\pi G \rho}{3} \int \left( 1 - H_o \tau \right) R dR. \]  
(32)
We now need to know how to integrate the right hand side of Equation (32). Suppose we consider the time it takes for light to travel from the distant star to Earth, or \( t = -\frac{a}{c} \), where \( a \) is the distance from the star to Earth and the minus sign comes from looking backwards in time. The radius of the universe now has two parts. The first part is the radius of the universe when the light left the star on its journey to the Earth. Let this time be \( R_o \). Thus we see that

\[
R = R_o + a
\]  
(33)

and

\[
\dot{R} = \dot{a}
\]  
(34)

Further, from considerations of the dark matter it was determined that the world time was given by

\[
\tau = \tau_o - \frac{H_o GM}{2c^2 R} t^2 + \left( \dot{\tau}_o + \frac{H_o GM}{c^2 R} t_o \right) t. 
\]  
(35)

When we set both initial times to zero and use the value of

\[
\lambda_U = \frac{GM}{c^2}, \quad (36)
\]

Equation (35) becomes

\[
\tau = -\frac{H_o \lambda_U}{2R} t^2 + \dot{\tau}_o t. 
\]  
(37)

Now we find that Equation (32) may be written as

\[
\dot{a}^2 = \frac{8\pi G \rho}{6c^2} \left\{ 2R_o c^2 a + \left( c^2 + H_o \dot{\tau}_o c R_o \right) a^2 \right\} + K. 
\]  
(38)
If we set the constant of integration, $K$, to zero, then Equation (38) becomes

$$\dot{a}^2 = H_o^2 \Omega'_M \left\{ R_o a + \frac{1}{2} \left( 1 + \frac{H_o \dot{R}_o}{c} \right) a^2 + \frac{1}{6} \left( \frac{H_o^2 \lambda_U}{c^2} + \frac{2H_o \dot{R}_o}{c} \right) a^3 \right\}. \quad (39)$$

where we have used the definitions

$$\rho_c \equiv \frac{3H_o^2}{8\pi G}, \quad \text{and} \quad \Omega'_M \equiv \frac{\rho}{\rho_c}. \quad (40)$$

In Equation (39) we find that the mass density term splits into three terms for a time-dependent gravitational field. For a time-independent gravitational field there was only one term.

An interesting aspect of Equation (39) is that the two new mass terms both involve the same time dependence factor as the one that causes the tangential velocity of the arms of spiral galaxies to differ from Newtonian behaviour. That is to say that should the two new terms provide a basis for the current experimental evidence for dark energy it comes from the same source as the basis for dark matter. The time dependence of the gravitational field explains both phenomena.

Consider Equation (39) again and add the usual term for radiation so that we find

$$\left( \frac{\dot{a}}{a} \right)^2 = H_o^2 \Omega'_M \left\{ \frac{R_o}{a} + \frac{1}{2} \left( 1 + \frac{H_o \dot{R}_o}{c} \right) + \frac{1}{6} \left( \frac{H_o^2 \lambda_U}{c^2} + \frac{2H_o \dot{R}_o}{c} \right) a \right\} \quad (41)$$

where we did not add a term for the cosmological constant. If this is to compare with the usual expression we could write
\[
\left( \frac{\dot{a}}{a} \right)^2 = H_o^2 \left\{ \Omega'_M (1+z)^3 + \Omega'_{DM} (1+z)^3 + \Omega'_{DE} (1+z)^3 + \Omega_{RO} (1+z)^4 \right\}
\]

wherein the sum of the terms are taken to be unity at \( z=0 \) and the integration constant has been taken to be zero. Equation (41) and (42) would require

\[
\Omega'_M \left[ \frac{R_o}{a} + \frac{1}{2} \left( 1 + \frac{H_o \dot{t}_o R_o}{c} \right) + \frac{1}{6} \left( \frac{H_o^2 \lambda_U}{c^2} + \frac{2H_o \dot{t}_o}{c} \right) a \right] + \Omega_{RO} = 1. \quad (43)
\]

The relation between the red shift and \( a \) is

\[
1 + z = \left[ \frac{a(t_{obs})}{a(t_{em})} \right] = \frac{R_o + a}{R_o}
\]

By putting Equation (44) into (41) we find

\[
\left( \frac{\dot{a}}{a} \right)^2 = H_o^2 \left\{ \Omega'_M \left[ z + \frac{1}{2} \left( 1 + \frac{H_o \dot{t}_o c^3 a}{zc^4} \right) + \frac{1}{6} \left( \frac{H_o^2 GM}{c^4} + \frac{2H_o \dot{t}_o}{c} \right) a \right] + \Omega_{RO} \right\}
\]

The fact that these terms have expressions relating them argues that their relative values may be determined.

For example, if \( \Omega_{RO} \) is taken to be small compared with the mass terms and \( \Omega_M \) is set at the typical value of 0.25, then we would require

\[
\dot{t}_o = \frac{zc \left( 21 - 6z - \frac{H_o^2 \lambda_U a}{c^2} \right)}{H_o a \left( 3 + 2z \right)}.
\]
Since the source of the light being measured left its origin some time after the universe completed the exponential inflationary expansion early in universe time, we would have

\[ \dot{\mathcal{r}}_o = \left( \frac{zc3(7-2z)}{H_o a(3+2z)} \right). \] (47)

Putting this back into Equation (45) we find

\[ \left( \frac{\dot{a}}{a} \right)^2 = H_o^2 \left\{ \Omega'_M \left[ z + \frac{12-2z}{(3+2z)} + \frac{z(7-2z)}{(3+2z)} \right] + \Omega_{RO} \right\}. \] (48)

There are three terms for the mass with different functions of \( z \).

Now we would have

\[ \Omega'_M = \frac{(3+2z)}{4(3+2z)} = 0.25 \] (49)

as set above and we can then evaluate each term when \( z=0 \). Let us associate the middle term with \( \Omega_M \), the first term with \( \Omega_{DMO} \) and the remaining term with \( \Omega_{DEO} \). We would then have the values

\[ \left( \Omega'_M \right)_{z=0} = \left( \Omega'_M \frac{12-2z}{(3+2z)} \right)_{z=0} = 1 \]
\[ \left( \Omega_{DM} \right)_{z=0} = \left( \Omega'_M z \right)_{z=0} = 0 \]
\[ \left( \Omega_{DE} \right)_{z=0} = \left( \Omega'_M \frac{z(7-2z)}{(3+2z)} \right)_{z=0} = 0 \] (50)

for \( z=0 \).

Our overall equation would then be
\[
\left( \frac{\dot{a}}{a} \right)^2 = H_o^2 \left\{ \left[ \Omega_M z + \Omega_M \frac{12 - 2z}{3 + 2z} + \Omega_M \frac{z (7 - 2z)}{(3 + 2z)} \right] + \Omega_{RO} \right\}, \tag{51}
\]

where \( \Omega_M \) varies as \((1 + z)^3\).

**Comparing with Experiment**

The expansion of the universe means the distance between two distant galaxies varies with time as

\[
L(t) \propto a(t). \tag{52}
\]

The rate of change of the distance is the speed

\[
v = \frac{dl}{dt} = Hl, \quad H = \frac{\dot{a}}{a} \tag{53}
\]

where \( H \) is the time dependent Hubble parameter.

A method of measuring of the expansion of the universe comes from measuring the shift of frequencies of light, the red shift, coming from distant stars. The observed wave length, \( \lambda_r \), of a feature in the spectrum that had wavelength \( \lambda_e \) at emission is given by the relation

\[
1 + z = \frac{\lambda_r}{\lambda_e} = \frac{a(t_r)}{a(t_e)}. \tag{54}
\]

When the velocity is given by \( c \dot{z} \) then Hubble’s law is written as

\[
c \dot{z} = Hl \tag{55}
\]

from which we see that

\[
z = \frac{HL}{c}, \quad \text{or} \quad H = \frac{cz}{L}. \tag{56}
\]
An additional feature of the new theory presented here is the expression for the red shift of light from distant stars. This has been shown to be [12]

\[
\frac{\Delta \lambda}{\lambda_e} = \exp \left\{ \left( -\frac{G}{c^2} \right) \left[ \frac{\lambda_r}{R_r} - \frac{\lambda_e}{R_e} \right] + \left( \frac{HL}{c} \right) \left[ \frac{M_r}{R_r} - \frac{M_e}{R_e} \right] \right\} - 1, \tag{57}
\]

where the subscript \( r \) designates values at the time and point of reception, the subscript \( e \) represents values at the time and point of emission and the quantities, \( M_E \) and \( R_E \), represent the mass and mean radius of the Earth. We have also used the subscript ‘exp’ on the red shift to indicate that it is the experimental value of red shift measured at the receiving location.

There are two parts to the red shift. One part is due to the gravitational fields at the points and time of emission and reception and the other part is due to the travel time between emission and reception. This is the part that involves the expansion of the universe. Therefore let us rewrite Equation (57) as

\[
z_{\text{exp}} = \exp \left\{ \left( -\frac{G}{c^2} \right) \left[ \frac{M_r}{R_r} - \frac{M_e}{R_e} \right] \right\} \exp \left\{ \left( \frac{HL}{c} \right) \left[ \frac{M_r}{R_r} - \frac{M_E}{R_E} \right] \right\} - 1, \tag{58}
\]

and then rearrange it to get
\[ z \equiv \frac{HL}{c} = \left\{ \left( \frac{G}{c^2} \right) \left[ \frac{\frac{\lambda_r}{R_r}}{M_r e^R_r} - \frac{\frac{\lambda_c}{R_c}}{M_c e^R_c} \right] + \log \left( 1 + z_{\text{exp}} \right) \right\} \left( \frac{M_E}{R_E} \right) \left( \frac{M_r}{R_r} \right) \] (59)

which is the red shift of the universe expansion.

Two simplifications may now be made. First, in many cases the gravitational component of the experimental red shift may be ignored. Secondly, if we are only using the red shift data measured at the Earth’s surface then Equation (59) reduces to

\[ z \equiv \frac{HL}{c} = \log \left( 1 + z_{\text{exp}} \right). \] (60)

This is the red shift value to be used in the expansion velocity of the universe, Equation (51), so that we may write

\[ \left( \frac{\dot{a}}{a} \right)^2 = H_o^2 \left\{ 0.25 \left( 1 + \log \left( 1 + z_{\text{exp}} \right) \right)^3 \log \left( 1 + z_{\text{exp}} \right) \\
+ 0.25 \left( 1 + \log \left( 1 + z_{\text{exp}} \right) \right)^3 \frac{12 - 2 \log \left( 1 + z_{\text{exp}} \right)}{3 + 2 \log \left( 1 + z_{\text{exp}} \right)} \\
+ 0.25 \left( 1 + \log \left( 1 + z_{\text{exp}} \right) \right)^3 \frac{z \left( 7 - 2 \log \left( 1 + z_{\text{exp}} \right) \right)}{3 + 2 \log \left( 1 + z_{\text{exp}} \right)} \right\} + \Omega_{RO} \] (61)

If one can simultaneously measure the red shift and the distance to the object then Equation (56) gives a value of Hubble’s parameter
that may be used in Equation (53) to get the universe expansion velocity.

**Standard Candles**

One reason for choosing the Type Ia supernova in the universe expansion research is the assumption that the mass of this type supernova are all the same; roughly the Chandrasekhar Limit mass of 1.39 solar masses. However, a time dependent gravitational field causes this limit to change with time. This may be seen by considering the Newtonian equation of hydrostatic equilibrium known as the Tolman-Oppenheimer-Volkov [TOV] equation, or

\[
\frac{dp}{dr} = -\frac{GM(r)(1 - H_o \tau)}{r^2} \rho. \tag{62}
\]

The gravitational field that is holding the star together against the internal pressure is diminishing in time. This means that the limiting mass increases in time. Supernova found closer to Earth will have more mass and therefore greater luminosity, than more distant supernova. A reduction in luminosity from the assumed constancy would show up in an analysis by making the more distant supernova appear further away than it really is. The natural conclusion, based on the time-independent gravitational field that produces the constant Chandrasekhar limiting mass, would be that the expansion of the universe is accelerating.

Using the Virial Theorem development by Collins [13] who arrives at the Chandrasekhar limiting mass with the equation

\[
\frac{R_o}{2GM} > 228 \left( \frac{M_{Sun}}{M} \right)^{\frac{4}{9}} \approx 200 \tag{63}
\]
the time dependent gravitational field requires that this relation become

\[
R_o > 228 \left( \frac{M_{\text{Sun}}}{M} \right)^4 \approx 200.
\]  

(64)

This gives the limiting mass as

\[
M_L = M_{\text{Ch}} \left( 1 - H_0 \tau \right)^{-\frac{9}{4}},
\]  

(65)

where \(M_{\text{Ch}}\) is the Chandrasekhar limiting mass. By differentiating Equation (65) with respect to universe time we find the limiting mass for the type Ia supernovae to change according to

\[
\frac{dM_L}{d\tau} = \left( \frac{9H_0}{4} \right) M_{\text{Ch}} \left( 1 - H_0 \tau \right)^{-\frac{13}{4}}.
\]  

(66)

Conclusions

A time-dependent gravitational field, that gets weaker in time, shows the physical effects of this past, stronger field in the dynamics of spiral galaxies. This weakening gravitational field also shows up in the analysis of the distances to, and red shift of light from, supernovas. Here it adds terms to the universe expansion velocity relations that are not present in the analysis of time-independent fields. It also changes the luminosity of the supernovas that were assumed to have constant luminosity. These effects of the time dependent gravitational field remove the need for hypothesizing new matter or energy to explain these effects.

There have been many attempts in the past to find different solutions to Einstein’s field equations and to show how an expanding universe may be viewed in different ways. Portions of the above may
be reminders of prior approaches. Therefore, it may prove useful to point out what is new in this article.

Fundamentally there are three things that are new in this article. First, the fifth dimension is considered to be a real physical entity. All five dimensional theories that I know of in the past, whether by Kalusa-Klein, Einstein with his many collaborators, and others, did not consider the fifth dimension to be real and, therefore, required several terms in the resulting gauge field equations to be zero. Here these terms are non-zero and require that the gravitational potential and field be time-dependent. Second, this article uses the Weyl Gauge Principle as its basis for quantum theory and this requires that the gravitational potential be a non-singular potential. These two things require the gravitational field to be a time-dependent, non-singular, gauge field not seen previously. The third aspect of the article is that the Weyl Gauge Principle requires that the unit of action be dependent upon the gauge function. This requires the red-shift from distant objects to have an exponential dependence upon both the time and distance between emission and reception. The new red-shift relation becomes important in both dark matter and dark energy predictions because both phenomena are witnessed by red-shifted light. The time dependence, or weakening, of the gravitational field is the major factor in predicting effects interpreted as dark matter. The time dependence of the gravitational field also provides the major factor in predictions with respect to dark energy as it is responsible for the diminishing of the luminosity of the distant supernovas used as standard candles and the expression for the expansion of the universe.

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