Influence of the Soup-Bubble Structure on the Stability of a Static, Flat Universe Consisting of Matter and a Repulsive with 1/R Decaying Scalar Field

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The influence of the soup-bubble structure on the stability of a static, flat Universe consisting of matter and a repulsive with 1/R\(_U\) (R\(_U\) = radius of the Universe) decaying scalar field \(\Psi\) has been studied. It is shown that due to the locally heterogeneous mass distribution the bubbles are subject to an instability driven by the repulsive force of \(\Psi\). The excess of repulsive forces inside the voids causes a true instability which can drive the cosmic expansion up to radii comparable in scale to the voids in the large scale galaxy distribution.

The tentative conclusion is discussed if the background radiation possibly could exert properties similar to those of a repulsive scalar field.
Keywords: repulsive scalar field, dark energy, soup-bubble structure, cosmic microwave background, static Universe, universal expansion, redshift, energy conservation, general relativity.

1 Introduction

It is generally believed that the Universe began as a hot big-bang from an instantaneously expanding point and it has been expanding and cooling ever since. The tearing force of the expansion is supposed to be the outward impulse of the primordial big-bang and it is thought that the kinetic energy ($E_{\text{kin}}$) of the expansion is exactly balanced by the gravitational potential energy ($E_{\text{pot}}$) of matter:

$$E_{\text{tot}} = E_{\text{kin}} + \frac{1}{2} E_{\text{pot}} = 0 \quad (1)$$

Another possibility for explaining expansion is that the Universe started off in a steady state situation and then changed to an expanding phase. Eddington (1930) showed that Einstein’s static Universe is unstable against spatially homogeneous and isotropic perturbations and consequently unstable to gravitational collapse or expansion. Expanding solutions of steady state models are still of astronomical interest and the expansion of the family of Einstein static Universes has recently been investigated (Barrow et al. 2003).

In this paper I will show that a formally static, flat Universe consisting of matter and a repulsive with $1/R_U$ decaying scalar field $\Psi$ in which the gravitational potential energy of the baryonic matter keeps exact balance with the energy of the repulsive scalar field $\frac{GM_U^2}{R_U} = \Psi_{\text{today}}$ would expand in a natural way due to its soup
bubble structure. \((R_U = \text{Radius of the Universe}; M_U = \text{Mass of the Universe}; \Psi = \text{Energy of the scalar field})\)

2 The Cosmological Constant Problem

The Einstein field equation read:

\[
R_{\mu\nu} - \frac{R g_{\mu\nu}}{2} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} \times T_{\mu\nu}
\]

where \(\frac{R_{\mu\nu} - R g_{\mu\nu}}{2}\) is the Einstein tensor and \(T_{\mu\nu}\) is the energy-momentum tensor and \(\Lambda\) is Einstein’s cosmological constant.

Einstein did not consider the cosmological constant to be part of the energy-momentum tensor. The left hand side of eq. (2) contains the metric tensor and its derivatives and the new constant \(\Lambda\) appears in addition to this term. One can equally put \(\Lambda\) on the right hand side of eq. (2) and view the cosmological constant as an additional matter tensor \(T^*_{\mu\nu} = \frac{c^4}{8\pi G} \times \Lambda g_{\mu\nu}\) and thus,

\[
R_{\mu\nu} - \frac{R g_{\mu\nu}}{2} = \frac{8\pi G}{c^4} \left( T_{\mu\nu} + T^*_{\mu\nu} \right)
\]

More recently, it has become increasingly common (Misner, Thorne, Wheeler 1973) to interpret \(\Lambda\) as a form of energy present in the empty space (vacuum energy) and consequently as part of the matter tensor \(T_{\mu\nu}\).

Because General Relativity (GR) is not a self containing theory there is no scientific proof that one or the other of these interpretations is the correct approach to use in cosmology, both of the interpretations described above are compatible with GR.
In the following discussion I will use Einstein’s original notation assuming $\Psi$ as an additional matter tensor $T^*_{\mu\nu}$ as defined in eq. (3).

3 Expansion of the Baryonic Universe

On the assumption that expansion is governed by the gravitational attraction of matter against the outward impulse of motion the solution of the field equation ($k = 0; \Lambda = 0$) leads to

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3} \rho$$

(4)

For this case we need only insist on large scale uniformity, i.e. we suppose that the Universe is filled with galaxies which are fairly regularly distributed over space.

4 Friedmann Solution Including $\Psi$

4.1 Homogeneous Mass Distribution

If we, in addition to matter, simply postulate a repulsive with $1/R_U$ decaying scalar field the Friedmann solution of eq. (3) ($k=0; \Lambda = 0$) obtains the following form:
(\kappa \text{ is a constant converting the total energy of } \Psi \text{ into the energy per unit mass corresponding to any given radius } R_U. \text{ With respect to the additional ‘matter’ tensor } T^* \text{ in eq. (3) the attractive and repulsive potential terms are included with opposite signs.})

According to eq. (5) the gravitational potential amounts to \(-2GM_U/R_U\) and the value of the potential due to the repulsive scalar field \(2\kappa\Psi/R_U\) has the same value by definition. At any point of such a Universe the attractive repulsive forces all cancel out due to homogeneity and symmetry.

In equation (5) the first time derivative of \(R\) seems to vanish for all times, implying a static Universe.

4.2 Heterogeneous Mass Distribution, Equations of Expansion

In today’s Universe, however, matter is not homogeneously and isotropically distributed over space, the 3-dimensional distribution of luminous matter has a soap-bubble (SB) appearance with the visible galaxies on the surface of the soap bubbles (Landy 1999). The bubbles occur on scales of 100 Mpc and are entirely free of matter. An important feature of the SB - Universe is that due to its still homogeneous and isotropic large scale structure the amount of mass \(m_v\) containing in the thin outer shell of the voids exactly corresponds to the mass \(m_v = \frac{V_v}{V_U} \times M_U\) \((V_v = \text{Volume of the void with radius } r_v, V_U = \text{Volume of the Universe})\).
In contrast to equations (4) and (5), in presence of the postulated scalar field local heterogeneities in mass distribution due to the SB - structure can not be neglected and the matter term in eq. (5) has to be modified by a structure factor $S$ taking into account local heterogeneities of the corresponding with $1/R$ decaying gravitational field:

$$\left( \frac{\dot{R}}{R} \right)^2_{sb} = \frac{S}{R^2} \frac{GM_U}{R_U} + \frac{2}{R^2} \frac{\kappa \psi}{R_U}$$

(6)

The structure factor $S$ can be calculated from the decrease of gravitational potential inside the hollow spheres using equations of Newtonian dynamics for spherical bodies (Bergmann, Schäfer 2002) making the usual assumption of homogeneous and isotropic mass distribution outside the voids. Results are summarized in Table 1. Details of calculation are given in the Appendix.

Table 1

<table>
<thead>
<tr>
<th>D</th>
<th>0.05</th>
<th>0.10</th>
<th>0.15</th>
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</thead>
<tbody>
<tr>
<td>S</td>
<td>1.367</td>
<td>1.402</td>
<td>1.438</td>
</tr>
</tbody>
</table>

(Thickness of the shell = $d \times r_v$)

Because the potential of $\Psi$ remains unchanged $\frac{2\kappa \psi}{R_U} \left( = \frac{2GM_U}{R_U} \right)$, in eq. (6) the repulsive potential overwhelms the attractive gravitational potential and the system consequently expands. The excess of repulsive forces inside the voids causes a true instability which can drive the cosmic expansion due only to the surplus of repulsive force inside the voids exerted by the homogeneously distributed repulsive scalar field. No matter how small the bubbles are when they begin, such bubbles will grow up to a radius
comparable in scale to the voids in the large scale galaxy distribution.

The structure factor for the SB - Universe containing $10^6$ voids with radius $10^{-2} \times R_U$ and $d = 0.1 r_v$ takes the value 1.402. From eq. (6) the velocity of expansion can be calculated to $v \approx 2 \times 10^9$ cm/s.

5 Speculations on the Physical Nature of Variable Cosmological Constants

5.1 Dark Energy

Einstein’s cosmological constant is one special case for that an attempt of physical interpretation, namely as energy containing in empty space, exists. Unfortunately, the estimated energy of the vacuum exceeds the observed value by 120 orders of magnitude. In order to overcome this problem in recent approaches, including this work, several forms of varying cosmological constants were introduced into the Einstein equations (Caldwell, Dave, Steinhard 1998; Peebles, Ratra 1988, 2003) and a number of authors constructed models, in which specific decay laws are postulated for $\Lambda$. Examples for phenomenological $\Lambda$-decay laws are summarized in Overduin, Cooperstock (1998). These theories, however, are entirely speculative as much as it is not clear what the physical nature of the dark energy should be.

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5.2 Correlation by Chance or Causality?

Finally, I would like to point out the striking fact that the energy of the background radiation containing in the Universe exactly fit the requirement for the energy of the scalar field $\Psi$ in eq. (5) necessary for the construction of a flat Universe. Due to the large number of photons compared to that of the baryons ($N_{ph}/N_B = 10^{10}$) the background radiation is absolutely smooth and can be considered as scalar and similarly to the energy of the presupposed scalar field $\Psi$ the energy of the photons vanishes with $1/R$.

The assumption that photon energy could replace $\Psi$ appears strange for the first sight and in fact, there are crucial arguments against this hypothesis.

First of all, the approach violates the Principle of Equivalence for photons. Although experimental constraints on such violations were claimed (Gasperini 1989), the only fact, however, that light does not interact with light by gravitation shows that in case of light rays the Principle of Equivalence is possibly not unrestricted applicable to gravitational phenomenon.

Another problem with this approach is that if it is applied to the gravitational field of a star or planet, this possibly would lead to observable differences in bending of light. The bending of light rays has been measured with great precision and the value observed corresponds exactly to that predicted by the Einstein equation.

However, according to equation (5) the net gravitational field of stellar objects is given by the sum of the attractive and repulsive
potentials. The attractive gravitational potential on the surface of the Sun amounts to $1.9 \times 10^{15}$ ergs g$^{-1}$. In comparison, the total energy of radiation emitted by the sun is $1.86$ ergs g$^{-1}$, 15 orders of magnitude smaller than the gravitational potential. Therefore, the curvature of space caused by the mass of the sun is not affected by the repulsive force of radiation within measurable limits.

On the other hand, some conspicuous peculiarities of the physical properties of radiation are in favour of above assumption:

Light is regarded as a kind of excitation of the empty space (Genz 1999, 312; Krasnoholovets 2002) and it is one of the most fundamental laws of physics that every energetically excited system relaxes to a state with lower energy, a less excited state or the ground state. A molecule, for instance, can relax by various competing pathways. For photons, however, the only way to relax to the ground energy state is to increase their wavelength.

In addition to this, the basic differences between the gravitational interactions of matter and radiation are obvious: Light does not interact with light by gravitation (Fritzsch 1997, 108; Faraoni & Dumse 1998); it is hard to imagine a gravitational condensation of pure radiation (Gamow 1948). Considering the different forms of energy involved in gravitation, the kinetic energy, thermal energy etc., it is obvious that all these energies represent a certain physical state of matter, or in other words, they are (changeable or temporary) properties of matter that is the carrier for these types of energy. Light, however, is not a property of matter, it is rather a physical property of the excited empty space that possibly could reveal tendency to relax to the ground energy state by increasing its wavelength, i.e. expanding space.

If we are completely unprejudiced we have to concede that there are no compelling physical reasons to assume that the energy of radiation has to be condensed in a general sum together with matter.
or with other forms of energy carried by matter and consequently, we can not rule out the possibility that photons could posses the properties of a repulsive scalar field by physical evidence.

6 The Flat Photon/Baryon Governed Universe

6.1 Homogeneous Mass Distribution

If we, if only for sake of curiosity, replace $\Psi$ by the energy of the photons present in the background radiation eq. (5) obtains the following form ($\Lambda=0$; $k=0$):

$$\left(\frac{\dot{R}}{R}\right)^2 = -\frac{2}{R^2} \frac{GN_B}{NL R} + \frac{2}{R^2} \frac{\kappa N_\lambda}{R} = 0 \quad (7)$$

The term $\frac{GN_B}{NL R}$ is equivalent to Einstein’s matter tensor in every respect, containing matter and all forms of energy/c$^2$ which is associated with a certain physical state of matter. The term $\frac{\kappa N_\lambda}{R_U}$ is a new term, a true energy term, containing those forms of energy which are considered to be a property of the excited empty space, the energy of photons and neutrinos.

$N_B$ is the total number of baryons and $N_\lambda$ the total number of photons in the Universe. $N_L$ is the Loschmidt number and $\kappa$ is a constant converting the total energy ($E_\text{tot}^\lambda$) of the microwave background radiation as observed today into the energy per unit mass ($E_\text{tot}^\lambda/M_U$) corresponding to any given radius $R_U$ of the Universe. The numerical value of $\kappa$ can be calculated to $1.4 \times 10^{-41}$ cm$^3$ s$^{-2}$
6.2 Cosmological Parameters

In order to construct a solution which corresponds to a flat Universe the mass $M_U$ and the radius $R_U$ have to be adjusted to the proper value. For parameterization the following data were used: $R_U = 1.4175 \times 10^{28}$ cm (corresponding to an age of the Universe of 15 billion years), temperature of the cosmic background radiation $T_\lambda = 2.726$ K, corresponding to the total number of photons of $N_\lambda = 4.898 \times 10^{87}$ (other sources of radiation were not taken into account). From this the total mass can be calculated to $M_U = 1.028 \times 10^{54}$ g, in best accordance with literature data (Narlikar 1993; Turner 2001). With above parameter both the gravitational energy and the energy of the cosmic background radiation amount to $4.84 \times 10^{18}$ ergs $g^{-1}$ and according to eq. (7) the exact balance between these two opposite forms of energy remains preserved for any given radius $R_U$ of the future expansion.

The expansion follows the schema as described in Section 4.2 and needs no further explanation.

7 The Influence of the Photon Term on Local Dynamics

Once introduced the additional photon term $T_{00}^* = \frac{c^4}{8\pi G} \times E^{tot}_\lambda g_{00}$ in eq. (3) the question arises as to whether the new term affects the dynamics of local systems. Since we assume that the Universe is homogeneous and isotropic on the cosmic scale and contains positive and negative energy in the same amount, the positive and negative energies all cancel out due to the uniform density and symmetry. As described in Section 4 the mass $M_U$ of the Universe is condensed on the surface of large empty spaces (voids) in a thin
outer shell with $\rho_{\text{shell}} >> \bar{\rho}$. Formation of galaxies will cause a local increase in gravitational potential above the overall net zero level according to the Einstein equation. The potential of the repulsive photon term inside the galaxies can be neglected from reasons of magnitude. All appearances of local gravity obey the Einstein equation. Expansion due to the repulsive force occurs strictly on the cosmic scale.

8 Redshift and Energy Conservation

The difficulty with the law of energy conservation in an expanding Universe was first noticed by Hubble (1936). He wrote that “redshift by increasing wavelengths must reduce the energy in the quanta. Any plausible interpretation of redshifts must account for this loss of energy.” The conservation of energy of light photons in transit has been a problem for cosmologists ever since and some of them take the view that in this one case the energy in the Universe as a whole is not conserved (Harrison 1981, 275)

According to eq. (8) the energy required for the expansion of the mass $M_U$ of the Universe versus its own gravitational attraction is supplied by the radiation energy. The energy of the cosmic background radiation decreases by the amount of the work done in the expansion of volume of the Universe.

$$\Delta E = - \int_{r_1}^{r_2} \frac{GMm}{r^2} dr = - \left( \frac{GMm}{r_1} - \frac{GMm}{r_2} \right)$$

The expansion is accompanied by an equivalent redshift and the law of energy conservation is not violated.

9 Concluding Remarks
(i) I have examined the stability of a formally static Universe in the presence of a variable cosmological term $\Psi$ that scales $\Psi \propto \frac{\Psi}{R}$. I have shown that due to the SB - structure the potential field of such a Universe consists of expanding and contracting regions leading to a global expansion of the whole system.

The model includes an explicit physical mechanism between the shrinking cosmological term and the gravitational energy of matter to the extent as the energy of the scalar field decreases by the amount of the work done in the expansion of volume of the Universe.

(ii) The tentative approach that the photon field could act like a repulsive scalar field differs from previous discussions. Instead of presupposing repulsive scalar fields of unknown nature, I introduced a homogeneous density of radiation sources. An advantage of this approach is that it works with forms of matter and radiation that are known to be present in the Universe today and would provide a natural explanation of the flatness or fine - tuning problem without the need for dark matter and dark energy and, in addition to this, the model could explain the redshift of the cosmic background radiation in transit in accordance with the law of energy conservation.

That redshift is caused by the expansion of space time due to the outward impulse of a primordial big - bang event is merely a result of interpretation. The opposite view, however, that light itself expands space can not be ruled out by known experimental evidence. As far as the origin of redshift is concerned, the only indubitable observational fact is that the wavelength of radiation increases in the same extent as the radius of the Universe expands. At present time there is no scientific proof if one or the other of
these approaches is the correct approach to use in cosmology and, therefore, the basic postulate of the new model has to be accepted without further evidence. Nevertheless, the model represents an attractive alternative to dark matter and dark energy hypotheses. The underlying assumption appears consistent with theories on the physical nature of light and the general behaviour of excited energy states and could possibly be proved or disproved by observation of the expansion rate in near regions of supernovae bursts.

Appendix

Calculation of the Structure Factor for the SB - Universe

For numerical calculation of the structure factor $S$ we hollow out a full sphere with radius $r_v$ ($r_v << R_U$) and uniform and isotropic mass distribution and disperse the mass $m_v$ of the sphere in a thin outer shell on the surface of the void. Continuing this procedure results in a number of voids with a thin shell of matter as observed in the large scale structure of the Universe.

The gravitational potential inside the hollow spheres can be calculated using the well known equations of Newtonian dynamics for spherical bodies (Bergmann, Schäfer 2002) making the usual assumption of a homogeneous and isotropic mass distribution inside the shell as well as outside the voids

$$ U^F_G = \frac{2}{1.5} \int_{x=0}^{x=r_v} \frac{x^3 - (x-1)^3}{x} dx $$(9)
\[ U^H_G = \frac{2}{1.5} \int_{x=r_v-d}^{x=r_v} \frac{x^3 - (x-1)^3}{x} \rho_{\text{shell}} \, dx \]  

\[ U^F_G = \text{gravitational potential full sphere}, \quad U^H_G = \text{Gravitational potential hollow sphere}, \quad d = \text{thickness of the shell}, \]

\[ \rho_{\text{shell}} = \frac{r_v^3}{r_v^3 - (r_v - d)^3} \quad (m_v = 1 \quad \text{and} \quad \frac{4\pi}{3} = 1). \]

The contribution of matter and the repulsive scalar field outside the shell with mass \( m_v \) to the forces inside the void can be neglected because all these forces cancel out due to symmetry and reversed signs. The multiplication factor \( \frac{2}{1.5} \) was introduced in order to remain in conformity with the value for the gravitational energy for expanding self gravitating systems predicted by the virial theorem and generally used in GR: \( E_{\text{kin}} = \frac{1}{2} E_{\text{pot}} = \frac{GM^2}{R} \).

Expectedly, the gravitational potential at the centre of the full sphere amounts to \( U^F_G = 2 \frac{Gm_v}{r_v} \) and calculation using eq. (10) leads to

\[ U^H_G = S \times \frac{Gm_v}{r_v} \]  

with \( S = 1.367, 1.402, 1.438 \) for shells with thickness of \( d = 0.05, 0.10, 0.15 r_v \), respectively.

With eq. (12) the sum of gravitational potential inside the voids containing in the SB - Universe can be calculated to
\[ U^\text{SB}_G = S \frac{G m_i}{r_i} \times N_{\text{voids}} \times \frac{r_i}{R_U} = S \times \frac{G M_U}{R_U} \]  

instead of \( \frac{2GM_U}{R_U} \) for a homogeneous and isotropic mass distribution. The multiplication factor \( \frac{r_v}{R_U} \) takes into account that the potential energy of a self gravitating sphere with radius \( R \), when calculated from the sum of potentials of the constituting individual spheres (voids) with radius \( r_v \) equals to \( U_G = \frac{r_v}{R_U} \sum_{\text{voids}} \frac{2Gm_v}{r_v} \).

From eq. (12), the structure factor for the soup-bubble Universe containing \( 10^6 \) voids with radius \( 10^{-2} \times R_U \) and a shell thickness of \( d = 0.1 r_v \) takes the value 1.4.

According to this schema the Universe can be thought of as being built up from expanding \( (U_\Psi > U_{\text{pot}}) \) and contracting \( (U_\Psi < U_{\text{pot}}) \) regions. Inside the voids the force of the repulsive scalar field overwhelms the gravitational attraction of matter whilst in regions with \( \rho_m > \bar{\rho} \) the attractive gravitational potential predominates.

It is important to note that expansion of empty spaces, inside galaxies for instance, can only occur when the mass of the surrounding sphere calculated from the total mass of the galactic system is less than \( \approx \frac{2}{S} \frac{V^\text{void}}{V_U} M_U \). Because the mass density of galaxies \( \rho_G \gg \bar{\rho} \) empty spaces inside galaxies are not subject to expansion.
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