# Tangible Grounds for Lorentz and Generalized Lorentz Transformations

C. P. Viazminsky
Email: Kayssarv@mail2world.com

It is shown that the distance between a moving object and an observer obeys longitudinal scaling transformations, which amounts to a contraction when the object and the observer approach each other and to an elongation when recede from each other. The contraction-elongation relation is extended to the scaling transformations that determine, for a general type of motion, the relation between the distances of an object and its location from an observer. It is shown that the scaling transformations can be interpreted as transformations between quantities pertaining to a moving body and its initial location in one inertial frame, or else, between the coordinates of the body in two inertial frames. Using the scaling transformations, phenomena such as Michelson and Morley experiment, lifetime of metastable particles, Doppler's effect, the drag effect, and the Sagnac's effect, all yield to simple and lucid explanations. Contrary to the relativistic prediction, the scaling relation implies a complete absence of traverse Doppler effect. The Lorentz transformations (LT) are derived, using the longitudinal scaling relation; the method of derivation restricts the domain of its validity to space-like and null intervals. Moreover, a generalized form of Lorentz transformations (GLT) will be given and briefly discussed. One consequence of the GLT is that the familiar LT is only one in a class of such, and that LT is valid only for motion on an axis.

*Keywords*: (scaling transformations, relatively absolute units, 3-physical space, Sagnac's effect, generalized Doppler's effect, generalized Lorentz transformations)

# 1. Introduction

In the special relativity theory (SRT) it is postulated that the velocity of light is a constant *c* that is independent of the relative motion between the source and the observe, or as to say, light's speed is the same in all inertial frames[1-4]. We start with a weaker postulate: the velocity of light within each inertial frame is a constant *c*. By this we mean that if light is emitted at an instant $t_0$ from the point $\vec{r}_0$ and received at an instant $t_1$ at the point $\vec{r}_1$, where $\vec{r}_0$ and $\vec{r}_1$ are stationary in an inertial frame *s*, then $\left|\vec{r}_1 - \vec{r}_0\right| / (t_1 - t_0) = c$. The starting point we have adopted can hardly be counted a postulate, for it is a consequence of the equivalence of inertial frames regarding length and time measurements within each inertial frame. We shall use tentatively the Galilean law of velocity addition to combine the velocity of a light's signal with the velocity of the emitter. It is clear however that a constant velocity of light within each inertial frame cannot be reconciled with the Galilean law of velocity addition unless length and time measurements in different inertial frames are subjected to revision in meaning and magnitude. In this article:

-We shall derive new transformations of distance and time intervals and show that these preserve the speed of light within each inertial frame and yield the same relativistic law of velocity addition.

-Using the resultant transformations the perplexities that arise in propagation of light are resolved. Indeed, the Doppler's effect, drag's effect, the negative results of Michelson and Morley experiment, the Sagnac's effect[5] , as well as the lifetime of meta-stable particles[1-4] are all explicable in a simple manner. Stellar aberration phenomenon[1,6], which is also neatly explicable by the current theory, will be the subject of a forthcoming article[7].

-Starting from the transformations of time and distance intervals, the LT transformations are derived in a transparent manner, with space-like intervals are excluded from the domain of its validity.

-A general form of LT is given and shown to tend to the corresponding LT for one dimensional motion; it dictates however that LT is applicable only within this subspace.

## 2. The Longitudinal Scaling Relation

Suppose that $s = bxyz$ is an inertial frame in standard configuration with the inertial frame $S = BXYZ$, and moving relative to *S* with a constant velocity $\vec{u} = u\vec{i}$, where $\vec{i}$ is the unit vector of the *X*-axis of *S* and $(u > 0)$. When at the point $B(0,0,0)$ in *S*, the body *b* emits a pulse of light in the +*X*-direction. The pulse of light reaches an *S* observer with coordinate $X(X > 0)$ (which we call the observer *rX*), and at the same time it reaches also the observer *rx* in *s* that is contiguous to *rX* at the moment of light's reception by both observers. The observers *rX* and *rx* that are contiguous when light reaches them are called *conjugate observers*. We take the instant at which light reaches *rx* and *rX* the mutual zero of timing in both frames, i.e. $T = t = 0$, when *rX* and *rx* are contiguous. We may conceive light as if emanated from the source *b* when was at the point *B* in *S*, or from the point *B* when was occupied by the source *b*, and subsequently received by the conjugate observers *rX* and *rx* at $T = t = 0$. Thus we may conceive light emanating from one and the same point in a 3-physical space, defined by the body *b* and the point that occupies in *S*, which is *B*, when light is emitted, and ending up at the same point defined by *rX* and *rx* at the moment of their contiguity. It follows that there is one and the same ray of light, whose path is to be described in two inertial frames *s* and *S*, or else, by using quantities pertaining to the body *b* and its location *B* in *S*.

When light is considered emanating from the point $B \in S$, the light's trip $(B \to X \; and \; x)$ may be viewed to take place within *S*, and hence its length *X* relates to its duration *T* by $X = cT$. Similarly, when light is considered to emerge from the source *b*, the trip $(b \to x \, and \; X)$ may be viewed to take place within *s*, and hence $x = ct$, where *x* and *t* are the length and duration of the trip respectively. Note that in each frame, *B* and *b* are coincident at the instant of light's emission whereas *X* and *x* are not, and that at the instant of light's reception, *X* and *x* are coincident whereas *B* and *b* are not. The equations

$$(i) \;\; X = cT, \qquad (ii) \;\; x = ct, \tag{2.1}$$

yield

$$\frac{X}{x} = \frac{T}{t} = \Gamma(u), \tag{2.2}$$

where $\Gamma(u)$ denotes the common value of both ratios. Since each frame can count itself stationary while the other is moving, the proportionality factor should be such that

$$\Gamma^{-1}(u) = \Gamma(-u) \text{ and } \Gamma(0) = 1. \tag{2.3}$$

To determine the factor $\Gamma(u)$ we use tentatively the Galilean law of velocity addition. To the *S* observers the light's trip $(b \to x \, and \; X)$, that yields equation (2.1ii) when viewed within *s*, is identical to the trip $(b\, at\, B \to X\; and\; x)$. Since $b$ however is moving with velocity $u$ relative to $S$, the $S$ observers relate the period $t$ of the trip to its length $X$ by the equation

$$X = (c+u)t\,. \tag{2.4}$$

Similarly, and since $B$ is moving relative to $s$ with velocity $-u$, the $s$ observers relate the period $T$ of the trip $(B\, at\, b \to x\, and\;\; X)$ to its length $x$ by the equation

$$x = (c-u)T\;. \tag{2.5}$$

Dividing the last two equations side to side and using (2.1) we obtain, on setting $\beta = u/c$, the relation

$$\Gamma = \frac{1+\beta}{1-\beta}\frac{1}{\Gamma}\,,$$

which determines the scaling factor:

$$\Gamma(u) = \sqrt{\frac{1+\beta}{1-\beta}}\,. \tag{2.6}$$

The transformations (2.2), which was derived in previous works[8-10] through different methods, will be referred to as the contraction-elongation relation, or the longitudinal scaling relation. The reason underlying this terminology will appear soon.

## 3. Remarks on the Contraction-Elongation Relation.

(i) It is important to note that the same transformations (2.2) holds if $B$ is the true source of light while $b$ is merely a virtual source. It is also clear that if $s$ in which $b$ is stationary is moving with velocity $(-u)$

relative to *S*, then $u$ is to replaced by $(-u)$ in (2.2). A similar replacement has be made if the coordinate $x$ is negative (but $u$ is still positive).

(ii) The transformations (2.2) guarantee that $X/T = x/t$, which in turn gives rise to the first postulate of SR in a specific sense: if light propagates in one frame with velocity $c$, it propagates also with the same velocity $c$ in the other frame. This does not mean however that the velocity of light is independent of the source's velocity[7].

(iii) At the beginning of the light's trip the true and virtual sources $b$ and $B$ are contiguous to each other while the observers $rx$ and $rX$ are not, and the converse is true at the end of the trip. In a given frame, say *S*, the distance $x$ between a body $b$ and an observer $rX$ is by definition their distance at the final instant $T = 0$, while the distance $X$ of its location from $rX$ is the distance between them at the instant light was emitted $(T = -X/c)$; or as to say, it is the distance between an *S* observer at $B$ and the observer $rX$. The relations (2.2) relate the distance separating the body $b$ and the observer $rX$ to the distance between its location and $rX$. It follows that one frame, say *s*, can be dismantled, while there exists in the other (here is *S*) two types of distance intervals:(i) the *geometric* (or stationary or static) distance which is the distance of the body from the observer at $(-X/c)$, and the "*proper*" (or mobile or kinematical) distance $X_p = x$ which is the distance of the body from the observer at $T = 0$.

(iv) Given two objects $A$ and $B$ moving on the same line, an inertial frame *S* can be chosen such that either object, say $A$, is stationary in *S* and taken as an observer, whereas the other, $B$, is a source. By remark (iii) and the transformations (2.2), the distance $D$ between the location of the object $B$ and the observer $A$ relates to the distance $d$ between the object $B$ and the observer $A$ by $D = \Gamma(u)d$ if

*A* and *B* approach each other with a relative velocity *u*, and by $D = \Gamma^{-1}(u)d = \Gamma(-u)d$ if *A* and *B* recede from each other with velocity *u*.

(v) Because of the concept of location in *S* is a primary one, it is quite natural for the *S* observers to consider light emanating from $B \in S$. But why do we require that the *s* observers should adopt *B* as a virtual source of light? Our requirement is justified by the fact that, if *B* was a true source whereas *b* was not, then *b* would be the location of the body *B* in *s* when light is emitted, and the *s* observers would certainly consider light emanating from *b* as well as from *B* and demand that the *S* observers should imagine *b* as a virtual source of light. This leads the *S* and the *s* observers, when both observe light's emission from a source, to treat virtual and real sources evenly.

# 4. Interpretations of the Scaling Transformations

## The Active View

In a given frame *S*, a unit of length (or distance) is presupposed and can be chosen arbitrarily in any convenient way, such as, the distance between any given two material points in *S*, or the length of a rod joining such, or the wave length of a specific spectral line characterizing stationary emission of some chemical element. After choosing a unit of length, the geometric distance in *S* between any two material points *A* and *B*, that are stationary in *S*, can be quantified. Moreover, the axes of a rectangular Cartesian system *OXYZ* in *S* can be calibrated by multiples and fractions of the chosen unit, and consequently, points in *S* can be specified by their coordinates. When *S* is endowed with the system *OXYZ*, the geometric distance between a material point on an axis and the origin *O* is the absolute value of its coordinate on that axis.

Reverting to the frame *S* considered above, we identify *X* as the "geometric" distance between an *S* observer at the location *B* of the body *b* and the observer *rX*. Being geometric, the distance *X* depends only on the two points *B* and *rX* in *S*, so that whenever these two points are specified the distance *X* can be determined by geometric means, and it is independent of the instant at which the measurement is carried out. On the other hand, the quantity *x* can be envisaged as the distance of the *moving* body *b* from the observer *rX when* light is received by *rX*. Equivalently, *x* is the distance between an *S* observer $B'$, that is contiguous to *b when* light is received, and the *S* observer *rX*, i.e., it is what we have already named the proper, or mobile, distance between the source *b* and the observer *rX*. Thus the relations (2.2) can be interpreted as transformations within the same frame *S*, between the geometric length *X* and the proper length *x* at any instant of observation. Note that in this interpretation *X* can be assumed already given or known, while *x* which is calculable by the transformations (2.2) has to coincide with its measured value in *S* if the theory is correct. According to the latter interpretation, the transformations (2.2) which hold within the same frame of reference *S*, there exists in addition to the usual geometric distance between a body and an *S* observer, a proper distance that depends on the velocity of the body, and these two types of distance are identical only for stationary bodies in *S*. On account of (2.1) parallel statements hold for the geometric and proper durations *T* and *t* respectively.

Alternatively, the transformations (2.2) hold within *s*, but with the rules of *b* and *B* as true and virtual sources are interchanged; *X* is the proper distance of a true body *B*, which is moving with velocity $-u$ in *s*, and *x* is its geometric distance from *rx*. Although it is convenient to consider the geometric quantities as given or known data, it is also possible to start from the measured mobile quantities and calculate the geometric quantities pertaining to the virtual source at the instant of emission.

In the active view, the stationary and proper distances are essentially coordinates. The distances *X* and *x* are the coordinates of the observers *rX* and *rx* in *S* at the instant of light reception. Or, we may view *x* and *X* as the coordinates of the body *b* and its location *B* in *S* if the origin of *S* is taken at the conjugate observers *rX* and *rx* and the *X*-axis is directed towards the body. We shall adopt the latter convention whenever we speak of the geometric and mobile coordinates of a moving body in *S*. For $u << c$, the transformations (2.2) are approximated by

$$X - x = X(1 - 1/\Gamma) = uT - (u^2/c)T + .. \approx uT .$$

It is noted that the active view "*decouples*" the two frames *S* and *s*, in the sense that one can be contented with measurements of quantities pertaining to a body and its location in one frame.

## The Passive View

In the active view no ambiguity arises regarding units, because the same units in a single frame *S* are used to measure the length and duration of the trips $(B \to X)$ and $(b \to X)$. When two frames are involved in measuring the length and duration of the same trip, it is necessary beforehand to specify the units of length and time in each frame.

We have seen in section 2 that light emitted from the body *b* and received latter by the conjugate observers $rX$ and $rx$ can be considered by all observers to emanate from the same point $(b\, at\, B)$ $\equiv (B\, at\, b)$ and to end up at the same point (*X and x*) in the 3-physical space. Thus, a single light's trip $(B\, or\, b \to rX\ and\, rx)$, which involves only one true source, is considered an absolute entity in the 3-physical space, and processes accordingly absolute length and duration, whereas the relations (2.2) are interpreted as transformations between units of length and time in *S* and *s*. Denote the units of length (time) in *S* and *s* by *SLU* (*STU*) and *slu* (*stu*) respectively, then by (2.2), $SLU/\Gamma = slu/1$. Hence, the units of lengths in *S* and *s* are in the proportion $\Gamma:1$, or, $SLU = \Gamma slu$. If the unit of time in each frame is set equal to the duration taken by light to cross the unit length trip, then $STU/\Gamma = stu/1$, and $c = 1$ in each frame. It is possible of course to adopt any *chosen length* (period) in one frame *as the unit* of length (time), but the choice of the unit of length (time) in any other frame must *respect the proportion* specified above. Units of length (time) in *S* and *s* that respect the proportion specified above will be called *relatively absolute units.*

Now, if *b* is the source of light, then the length of the trip $(b \to x)$ which occurs exclusively within *s* is equal to its geometric length $x = x_g.uls$ in *s*. In the passive view of the scaling transformations, a single light's trip is considered an absolute entity in the 3-physical space, and hence its length *X* in *S* must be equal to its length *x* in *s*, i.e. $X = x = x_g.uls$. The light trip $(b \to rx)$ which occurs within *s* can be viewed in *S* as starting from the *moving* source *b* in *S* and ending at the *S* observer *rX*, which is the trip $(b \to rX)$. The question now is that: if it is known that the length of a light's trip within *s* is $x_g$ (which means $x = x_g.slu$), then what would be its measured length $X_m$ in *S*? The answer is simple. Since the unit of length in *S* is $\Gamma$ times the unit of length in *s*, the length of the trip $(b \to rx)$ when observed in *S* is its length within *s* divided by $\Gamma$, i.e. $X_m = x_g / \Gamma$. Indeed, and if $X = X_m.ULS$ , then

$$x_g.uls = X_m.ULS \Leftrightarrow x_g.uls = X_m.(\Gamma uls) \Leftrightarrow X_m = x_g / \Gamma$$

Similarly, and if the *S* observers assign to a light's trip $(B \to rX)$ that occurs exclusively within *S* a length $X_g$, then its length, when observed in *s*, is $x_m = \Gamma X_g$.

In the remainder of this section we summarize the passive view of interpretation of the scaling transformations by the following elements

- The transformations (2.2) define relatively absolute units of length and time in *S* and *s* that are in the proportion $\Gamma : 1$.

- If it is known in *s* that the geometric length and duration of a given trip is $(x_g, t_g)$, then the observed values in *S* corresponding to these geometric data are given by

$$\frac{X_m}{x_g} = \frac{T_m}{t_g} = \frac{1}{\Gamma}. \tag{4.1a}$$

Similarly, there corresponds to the *S* geometric data $(X_g, T_g)$ the observed values $(x_m, t_m)$ in *s*, where

$$\frac{X_g}{x_m} = \frac{T_g}{t_m} = \frac{1}{\Gamma}. \tag{4.1b}$$

Therefore and when observed in *S*, the value of a geometric *s*-quantity is simply the *S* equivalents (means using *S* units) of its value in *s*, and vice versa.

- A concise form of (4.1) is

$$\frac{X}{x} = \frac{T}{t} = \frac{1}{\Gamma}, \tag{4.2}$$

provided we understand that

1. $X$ and $x$ refers to the coordinates of the 3-physical point $(b\,at\,B) \equiv (B\,at\,b)$ in the frames $S$ and $s$ respectively, or as to say, to the coordinates of one true source, either $b$ or $B$, (but not both) in $S$ and $s$ respectively.
2. Either the quantities in the numerator or the denominators, but not both, are geometrically measured. Precisely, the geometric quantities pertains to the frame in which the body is at rest.

Note that although the scaling transformations (4.2) between two frames have the inverse form of the contraction-elongation relations (2.2) which hold within one frame, both forms embody the same geometric and physical contents.

## 5. Conjugate Sources

If $B$ and $b$ are both true sources of light then we have two light's pulses emanating simultaneously from $B$ and $b$. Now each observer $rX$ and $rx$ receives two pulses but not simultaneously, and hence $rX$ and $rx$ are conjugate observers only for the pulse that is first received. To see that this is indeed the case, we use the fact that the length of a light's trip is absolute and equal to its geometric length. The length of the light's trip $(b \to rx)$ is $x = x_g uls$, and the length of the light's trip $(B \to rX)$ is $X = X_g.ULS$. It follows that

$$\frac{1}{\Gamma} = \frac{x}{X} = \frac{x_g.uls}{X_g.ULS} = \frac{x_g}{X_g}\frac{1}{\Gamma}, \tag{5.1}$$

and hence

$$x_g = X_g = a_l \,. \tag{5.2}$$

Similarly

$$t_g = T_g = a_t \,. \tag{5.3}$$

This means that the readings of the geometric lengths and durations of the true trips $(B \to rX)$ and $(b \to rx)$ in *S* and *s* respectively are equal. The lengths of these trips however and their durations are not equal. Indeed

$$x = a_l uls = a_l ULS/\Gamma = X/\Gamma \,, \tag{5.4}$$

$$t = a_t uts = a_t UTS/\Gamma = T/\Gamma \,. \tag{5.5}$$

The last relation means that the trip $(b \to rx)$ takes less time than the trip $(B \to rX)$ provided that $\Gamma(u) > 1$, and longer time provided that $\Gamma(u) < 1$. Confining ourselves to the case $\Gamma(u) > 1$, we see that the conjugate observers *rX* and *rx* receive first the pulse emanating from *b*, and after they individuate, each receive the pulse emanating from *B*. Each of the observers *rX* and *rx* can claim itself stationary while receiving the two pulses, whereas the other has moved a certain distance after receiving the first pulse. Assuming that the emission of the two pulse of light from (*b and B*) takes place at $T = t = 0$, the observer *rx* registers $t = a_t .uls$ at receiving the first pulse (from *b*) and $a_t \Gamma uls = t\Gamma$ at receiving the second (from *B*). The difference between these is

$$\Delta t(rx) = t(\Gamma - 1) = a_t (\Gamma - 1) uls \,.$$

The observer *rX* register $(a_t/\Gamma)ULS = T/\Gamma$ at receiving the first pulse (from *b*) and $a_t ULS = T$ at receiving the second (from *B*). The delay period between receiving these two pulses is

$$\Delta T(rX) = T(1-1/\Gamma) = a_t(1-1/\Gamma)ULS\ .$$

Thus, although the speed of light within each inertial frame is *c*, the velocity of light is not independent of the relative velocity between the source and the observer. Indeed, the two pulses emanated at the same time from two conjugate sources *b* and *B* do not reach *rX* (or *rx*) at the same time.

# 6. The Scaling Relations in General

Let $S \equiv OXYZ$ be an inertial frame endowed with a system of spherical coordinates $(R,\theta,\phi)$, with $\theta$ is the azimuth angle between the *OX* axis and the radius vector $\vec{R}$. Consider a body *b* moving relative to *S* with velocity $\vec{u} = u\vec{i}\ (u > 0)$, with $\vec{i}$ is the unit vector of *OX* and $(u > 0)$. It is shown[7,11] that the transformations from the geometric coordinates $(R,\theta,\phi)$ of the body *b* to its proper coordinates $(r,\theta',\phi')$ are given by

$$\frac{r}{R} = \Gamma(\theta,u),\ \ \theta' = \theta,\ \ \phi' = \phi,\ \ \frac{t}{T} = \Gamma(\theta,u), \tag{6.1}$$

with

$$\Gamma(\theta,u) = \frac{\sqrt{1-\beta^2\sin^2\theta} + \beta\cos\theta}{\sqrt{1-\beta^2}}, \tag{6.2}$$

and

$$\Gamma^{-1}(\theta,u) = \Gamma(\theta,-u) = \frac{\sqrt{1-\beta^2\sin^2\theta} - \beta\cos\theta}{\sqrt{1-\beta^2}}. \tag{6.3}$$

The transformations (6.1) will be referred to as the scaling transformations. On account of the equations

$$\Gamma(0,u)=\sqrt{\frac{1+\beta}{1-\beta}}\equiv\Gamma(u),\ \Gamma(\pi,u)\equiv\Gamma^{-1}(u), \tag{6.4}$$

the transformations (6.1) reduce to the contraction-elongation relations

$$X=\Gamma^{-1}(u)x,\quad Y=y=0,\quad Z=z=0\quad (X>0) \tag{6.5}$$

for $\theta=0$, and to

$$X=\Gamma(u)x,\quad Y=y=0,\quad Z=z=0\quad (X<0) \tag{6.6}$$

for $\theta=\pi$. The relation (6.5) corresponds to the case in which the body is receding from the observer, and the second, (6.6), to the case in which the body is approaching the observer. For $\theta=\frac{1}{2}\pi$, the transformations (6.1) reduce to the identity transformations in the plane containing the source and perpendicular to the velocity vector

$$X=x=0,\quad Y=y,\quad Z=z. \tag{6.7}$$

In the last case there is no contraction or elongation effect.

Using the passive view, the relations (6.1) hold between the units of length and time in *S* and *s*, and the scaling transformations between *S* and *s* assume the form

$$R/r=\Gamma(\theta,u),\qquad \theta=\theta',\quad \phi=\phi',\qquad T/t=\Gamma(\theta,u), \tag{6.8}$$

with $(R,\theta,\phi)$ and $(r,\theta',\phi')$ are the coordinates of the body *b* in *S* and *s* respectively.

# 7. Comparison with Experiment

## Lifetime of Meta Stable Particles

The $\mu -$meson particles have a short lifetime $\tau \approx 2.10^{-6} s$, during which and even it moves with the velocity of light, it can covers only a distance $d \approx c\tau = 0.6km$. The $\mu -$meson particles are generated at an altitude $D = 60km$ with a very high speed close to that of light. In spite of its short lifetime, these particles can be detected abundantly at the earth surface. According to the active view, the distance of an $\mu -$meson particle generated at an altitude $D$ and approaching the earth's surface shrinks to a value $d = \Gamma^{-1} D$, and in order to reach the earth surface the particle should possess a velocity $v$ such that

$$\sqrt{\frac{1+\beta}{1-\beta}}0.6 > 60. \tag{7.1}$$

This yields $\beta > 0.99980002$, which is a probable value for the speeds of such particles. Relative to an observer at an altitude *D=60km* and stationary with respect to the earth, the particles heading towards the earth surface are receding away from him, and hence $D = \Gamma d$, which is equivalent to the formula used above.

### The Drag Effect

This effect[3] is explained in the SRT using the law of velocity addition, which is also valid in the current theory as we show here: Let $x > 0$, and $S$ be moving with velocity $v(v > 0)$ relative to a third reference frame $S'$ whose origin is contiguous to $B$ at the instant of light's emission. Adopting the passive view, we find that the transformations from *s* to $S'$ is

$$X' = \Gamma^{-1}(v)X = \Gamma^{-1}(v)\Gamma^{-1}(u)x = \Gamma^{-1}(V)x, \tag{7.2}$$

where

$$\Gamma(V) = \sqrt{\frac{1+V/c}{1-V/c}}, \tag{7.3}$$

and

$$V = \frac{v+u}{1+uv/c^2}. \tag{7.4}$$

Equation (7.3) is interpreted as asserting that the frame *s* moves relative to $S'$ with velocity *V* given by (7.4), which is the same law of velocity addition in special relativity.

## Doppler's Effect

Let $S \equiv OXYZ$ and $s \equiv oxyz$ be inertial frames in standard configuration, and assume that *s* translates parallel to *OX* with a constant velocity *u* (*u*>0). Let *b* be a source of light that is stationary in *s*, and hence moving with a constant velocity $\vec{u} = u\vec{i}$ relative to *S*. Suppose that the source *b* is radiating a monochromatic light of a characteristic wave-length $\lambda_0$. The light emitted from *b* is received by any *s* observer and in particular by the observer *o*, as a monochromatic light of the same wave-length $\lambda_0$. If $(R, \theta, \varphi)$ and $(r, \theta, \varphi)$ are the spherical coordinates of *b* in *S* and *s* respectively, then at any instant of actual observation,

$$R = \Gamma(\theta, u)r, \quad \theta = \theta, \quad \phi = \phi, \tag{7.5}$$

where the passive view is adopted. If the distance *r* corresponds to one wave length $\lambda_0$ in *s*, then the distance *R* corresponds to one wave length $\lambda$ in *S* (which is the distance between two nodes for example). Setting $R = \lambda$ and $r = \lambda_0$ in (7.5) yields the generalized Doppler's formula

$$\lambda = \frac{\beta \cos\theta + \sqrt{1 - \beta^2 \sin^2\theta}}{\sqrt{1-\beta^2}} \lambda_0 \tag{7.6}$$

which determines the wave length as measured by the stationary observer *O*. Note that the radiating source here is at a position of azimuth angle $\theta$, and that the polar axis is *OX*. The last relation shows that $\lambda > \lambda_0$ for $0 < \theta < \frac{1}{2}\pi$, and $\lambda < \lambda_0$ for $\frac{1}{2}\pi < \theta < \pi$. The source *b* is receding from the observer in the first case, and approaching it in the second.

The generalized formula (7.6) reduces, for $\theta = 0$, to the red shift Doppler's formula

$$\lambda = \sqrt{\frac{1+\beta}{1-\beta}} \lambda_0 \tag{7.7}$$

corresponding to the source and the observer receding from each other. For $\theta = \pi$, the relation (7.6) reduces to the blue shift Doppler's formula

$$\lambda = \sqrt{\frac{1-\beta}{1+\beta}} \lambda_0 \tag{7.8}$$

corresponding to the source and the observer approaching each other. For $\theta = \frac{1}{2}\pi$, the relation (7.6) reduces to

$$\lambda = \lambda_0, \tag{7.9}$$

which, contrary to the relativistic prediction, shows that there is *no traverse* Doppler's effect.

## The Sagnac's Effect

Consider two electromagnetic waves emitted from a point at the earth's equator parallel to the equator and in opposite directions. It is experimentally verified that the spinning of the earth about its axis amounts for a complete round to approximately 207 *ns* advance (delay) for a wave propagating westward (eastward) parallel to the earth's equator[5].

Let $S \equiv OXYZ$ be the inertial frame of fixed stars with origin *O* at a point *o* of the earth's equator, and take *OX* tangent to the equator at *o* and directed eastward, so that the linear velocity *u* of *o* in *S*, when *o* is contiguous to *O*, is positive. Suppose that two pulses of light are emitted simultaneously from *o* in opposite directions parallel to the equator. Let $X_e$ ($X_w$) be an *S* observer that is contiguous to *o* when light emitted is received back by *o* and hence by $X_e(X_w)$. In other words, *o* and $X_e$ (*o* and $X_w$) are conjugate observers when light emitted eastward (westward) is received by *o*.

The path of light circling the equator can be decomposed into straight segments with two conjugate observers, an *S* observer and an equatorial observer, at the end of each segment. The problem can thus be visualized as a linear one. Let $s \equiv oxyz$ be an inertial frame in standard configuration with *S* and moving relative to *S* with velocity *u* $(u > 0)$. We may envisage the pulse emitted eastwards from (*o* and *O*) and received by (*o* and $X_e$), as if received by the conjugate *S* and *s* observers ($o'$ and $X_e$), with $o'$ on the negative *x*-axis of *s* and at a distance *x* from *o* that is equal to the circumference of the earth, as measured in *s*. For the eastward trip the emitter is receding from the observer $X_e$, and the relatively absolute units of time in *S* and *s* are in the proportion $1:\Gamma(u)$. The westward trip can be viewed as a trip starting from (*o* and *O*) and ending at ($o''$ and $X_w$), with $o''$ on the positive side of the *x*-axis and at a distance *x* equal to the earth circumference as measured in *s*. For the westward trip the emitter *o* is approaching the observer $X_w$, and hence the absolute units of time in *S* and *s* are in the proportion $1:1/\Gamma(u)$.

It is important to note that the *S* time is the time read by our clocks on earth. To the unit "second" of the *S* time, there correspond two relatively absolute units of time in *s*, which we name the east and west equatorial seconds. Let's denote the latter absolute units by *Esec* and *Wsec* respectively. According to the passive view of interpretation, the latter absolute units relate to the unit of time in *S* by

$$1\sec = \Gamma(u).W\sec,\ 1\sec = \Gamma(-u).E\sec. \tag{7.10}$$

Similar relations hold for the units of length, say "meter":

$$1m = \Gamma(u).Wm,\ \ 1m = \Gamma(-u).Em. \tag{7.11}$$

Let $t_e$ be the geometric duration of the eastward trip $(o\,at\,O \to o'\,and\,X_e)$ in *s*. Since the length of the trip in *s* is the earth's circumference in absolute *s* units, we have

$$t_e = \frac{circum}{c} = \frac{40{,}000\,(Ekm)}{300{,}000(Ekm/E\sec)} = \frac{2}{15}E\sec. \tag{7.12}$$

Similarly $t_w = \frac{2}{15}W\sec.$ The difference between these

$$\begin{aligned} t_e - t_w &= \frac{2}{15}(E\sec - W\sec) = \frac{2}{15}(\Gamma(u) - \Gamma(-u))\sec \\ &= \frac{2}{15}\frac{2u/c}{\sqrt{1-(u/c)^2}}\sec \approx 2\times 206.33n\sec, \end{aligned} \quad (7.13)$$

represents the delay period between receiving the two waves. When calculating (7.13), the linear velocity $u$ of a point of the equator was taken

$$\begin{aligned} u &= 40{,}000km/23h56\min\times 60(\min/h)\times 60(s/\min) \\ &\approx 0.46425km/s \end{aligned}$$

## Michelson and Morley Experiment

Perhaps there is no experiment in physics' history that was studied, analysed, and disputed as much as was the Michelson's and Morley's experiment (MM for short). The experiment was designed to detect the earth's motion through the ether by measuring the difference in time taken by light to make from a point, 2-way trips along two perpendicular axes, one of which points in the direction of the almost translational motion of the earth around the sun and the other is perpendicular to it. The observed effect was much less than the expected one. Similar experiments were carried out by other scientists[1], and the same result was found: the observed fringe shift is much less than the calculated one.

In this article we argue that the expected effect in the MM and similar experiments is due to the rotational motion of the earth about its axis, but not to its orbital motion around the sun. This argument is based on the following facts:

- There is no ether in the scaling theory,

- The time we use on earth is the time of the frame *S* translating with the earth in its orbital motion, and hence there can be no fringe displacement due to the earth's orbital motion.
- A frame with origin at the earth surface and rotating with the earth can be considered during a short period of time an inertial frame that is translating relative to S with the linear velocity of its origin, and hence its units of length and time are different from those of S.

In the following treatment, it is assumed that the reader is well informed of the MM experiment which can be found in most text books on special theory of relativity[1-3] .

The frame *S*, with origin *O* at the earth's center, which does not rotate relative to distant stars can be considered inertial, for it executes within a small period of time only a translational motion. Relative to *S* the earth spins about its axis with a constant angular velocity, and the linear velocity of the a point *o* of the earth's surface is $u \sin\theta$ where *u* is the linear velocity of a point of the earth's equator, and $\theta$ is the azimuth angle of the point *o*. For simplicity we assume temporarily that the experiment is carried out at the earth equator, with one of the arms is pointing eastwards and the other northwards. Let $s \equiv oxy$ be a frame rotating with the earth, with origin at the light's source *o*, the *x*-axis pointing eastwards, and *oy* northwards. For a trip of light along *oy*, southwards or northwards, the units of length in *S* and *s* are equal. The units of length (and time) in *S* and *s* are respectively in the proportion $1:\Gamma(u,0)$ for an eastwards trip and in the proportion $1:\Gamma(u,\pi) = 1/\Gamma(u,0)$ for an westwards trip. Assuming that the arms are oriented initially eastwards and northwards respectively, and that the length of each arm is *l*, then the difference in the light's path will be

$$\Delta = (l.Emeter + l.Wmeter) - 2l.meter\,, \tag{7.14}$$

or

$$\Delta = l\left(\sqrt{\frac{1+u/c}{1-u/c}} + \sqrt{\frac{1-u/c}{1+u/c}}\right) - 2l$$

$$= 2l\left(1-(u/c)^2\right)^{-1/2} \approx l(u/c)^2\ meter\,. \qquad (7.15)$$

When the interferometer is rotated by a right angle the difference doubles giving rise to a fringe shift

$$f = \frac{2l}{\lambda}\left(\frac{u}{c}\right)^2 . \qquad (7.16)$$

In the MM experiment

$$\lambda = 6\times 10^{-7}\ meter\,,\ l = 120\,cm\,.$$

Substituting $u = 0.46425\,km/s$ and $c = 298792.5\,km/s$ we obtain

$$f \approx 0.00001,$$

which is just (1/4000) of the commonly predicted value, and (1/1000) of the observed result, which is 0.01 fringe.

Going through the results of various trials of the MM experiments listed in Wikipedia, or in French[1], we note that the observed effect corresponds to a velocity *u* that is always greater than the rotational speed of the earth about its axis. According to the Wikipedia, the least upper limit of *u*, which is therein the orbital velocity of the earth, while it is the linear rotational speed of the earth in our argument, occurs in the Illingworth's trial (1927) with *u* is less than one kilo meter per second. The second reasonable upper limit is found in Joos' trial (1930), with $u = 1.55\,km/s$. In fact the MM experiment yields a fringe shift corresponding to an orbital speed $u = 4.75\,km/s$, which is still closer to the value of the linear rotational speed,

$(0.46425 \sin\theta) km/s$, of the location at which the experiment is carried out.

## 8. Lorentz Transformations

Let *S* and *s* be inertial frames as prescribed in section 2, and assume that *s* is moving relative to *S* with velocity *u* (*u*>0). Let *o* and *x* be points in *s*, with $x > 0$. At the instant $t = 0$ in *s* a source of light *b* at *x* emits two pulses, a pulse (+) in the +*x*-direction and a pulse (-) in the –*x*-direction. At an instant *t* in *s* the pulse (+) reaches a point $p_+$ in *s* with coordinate $x + ct$, while the pulse (-) reaches a point $p_-$ with coordinate $-x + ct$ in *s*. When the pulses ($\pm$) reach the observers $p_\pm$, at the instant *t* in *s*, there exist two *S* observers $P_\pm$ that are contiguous to $p_\pm$. The observers $p_\pm$ can consider light reaching them from *x* as emanated from *o* at the instants $\mp x/c$ respectively. When light is emitted from *x*, at *t*=0 in *s*, there exist two points *X* and *O* in *S* that are contiguous to *x* and *o* respectively. The light received by the *s* observers $p_\pm$ and by the conjugate *S* observers $P_\pm$ can be considered by the *S* and *s* observers as emitted from *x* or from the point *X* in *S* that was contiguous to *x* at the instant of light emission. The *S* observers are at liberty to start the clock at *X* at any time they wish, say $T = 0$, and synchronize the rest of their clocks with the *X*-clock in the natural procedure of light synchronization[3]. There corresponds to the period *t* in *s* during which the pulses emitted from *x* reached $p_\pm$ a period *T* in *S* during which the pulses emitted from *X* reach $P_\pm$. When the pulses reach $P_\pm$ the clock at *X* reads *T*. The observers *S*, can consider light reaching $P_-$ as emanated from *X* at an instant $T = 0$, or from *O* at an instant $T = X/c$. The coordinate of

$P_-$ when light is received is $-c(T-X/c)=X-cT$. The light reaching $P_+$ can be considered by the observers $S$ as had been emitted from $X$ at $T=0$, or from $O$ at $T=-X/c$. The coordinate of $P_+$ at light reception is $X+cT$. Thus the $S$ observers associate with the light trip $(o \to p_- \; and \; P_-)$ the period $t-x/c$ and with the light trip $(O \to p_- \; and \; P_-)$ the period $T-X/c$, and since $o$ is receding from $P_-$, the contraction-elongation relation yield

$$T-X/c=\Gamma^{-1}(t-x/c). \tag{8.1a}$$

The $S$ observers assign to the light trip $(O \to p_+ \; and \; P_+)$ a period $T+X/c$ and to the light trip $(o \to p_+ \; and \; P_+)$ the period $t+x/c$. Since $o$ is approaching $P_+$ the $S$ observers relate the latter periods, according to the contraction elongation relation, by

$$T+X/c=\Gamma(t+x/c). \tag{8.1b}$$

Solving equations (8.1) for $X$ and $T$ we obtain the Lorentz transformations (LT)

$$X=\frac{x+ut}{\sqrt{1-\beta^2}}, \qquad cT=\frac{ct+ux/c}{\sqrt{1-\beta^2}}. \tag{8.2}$$

The method used to derive LT promotes the following comments:

(i) Equations (8.1) are valid only after the observers $O$ and $o$ have received the wave front (-) mentioned above. Assuming $X \geq 0$, we should have $X-cT \leq 0$, which implies that $0 \leq X \leq cT$. The last inequality combined with the result of a similar argument concerning the case $X \leq 0$, renders LT valid in the domain

$$|X| \leq cT. \tag{8.3}$$

Therefore, LT is valid only for time-like and null intervals. The physical meaning of the latter statements is that, the pulses received by $P_+$ and $P_-$ can be considered as have been emitted from *any* $X$ that satisfies (8.3) for a given $T$.

The LT (8.2) reduce to

$$X = \Gamma x,\ X = cT \ \text{for}\ t = x/c\,(x > 0), \tag{8.4}$$

and to

$$X = \Gamma^{-1} x,\ X = -cT \ \text{for}\ t = -x/c\,(x < 0). \tag{8.5}$$

(ii) Since $T$ is always positive, it is not legitimate to consider the invariance of LT under time inversion.

(iii) Objections may rightfully be raised claiming that the constraint (8.3) is a consequence of the particular method followed to derive LT. In reply, we note that the LT (8.2) is algebraically equivalent to (8.1), and hence a tangible interpretation for (8.1) must be sought. In fact, the factors $\Gamma$ and $\Gamma^{-1}$ appearing in the form (8.1):

$$X + cT = \Gamma(x + ct), \quad X - cT = \Gamma^{-1}(x - ct)$$

of LT are precisely the factors sought by Einstein in his simple derivation[13] of LT.

## 9. The Generalized Lorentz Transformations

Let $S \equiv OXYZ$ be an inertial frame endowed with a system of spherical coordinates $(R, \theta, \phi)$, with $\theta$ is the azimuth angle between the $OX$ axis and the radius vector $\vec{R}$. Consider a body $b$ moving relative to S with velocity $\vec{u} = u\vec{i}\ (u > 0)$, with $\vec{\mathrm{i}}$ is the unit vector of $OX$ and $(u > 0)$. In a way very much similar to that followed in deriving the restricted LT, a general form of Lorentz transformation

(GLT) relating the geometric quantities $(R,\theta,\phi;T)$ and the mobile quantities $(r,\theta',\phi';t)$ can be obtained[12]; it is thus written

$$R=\frac{\sqrt{1-\beta^2\sin^2\theta}\,r+\beta\cos\theta\,ct}{\sqrt{1-\beta^2}} \tag{9.1a}$$

$$\theta=\theta',\qquad \phi=\phi' \tag{9.1b}$$

$$cT=\frac{\sqrt{1-\beta^2\sin^2\theta}\,ct+\beta\cos\theta.r}{\sqrt{1-\beta^2}}. \tag{9.1c}$$

The transformation (9.1) are valid in the domain

$$R=|\vec{R}|\leq cT\,. \tag{9.2}$$

One consequence of the last constraint on validity region of GLT is that, choosing a common origin of two frames of reference is not a passive process, for it entails a specific restriction on the possible values that can be assigned to spatial and time coordinates, which is due to mutual observation of the same light trip by *O* and *o*. Moreover, and as it is shown below, the relative motion of an observed object and an observer, if one dimensional, then LT is reduced to a transformation of motion in one-dimensional space. The GLT preserve the Minkowski metric. i.e.

$$c^2T^2-R^2=c^2t^2-r^2. \tag{9.3}$$

For $\theta=0\ or\ \theta=\pi$ the GLT reduce to

$$X=\frac{x+ut}{\sqrt{1-\beta^2}},\qquad cT=\frac{ct+ux/c}{\sqrt{1-\beta^2}}, \tag{9.4}$$
$$Y=y=0,\qquad Z=z=0.$$

For $\theta = \pi / 2$ , the LT assume the form

$$X = x = 0 \qquad Y = y \qquad Z = z \qquad T = t . \tag{9.5}$$

### Velocity Addition Once More

If a particle has an instantaneous velocity *v* along the radius vector in *s* then its instantaneous velocity in *S* is also radial and has the algebraic magnitude

$$V = \frac{\sqrt{1-\beta^2 \sin^2 \theta}\, v + u\cos\theta}{\sqrt{1-\beta^2 \sin^2 \theta} + uv\cos\theta / c^2} . \tag{9.6}$$

The relation (9.6) is obtained through dividing the differentials of (9.1a) and (9.1c), with $\theta', \phi'$ kept constants because the motion is radial. The relation (9.6) reduces to the familiar formula (7.4) of velocity addition in SRT for $\theta = 0$ or $\theta = \pi$ . For $\theta = \pi / 2$, we have $V = v$. Moreover, the relation (9.6) guarantees that *c* is invariant. Indeed and setting $v = c$ in (9.6) gives $V = c$. The last result, by no means, implies that *c* is the maximum speed in nature; it implies however that (9.6) is valid, in the same way as the theory itself, for motions with velocities that doesn't exceed *c*. The speculation that *c* is the maximum speed in nature should be postulated independently.

## Conclusion

We have shown that the postulate of SRT regarding light velocity is a result of a weaker postulate, and that optical effects that formed a challenge to the pre-relativistic era can be explained in a simple manner using the scaling formulae obtained above. It was also demonstrated that the same formulae lead to LT confined to the region of time–like and null intervals. The expressions of GLT were listed and briefly commented on. The concept of the 3-physical space introduced in this work is the subject of an expound study in subsequent works[7,12].

## References

[1] A.P. French. *Special Relativity*, Ch 2&3, Butler & Tanner Ltd. (1968).

[2] D.F. Lawden. *Special Relativity and Tensor Calculus*, Ch 2, Chapman and Hall. (1974).

[3] Richard A. Mould, *Basic Relativity*, Ch 1& 2, Springer-Verlag. (1998).

[4] W. Rindler, *Essential Relativity*, Springer-Verlag, Ch 3. (1977) 28pp.

[5] Daniele Russo, "A Critical analysis of special relativity in light of Lorentz's and Michilson's ideas", Apeiron **13** (July 2006).

[6] Daniele Russo, "Stellar aberration: the contradiction between Einstein and Bradley", Apeiron **14** (August 2007).

[7] C.P. Viazminsky, "The scaling transformations: theory and applications (I)", Research Journal of Aleppo University, to appear.

[8] C.P. Viazminsky, "Implications of the contiguous equivalence on inter-bodies time and distance", Research Journal of Aleppo University **38** (2003).

[9] C.P. Viazminsky, "Inter-bodies time and distance", The General Science Journal (16 August 2003).

[10] C.P. Viazminsky, "Generalized Lorentz transformations and restrictions on Lorentz transformations", Research Journal of Aleppo University **54** (2007).

[11] C.P. Viazminsky, " The Scaling theory", to be submiteed to Apeiron.

[12] C.P. Viazminsky, "The scaling transformations: theory and applications (II)", submitted to Research Journal of Aleppo University.

[13] A. Einstein, *The Special and General Relativity,* Appendix 1 (simple derivation of Lorentz transformations) ,The Project Gutenberg eBook of Albert Einstein Reference Archive. (2004).