Mechanical Problem of Ether

Nicolae Mazilu, PhD|
Silver Lake, Ohio, USA

The Ether is, as it always was, unbearable because it has contradictory properties: incompressible, having a huge stiffness in order to account for the speed of light, yet not opposing to the motion of bodily Matter. The stiffness of Matter has always been associated by our senses with its impenetrability. The prospect of reality as we know it strongly indicates that we can be well misled in our conclusions when mixing criteria based on senses with criteria of pure reason. The concept of Ether serves as a lesson whose introduction was written by Huygens and whose content was developed by Fresnel. We show here that the electromagnetic structure of Ether is unavoidable, and that it has actually a deeper meaning than the electromagnetism, as we know it today, is capable to show.

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Introduction

One might say that Fresnel moment in human knowledge has a meaning: to eliminate altogether the mechanical description from the considerations concerning the Ether. This task has apparently not been accomplished, or at least it has not been accomplished thoroughly, and this is one of the main reasons the problem of Ether recurs at times. Our concern in this essay is if Ether should stay. The Special Relativity says no, the General Relativity says yes, and these are basically the two fundamental lines of thought involved in the argument. Electrodynamics can follow either one of them.

It should then come as no surprise that, while Electrodynamics provided substantiation in rejecting the Ether at the beginning of the century past, it can turn into an argument in accepting the Ether at the end of that century. One of the proponents of this thesis about Ether is H. E. Wilhelm [14, 15]. The most important programmatic point brought by Wilhelm into argument is, in our opinion, the connection of the problem of Ether with the Cosmic Background Radiation: this requires a radically different approach, indeed more on the side of Electrodynamics, making this science one of the fundamental lines of thought ranking equal with Special and General Relativity. The ensuing polemics or the lack thereof, shows that Wilhelm's attempt was not clearly understood. One can clearly account for this by the fact that any point of view in this problem is attached to either Special or General Relativity lines of argument as the fundamental ones.

We need to recall however that at the time when the concept of Ether was rounded up, Electrodynamics was one of the fundamental ideas related to it. So there should come as no surprise the fact that it aspires to the place it naturally had as a birth right. On the other hand, this very fact seems to indicate that the problem of Electrodynamics is as deep as the idea of Ether is, and therefore it needs to be traced back to the very origins of this concept. And as these origins are tied up with the phenomenon of light, it seems indeed only natural to start with one of the first dynamical theories of light, that of Fresnel [3, 4, 12].

It is here the point where, for the first time, the very principles of Newtonian Dynamics have been put to stand for a trial, but the essentially accountable party, the concept of force, has always managed to escape this trial. For, at the time when the concept of force in problems of refraction has been considered by Fresnel, it has been approached not as such, but exclusively in relation with the Second Principle of Newtonian Dynamics, which was in the epoch the main line of argument. However, the Newtonian Dynamics had resources to strike back in a hidden way. First of all it has been recognized that there is no human possibility of describing the situation other than by Mechanics. Then, as only the Second Principle of Dynamics seemed to be involved here, the most obvious thing to do is to extend the idea of inertia. This has been done with the so-called gyrostatic model of Ether. However, another important thing done within the framework of Mechanics was to realize that Dynamics has also some other principles suited to the task. All things considered, the Mechanics was therefore resourceful in the problem of Ether so that, while it was imperatively necessary for it to quit the stage, it actually found itself even more deeply involved.

In order to better understand the issue one has to refer the discussion to an appropriate work, which was a product of the debating on the problems regarding the Ether. Too early works might not be suitable, for in such times the concept was not rounded up so to speak. On the other hand, contemporary works might not be suitable either, for they are certainly biased by the mirage of Mathematics. So we have to refer to a work that intended to be a summarization of the status of Ether problem when the intellect realized that the time was ripe for such an action. We think that the champion of the problem of Ether in its entirety was Larmor: one can find no other among the classics more to the point than him in revealing all sides of it. Larmor captured exactly the moment when the Mechanics was on the brink of yielding and passing the problem to Electrodynamics, and that moment was fixed, as it should naturally be, by a thorough assessment of the contribution of Mechanics to the problem. This is why we start our discourse from some of Larmor's observations regarding the Ether.

Concerning *the physical grounds* of the problem in general we have to mention that it hardly appears explicitly in the literature: rather, one talk indeed only of the mechanical or electromagnetic grounds. It was clearly understood from the beginning that the problem has a natural connection with Newtonian Dynamics. One has to insist on reading between the lines, so to speak, in order to make up an idea. The insistence is certainly rewarding, for once in a while there are observations, stating it almost explicitly. Case in point: consider this footnote of Larmor in his Preface to "Aether and Matter" [6]:

> *"It is not superfluous to repeat here that the object of a gyrostatic model of the rotational ether is not to represent its actual structure, but to help us to realize that the scheme of mathematical relations which defines its activity is a legitimate conception. Matter may be and likely is a structure in the aether, but certainly aether is not a structure made of matter. This introduction of a suprasensual aethereal medium, which is not the same as matter, may of course be described as leaving the reality behind us: and so in fact may every result of thought be described which is more than a record or comparison of sensations." (lc. cit. Preface p. vi).*

This quotation requires further explanation to be developed in this essay. For the record though, regarding the physical ground of our problem, the Ether is a category not given by sensations, but nevertheless making its presence known to reason. In this respect the previous extract from Larmor is a genuine program that unfortunately has not been followed ad litteram. Here, he is specifically addressing the case of the gyrostatic model of Ether, arduously promoted by Lord Kelvin. One might say that through it the Second Principle of Dynamics is still in force inasmuch as this mechanical model avoided explicitly only the polar inertia, i.e. that inertia due to the direct action of a force on a material point, and replaced it with a rotational inertia: the gyrostatic Ether is a medium inertially sensitive to rotations. This is one of the backlashes of the Second Principle of Dynamics we were just mentioning above.

On the other hand, it becomes more obvious from the quotation above that what one might dispute here, from a *general physical point* of view, is the right to mix the reasons. Since the Classical Physics, as represented by Newtonian Dynamics, has filled these reasons with just their practical side essentially represented by the concept of force, it seems to be no question about the legitimacy of the mechanical approach. It is nonetheless our opinion that the pure reason – to speak in Kantian terms – should have a substantial share when it comes to the description of the Ether because, as Larmor says, it is suprasensual. In other words, the force is a quintessence of the reality as perceived by man, but the Ether should be described "as leaving the reality behind us"; therefore, by a common logical inference, leaving the reality means leaving the force.

Another point to be noticed here is the definition of Ether by its relation to Matter: this last one is a structure in Ether. This is a unilateral relation, is true, but nevertheless it indicates the fundamental fact that the concept of Ether has two main determinations: Ether in Space and Ether in Matter. It is only the first one that is not accessible altogether to our senses, while the second determination shows in what circumstances it is accessible: only second-hand so to speak, through Matter.

Larmor's Aether and Matter, Appendix B

Because we started with Larmor, let's continue on with him, for he seems indeed the most explicit among the classics in regards to human possibilities when it comes to the description of Ether. Not just by chance, Larmor chooses the Mechanics as the science fit to do the job: as a matter of fact this was the universal instrument of Physics of the 19th century.

A fact never mentioned is that the concept of Ether revealed the very limits of Mechanics, whose whole content is most clearly expressed in the Appendix B of the Larmor's very "Aether and Matter": ([6], pp. 269 – 288). It is to be stressed from the very beginning that Larmor understands by Mechanics *"...dynamics of matter in bulk, in contrast with molecular dynamics"* (lc. cit. Preface, p. xiii). He recognizes that a mathematical theory of Ether within this general Mechanics could well go beyond the Second Principle but couldn't go however beyond the *Third Principle of Dynamics*. Indeed, it is this principle and that of D'Alembert that are taken as fundamental in description of the "material systems treated as continuous systems instead of as molecular aggregates". And these seem to be the only principles left when dealing with matters unknown to our senses, such as the Ether. They are then reformulated by Larmor as follows ([6], pp. 268 – 269):

> *1) "The mechanical action and reaction between any two parts of a material system, which are capable of separate permanent existence, must compensate each other, and therefore must have for their statical resultants equal and opposite wrenches on the same axial line"*
>
> *2) "... if we set down the effective forces which would directly produce (...) motions in (...) separate parts or*

> *differential elements of volume (...) considered by themselves as individually continuous but mutually disconnected, then for each part finite or infinitesimal (...) these forces are the statical equivalent of the actual forces acting in or on that part either from a distance or through the adjacent parts".*

In other words, all things considered, the concept of force is maintained through any theory of Ether. *"The foundation on which the whole subject is developed lies in the notion of force"* (ibidem, p. 271). This may come as no surprise when one talks about Mechanics. However, in our opinion it is this concept of force which has been attacked by Fresnel, for it is not quite convenient from a general *physical* point of view. Only, Fresnel made this attack within the limits of the Second Principle of Dynamics, inasmuch as he challenged the correctness of the current mechanical description of Ether in that epoch. Thus, apparently the concept comes back with the Third Principle: the instrument through which the force is "interrogated" so to speak is the action-reaction principle.

One might say that this fact confers some kind of transcendence to this principle: it does not belong just to Mechanics but to the human experience in its entirety. From this point of view the D'Alembert principle is only an addition allowing us to broaden the realm of the concept of force itself. This is, for instance, why Larmor finds necessary to explain the difference between the two kinds of forces entering the D'Alembert's principles: external or impressed forces and internal forces, of the inertia kind. These last forces are linked directly by Classical Mechanics to virtual displacements. While this distinction is truly necessary for the development of any explanatory theory, the essential fact to be retained here is that, no matter if internal or external, these categories are nevertheless forces; their action is identically described, the main characteristic of this action being its polarity. Maintaining forces allows by no means "leaving the reality behind us".

It is then only natural that we should know about Ether just as much as we know about inertia itself: almost nothing! Moreover, insisting upon the extension of the realm of concept of force, not only, as it seems to be philosophically clear and sound, every action has to have a reaction according to the determination of the concept of force contained in the Third Principle, but this reaction has to be of the nature revealed to us by senses – a force. One might say that, because the Third Principle of Dynamics is a kind of general principle transcending Mechanics, this last one has a lot of room to go around, so that it cannot be eliminated from the philosophical discussion of Ether. However, it should be emphasized that its hard currency here is the notion of force, and this is first and foremost the point that needs to be revised.

Going in a greater depth, one can say that even today the background idea in the problem of Ether is that, whenever the intellect describes this “result of thought”, recognized to be a “suprasensual medium”, it does so by concepts built as results of pure sensations, once they are based on the notion of force. While this is classically seen as the advantage of a sound philosophical attitude, we are nowadays in position to disagree. There was a contradiction here, and all we can say is that the spirit of this contradiction is maintained all through the history of electricity and magnetism, so that the discussion about Ether have never ceased. Lately it has been renewed but, as always when such a work has been undertaken, with attacks upon Special Relativity which, in our opinion, has nothing to do with the subject. But, what can we say? Special Relativity is a fashion, and it is only fashionable to attack it any way it can be done, even politically!

Poincaré's Critique and a Programmatic Conclusion

Among the few of the lucid critical studies, we hold that of Poincaré [8] in very high consideration. Not only for its minute detail in everything regarding the subject, but for discussing the problem thoroughly from the side of Electrodynamics, and for his close-keeping with principles just enunciated of the trade. Specifically Poincaré strives to judge every theory to date from three points of view:

The dragging of light waves

Conservation of electricity and magnetism

Mechanical principle of equality of action and reaction

The first two criteria are specific to the field of Electrodynamics and it may be interesting to note in passing that Poincaré's basis for judgment is a transport theorem. This started to be closely reconsidered in some new versions of Electrodynamics. However, the third criterion does not belong to Electrodynamics: it comes from Mechanics and, as we noticed before, it transcends this science, thus giving to Poincaré full right to use it as it stands for critical purposes.

It is here, however, the point where Poincaré went far beyond Larmor, or anyone of the classics for that matter, and we need to emphasize this moment of knowledge while assigning to it a special significance. The results of Poincaré show that Lorentz's theory of Electrodynamics is the only one which *does not satisfy* this essential Principle of Dynamics, according to which the action of Ether should be equal to its reaction from within Matter formations that are "capable of separate existence" as Larmor puts it. However, everyone knows that Lorentz's theory has plainly other virtues and it is worth saving it. Following the Poincaré's line of reasoning, one of attempts to save it, besides Special Relativity of course, is to update it in order to satisfy the action-reaction principle. It is at this juncture that one can place by now the notorious fact which, in our opinion, carries a special significance for the problem at hand, that Poincaré introduced some stresses of mechanical nature – the Poincaré stresses – in order to explain the Lorentz contraction and thus save the theory [9].

Come to think of it: the Continuum Mechanics of the last century teaches us that there is no deformation of material structures without accompanying stress. As a matter of fact Lorentz himself left his contraction without further specifying it as being a deformation, inasmuch as it is unaccounted for by a corresponding stress. Poincaré's procedure is therefore actually one of the classical examples of this sort, and perhaps the first of its kind at that, whereby Lorentz's transformation is given the determination of what it was first intended to be: a deformation. This logical step is, as the history shows, apparently not sufficient. As far as we can understand the issue, it just saves a dynamical principle which does not belong there: equality of action and reaction. *For there is no reaction*!

Indeed, it seems to us that if the Ether can be labelled as "suprasensual" it is especially because it has the capability of penetrating the Matter, i.e. because the Matter has no response to its extension. It is this response of the Matter that would mechanically qualify as reaction. And if, further, we call the Ether extension a strain, then it is a strain not accompanied by stress. Thus the Ether, inasmuch as it is characterized as a continuum, is a continuum that strains without producing a stress. On the other hand, what differentiates the Matter from Ether is the fact that it can stand stresses without apparent strain: those stresses producing strain only when 'relieved' so to speak. Now, as Larmor says, we can image the

Matter as made of Ether, but not reciprocally. Thus there is Ether in the form of Matter and Ether in free space, and we are in need of characterization of this continuum in its two instances. Therefore the concepts of stress and strain seem to be specially suited for pursuing Larmor's program of "leaving the reality". The Continuum Mechanics has here an instrument that seems to be perfectly fit for the job: *the constitutive law*.

Constitutive Characterization of Ether

The problem is, therefore, to characterize a continuous medium capable of withstanding stresses without strain when in Matter, and to exhibit strain under no stress when in free space. And we have nothing better at our disposal than the theory of stresses and strains, which is specially designed to deal with continuum problems.

Fact is that in Continuum Mechanics we work with second order tensors or, more general, matrices in order to represent stresses and deformations. These are strongly non-polar mathematical things, at least as long as we do not specify them in terms of fields of displacements and forces. Furthermore, when it comes to the reality of these things, it is guaranteed by the so-called constitutive law.

Let us elaborate a little on this concept. In broad terms, the constitutive law is a relationship between stresses and strains. As our representations of these concepts are by matrices, a constitutive law is simply a mathematical relation – algebraic or analytic – between two 3×3 matrices. If we denote by $\boldsymbol{\sigma}$ the matrix of stresses and by $\boldsymbol{\varepsilon}$ the matrix of strains, then a constitutive law is a relation of the form $\boldsymbol{\sigma} = \boldsymbol{\Sigma}(\boldsymbol{\varepsilon})$ where the function $\boldsymbol{\Sigma}$ is accessible to evaluation. Here we insist upon the meaning according to which $\boldsymbol{\sigma}$ is the applied stress while $\boldsymbol{\varepsilon}$ is the resulting strain. The reality we just mentioned above then relates to the identity of the material characterized by the constitutive law.

Indeed, it is claimed, in the modern science of materials, that the stress and strain matrices are universal mathematical tools while the function **Σ** is specific to the material upon which they are applied. One can see in the concept of stress, extended beyond the applied stress, a mean to *eliminate force in general*. Indeed, it is only the applied stress the one which is intimately tied up with the idea of force. Otherwise the stress can be thought of as a density of energy characterizing the Matter, occasionally quite independently of force. Therefore, if it is to extend conclusions due entirely to our senses to the description of a "suprasensual" Ether, then it is more appropriate to accept the idea that free space Ether deforms in any conditions, and if the Ether from Matter is acted upon in any way it responds by deformation which we describe by a matrix designated **ε**. Thus we are bound to find a function **Σ** that implicitly contains the physical nature of this continuum.

Now, a deeper insight into problem shows a specific feature of it: one has to deal here with *uncontrollable manifestations*. This is perhaps the main deep reason of maintaining the mechanical manner of thinking. It is indeed true that every action of ours is done by forces; in other words we cannot control but forces and, if anything else, through forces. It is, however, seldom noticed that in the framework of Mechanics, because of maintaining the forces as essential theoretical tools, there can be no uncontrollable quantities. This is exactly what has happened with the Ether along the time, as we showed in the succinct appraisal above. We think that the Ether theory is a critical field where we must recognize the existence of uncontrollable quantities and, most importantly, we must not describe them in the manner we describe the controllable quantities. In other words, we need to replace by something else the internal forces acting inside the “material systems treated as continuous systems” of Larmor. What can be done?

Well, it just happens that the most general idea of uncontrollability comes very naturally with a … natural constitutive law. Indeed, a constitutive law relating the stress and strain, must be of the form

$$\boldsymbol{\sigma} = p_0\mathbf{e} + p_1\boldsymbol{\varepsilon} + p_2\boldsymbol{\varepsilon}^2 \quad (1)$$

where **e** is the unit 3×3 matrix. We call this equation a natural constitutive law, on the grounds that it can be derived from the very basic considerations on our representations of stresses and strains. Indeed, if our models of stress and strain are 3×3 matrices and if the constitutive law is analytic, the equation (1) must be automatically in effect. For then the relation between the two matrices can be represented by a formal series reducible to a second order polynomial through Hamilton-Cayley theorem. By the same token, that relation can just as well be written with the places of stress and strain matrices interchanged. Thus, strain as a function of stress is also a quadratic function, only with other coefficients.

Now, the material has here a precise identity, for we can identify it by the coefficients p_0, p_1, p_2 which are accessible to experiments – the so called loading experiments. This is what one actually means by 'material characterization'. Often times in the actual practice these coefficients are considered *pure material* properties, but this restriction confuses the issues, sometimes with serious consequences mostly in engineering problems. Let us make this statement a little more explicit. No matter what these material properties are, equation (1) shows that in each and every one of the loading experiments the principal directions of stress coincide with the principal directions of strain. On the other hand, if $\sigma_{1,2,3}$ are the principal values of stress matrix, and $\varepsilon_{1,2,3}$ those of the strain matrix, according to the constitutive law (1) we must have satisfied the system

$$\begin{aligned}\sigma_1 &= p_0 + p_1\varepsilon_1 + p_2\varepsilon_1^2\\ \sigma_2 &= p_0 + p_1\varepsilon_2 + p_2\varepsilon_2^2\\ \sigma_3 &= p_0 + p_1\varepsilon_3 + p_2\varepsilon_3^2\end{aligned} \tag{2}$$

Assume, for the sake of argument, that we are able to perform experiments (which is actually an impossible task) allowing us to measure *all three* principal values of strain and stress simultaneously. Their outcome will then further allow us to calculate the material properties embodied in the coefficients $p_{0,1,2}$ from system (1). This system has a nontrivial unique solution if, and only if, the determinant

$$\begin{vmatrix} 1 & \varepsilon_1 & \varepsilon_1^2 \\ 1 & \varepsilon_2 & \varepsilon_2^2 \\ 1 & \varepsilon_3 & \varepsilon_3^2 \end{vmatrix} = (\varepsilon_2 - \varepsilon_3)(\varepsilon_3 - \varepsilon_1)(\varepsilon_1 - \varepsilon_2) \tag{3}$$

is non-null. Thus, the parameters p_0, p_1, p_2 are uniquely determined, regardless of the character of imposed stress, by the solutions of the system (2) if, and only if, the resulting principal deformations are different from each other.

However unique, and thus well suited for characterizing the material, the coefficients thus obtained are by no means *pure* material properties, inasmuch as they depend on the *impressed state of stress*. Therefore we are further required to make more precise what we understand by pure material properties, and this is, and indeed always was, an issue. However, this issue can be addressed by noticing that there are deformations even in case where there are no impressed stresses acting on our material. Inasmuch as we don't know their origin, *these deformations are some intrinsic properties the material*. They can be generated by forces of the presence of which we have momentarily no idea, therefore by those forces termed by Larmor as "internal", or else can be indeed true intrinsic properties that we still can model as stresses – internal stresses – having the physical meaning of energy densities. Limiting, when it comes to the description of Ether, the Mechanics as understood by Larmor only to external or impressed forces, i.e. accepting that there is no possibility to describe the action upon continuous parts by forces, leaves no alternative but to consider them as intrinsic properties of the continuum. In terms of system (2) they can be described by the system of equations:

$$\begin{aligned} 0 &= p_0 + p_1\varepsilon_1 + p_2\varepsilon_1^2 \\ 0 &= p_0 + p_1\varepsilon_2 + p_2\varepsilon_2^2 \\ 0 &= p_0 + p_1\varepsilon_3 + p_2\varepsilon_3^2 \end{aligned} \tag{4}$$

Then the material characterization by experiment is transferred to finding the solutions of this homogeneous linear system, in case they exist. As a matter of fact they always exist, we only have to decide just how many and this fact depends on what we really can always measure.

If we always measure *three* different deformations of the Ether in three orthogonal directions in space, then the Ether is not responsive to impressed stresses. That much we know from our historical experience: this is the main quality of the Ether that propagates light! However, there are also possibilities of solutions in which the Ether may be responsive to stresses, in other words its deformation is accompanied by stresses. Thus if we measure *one* and the same strain value in any direction, we have a double infinity of states of stress of Ether, depending on two material parameters. If we measure *two* strain values, and only two, in a direction and its perpendicular plane for instance, then we have states of stress of the Ether depending on one material parameter. Granting that we can include one of the material parameters into a measurable quantity, the most general constitutive law satisfied by the Ether exhibiting stresses under strain will be

$$\boldsymbol{\sigma} = K\left(\boldsymbol{\varepsilon} - \varepsilon_1 \mathbf{e}\right)\left(\boldsymbol{\varepsilon} - \varepsilon_2 \mathbf{e}\right) \tag{5}$$

where K is an arbitrary constant. Such a material has three uncontrollable quantities, out of which two are measurable.

In closing here, notice that as long as we are interested in just the measurable quantities, a convenient way to express a characteristic deformation matrix of a material exhibiting uncontrollable strains, is in the form of the tensor

$$\varepsilon_{ij} = \varepsilon_2 \delta_{ij} + \left(\varepsilon_1 - \varepsilon_2\right) l_i l_j \,, i, j = 1, 2, 3 \tag{6}$$

where $\hat{l}$ is a unit eigenvector, corresponding to the eigenvalue ε_1. Such a material has distinguished directional properties, with respect to the direction $\hat{l}$, and these properties are given by the eigenvalues ε_1 and ε_2. As a matter of fact, the equation (6) does contain both of the previous two cases as particulars, if we agree to characterize the intrinsic material properties as deformations. Notice that this is an assumption independent of the constitutive description and must be secured by our measurement capabilities. Thus we have this general conclusion: whenever a material deforms freely, i.e. under the action of no noticeable forces, its deformation matrix must be of the form given by equation (6), all the particular cases included. The deformations as well as the stresses are then manifestly tensors.

By the same token we can discuss the Ether in Matter: that category of Ether capable of sustaining stresses and exhibit no strain. It is indeed by this essential property that Matter comes first to our senses in the form of impenetrability. For this the converse constitutive law must be taken into consideration, namely

$$\boldsymbol{\varepsilon} = q_0 \mathbf{e} + q_1 \boldsymbol{\sigma} + q_2 \boldsymbol{\sigma}^2 \tag{7}$$

This time, however, $\boldsymbol{\sigma}$ may only abusively be called stress; let us just say that it is a tensor representing the internal energy in Matter. Then the defining state of such Ether will be characterized by the system of equations

$$\begin{aligned} 0 &= q_0 + q_1\sigma_1 + q_2\sigma_1^2 \\ 0 &= q_0 + q_1\sigma_2 + q_2\sigma_2^2 \\ 0 &= q_0 + q_1\sigma_3 + q_2\sigma_3^2 \end{aligned} \tag{8}$$

corresponding to no strain response. Again, the characterization of this Ether depends upon the number of solutions of this system. And the most general strain it exhibits is of the form

$$\boldsymbol{\varepsilon} = K_1^{-1}\left(\boldsymbol{\sigma} - \sigma_1\mathbf{e}\right)\left(\boldsymbol{\sigma} - \sigma_2\mathbf{e}\right) \tag{9}$$

where the constant K_1 has dimensions of a stress. It is perhaps of significance that the relation (9) with $\sigma_1 + \sigma_2 = 0$ has been found by Bell [1, 2] to be characteristic for metals, in large as well as small deformations: metals always struck our senses by their hardness.

Again, as long as we are interested in just measurable quantities characterizing such a material, then its intrinsic stress tensor assumes the following convenient representation, similar to (6)

$$\sigma_{ij} = \sigma_2\delta_{ij} + \left(\sigma_1 - \sigma_2\right)m_i m_j \, , i, j = 1,2,3 \tag{10}$$

where $\hat{m}$ is a unit vector corresponding to the eigenvalue σ_1. One can say that the general characteristic of materials exhibiting no strain under stress is of the form (9), all the particular cases included.

Huygens and Fresnel's Ether

It is not necessary to go in deeper details in order to see that the Ether as the carrier of light is a continuous medium of the kind represented by either one of the equations (6) and (10), or by both of them in certain conjunctions. One of these conjunctions will be described momentarily. Before going into details, however, a little algebraical digression is in order, about the meaning of the mentioned equations.

The case of equations (6) and (10) is specific for matrices that we term here as equivalent to a *vector field*. We understand this equivalence in the following way: let $\vec{v}$ be a vector field, and let us construct the following matrix

$$v_{ij} = \alpha\delta_{ij} + \beta v_i v_j \tag{11}$$

Now, it is clear that, because v_k are the components of a vector, and supposing α and β scalars, gives v_{ij} as the components of a tensor. One of the principal values of this tensor, namely α, is double. The other principal value, different from α, is given by

$$\alpha = \alpha + \beta v^2 \tag{12}$$

Notice some interesting features of this kind of tensor. First of all, if either β or v_k is null, v_{ij} is a purely spherical tensor. Secondly, if we calculate the eigenvector of **v**, corresponding to the eigenvalue (12), we find out that this eigenvector is $\vec{v}$, up to a normalization factor. This property is independent of the parameter α, and this is what we mean by the above mentioned equivalence: given the vector $\vec{v}$ we can directly construct the tensor **v** as a family of two-parameter tensor matrices having it as an eigenvector. One can say that **v** is a kind of action that points in the "general direction" of $\vec{v}$, not exactly in that direction.

Now, in order to convey what we think is the right meaning to the historical facts, we need to recall that the algebraic representation of a second order tensor is a quadratic form having space representation as a quadric (ellipsoid, hyperboloid or paraboloid). This was actually the classical way of description of light to Huygens and Fresnel [4, 5, 13 Volume I, mainly Chapter IV]. The light was seen as Ether in extension; it is only afterwards that it has been characterized as a perturbation propagating in Space. Therefore it was important for the genius of a geometer like Huygens to characterize the space form of extension, i.e. the form of wave as we say nowadays. It was a sphere or an ellipsoid of revolution, and these are quadratic forms associated to tensors of the form given in equation (11). Indeed, in case where the quadratic form of a tensor is positively defined, its space representation is always an ellipsoid whose semi axes are given by the principal values of the tensor. It is clear that a spherical tensor [$\beta = 0$ in equation (11)] represents a sphere, while the tensor (11) itself represents an ellipsoid of revolution (spheroid).

This is the way Huygens characterized the propagation of light in vacuum and the phenomenon of double refraction. In modern terms, he just noticed that the Ether entering the structure of Matter is characterized by a tensor like (11). Huygens considered both cases, β zero and nonzero separately, thus accounting for the strange phenomenon of double refraction. It was the merit of Fresnel to notice that a single space form as related to the general tensor (11) is quite sufficient in order to characterize the double refraction. In doing this he just noticed the important fact, taken for granted nowadays, that the thing we are after in this construction is actually the eigenvalue of the tensor representing the ellipsoid, for it is in relation with the speed of light. And, according to the classical principles, it is the speed of light that changes in the phenomenon of refraction. Only, it has been noticed that this change is not done according to the rules of Classical Dynamics, which are mainly vector rules.

Speaking of this moment in Fresnel’s thinking, let us notice a fact that we find to be of special importance for what has followed afterwards. Namely replacing the Huygens’ double construction – sphere and spheroid – by a single general construction – spheroid – carries over into space forms the mark of a certain property of linearity of the tensors from the family given by equation (11): the linear combination of any number of tensors belonging to the same vector is always a tensor of the same family. It is indeed this property that allowed Fresnel to see that in the general case of crystals having two axes of double refraction, the right description is that by a quadratic form representing not a spheroid but a general ellipsoid [13].

Relegating the reader to the historical works already indicated, we try here a more limited task, namely to answer the question: what would have happened if Fresnel would continue this logic, based however upon the existence of two kinds of tensors (11) one related to Ether in Space the other related to Ether in Matter, as described above?

A Neo-Fresnelian Point of View

Therefore, the only way to get the characterization of Ether inside a biaxially birefringent crystal is by simply entering into play the fact that there exist Ether in Space and Ether in Matter, i.e. by admitting that the Ether is characterized not by one tensor of the general type (11) but by two, with two characteristic vectors $\vec{u}$ and $\vec{v}$ say. According to the Fresnel subjacent logic outlined right above, the complete tensor describing the Ether would then be:

$$w_{ij} = \alpha\delta_{ij} + \beta u_i u_j + \gamma v_i v_j \tag{13}$$

Somewhere along the line Fresnel – or someone else, doesn't really matter – would have noticed that the calculations are more symmetrical in case we write (13) in a more convenient way as

$$w_{ij} = \lambda u_{ij} + \mu v_{ij} \tag{14}$$

where λ and μ are real parameters, describing the degree of “Space or Matter” of the Ether, with the matrices **u** and **v** defined by

$$u_{ij} = u_i u_j - \frac{1}{2} u^2 \delta_{ij}, u^2 \equiv \sum u_j^2$$
$$v_{ij} = v_i v_j - \frac{1}{2} v^2 \delta_{ij}, v^2 \equiv \sum v_j^2 \tag{15}$$

We recognize in equation (14) the form of Maxwell stress tensor for the electromagnetic field, with an appropriate interpretation of the matrices **u** and **v**. This tensor contains eight measurable quantities: λ, μ, and the two intrinsic vectors. Written at length, the tensor (14) is

$$w_{ij} = \lambda u_i u_j + \mu v_i v_j - \frac{1}{2}\left(\lambda u^2 + \mu v^2\right)\delta_{ij} \tag{16}$$

It is easy to see that it has three real eigenvalues. Indeed, its orthogonal invariants are

$$I_1 = -e, I_2 = -e^2 + g^2, I_3 = -e\left(e^2 - g^2\right) \tag{17}$$

where we denoted

$$e \equiv \frac{1}{2}\left(\lambda u^2 + \mu v^2\right); \vec{g} \equiv \sqrt{\lambda\mu}\left(\vec{u} \times \vec{v}\right) \tag{18}$$

The eigenvalues of **w** can then be calculated as the roots of the corresponding characteristic equation, and they are

$$w_1 = e, \qquad w_{2,3} = \pm\sqrt{e^2 - g^2} \tag{19}$$

It turns out that the pair from (18) gives one eigenvector of **w** and the corresponding eigenvalue. The other two eigenvectors of **w** are orthogonal, and located in the plane of the vectors $\vec{u}$ and $\vec{v}$.

At this point Fresnel would have to give an explanation to scientific community. Indeed, the general definition (13) of the Fresnel tensor involves many quantities in order to establish it by measurement: the constants α, β, γ, the lengths of the two vectors and their orientations; a total of nine quantities. However, this fact is only apparently true, for we have to deal with a symmetric matrix, having therefore only six independent components; as a matter of fact the representation in equation (16) has only eight quantities. As the three eigenvalues seem to be mandatory, for the two vectors remains a need for only three quantities, leading us to the idea that three of the parameters are redundant.

The problem popped up even from the pioneering works of Fresnel, in the form of representability of the elliptically polarized light. Its solution took different forms along the time leading eventually to the science of Ellipsometry, whose first champions were apparently Stokes and Verdet [10, 11, 12]. Especially Verdet insisted at length upon statistical aspect of the problem which, according to any imaginable criterion, seems to be indeed its essential nature. Here we give an inedited shade to this statistical aspect.

Light as an Essentially Statistical Process

The tensor characterization of the Space forms of Ether as it comes out of light measurements is a highly idealized situation. What one can actually measure in a point in Space is an average of the influences of Ether and Matter, and it is this fact that has to be taken into consideration, when talking of the Cosmic Background Radiation for instance. Now, as long as we represent these influences by a tensor, this representation has two kinds of space averages attached to it: one of them is the *average normal component* the other is the *average tangential component* of the tensor in a space point.

Indeed, for every plane in space through a certain point a tensor has two characteristic scalar intensities associated: the normal and in-plane (tangential or shear) intensities. In the case of locally isotropic space, V. V. Novozhilov has shown [7] that the space averages of these components with respect to the ensemble of planes through a point are invariant quantities that can be written only in terms of the principal components of the tensor. For our tensor **w** they are

$$\begin{aligned} w_n &= \frac{w_1 + w_2 + w_3}{3} \\ w_t^2 &= \frac{1}{15}\left[\left(w_2 - w_3\right)^2 + \left(w_3 - w_1\right)^2 + \left(w_1 - w_2\right)^2\right] \end{aligned} \tag{20}$$

There is a subtle point here, allowing us to say that there is no contradiction in terms between the tensor model of Ether and this statistical image of local measurements. It so happens that these quantities can be described in vector terms in the reference frame given by the eigenvectors of our tensor. Indeed, in this reference frame the tensor can be simply represented as a vector having the three eigenvalues as components:

$$\left|w\right\rangle \equiv \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} \tag{21}$$

The matrix notation here is meant to show that this vector is a kind of strange one, for which *we only can infer* the components; the quantities measured are actually those from equation (20). Those quantities can be taken as magnitudes of the components of vector (21) with respect to a special plane, the so-called *octahedral plane* of the reference frame given by eigenvectors. This is a unique plane in a certain octant of the reference frame cutting all axes at unit distance from origin, thus having, in the first octant for instance, the normal

$$\left|\vec{n}\right\rangle \equiv \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix} \tag{22}$$

An arrow above the letter shows here that, while a vector is represented as a matrix, it also has the intuitive meaning of a space vector of the kind we are accustomed with. Then w_n from equation (20) is the projection along the normal of the octahedral plane of the vector

$$\left|\frac{w}{\sqrt{3}}\right\rangle \equiv \frac{1}{\sqrt{3}}\begin{pmatrix}w_1\\w_2\\w_3\end{pmatrix} \tag{23}$$

The other quantity from equation (20) comes around if we consider in-octahedral-plane (shear) components of the vector (21). These components are given by the vector

$$\left|w_t\right\rangle \equiv \left|w\right\rangle - \left|\vec{n}\right\rangle\left\langle\vec{n}\right|\left|w\right\rangle = \frac{1}{3}\begin{pmatrix}2w_1 - w_2 - w_3\\-w_1 + 2w_2 - w_3\\-w_1 - w_2 + 2w_3\end{pmatrix} \tag{24}$$

Then, a simple calculation gives

$$\left\langle w_t \middle| w_t \right\rangle \equiv \frac{1}{5} w_t^2 \tag{25}$$

For the specific forms of eigenvalues given by equations (19), the two octahedral plane components are

$$w_n \equiv \langle w | \vec{n} \rangle = -\frac{2}{\sqrt{3}} e ; | w_t \rangle = \frac{2}{3} \begin{pmatrix} -2e \\ 3\sqrt{e^2 - \vec{g}^2} + e \\ -3\sqrt{e^2 - \vec{g}^2} + e \end{pmatrix} \tag{26}$$

As long as only the values (20) are measured, the orientation of the vector from (26) in the octahedral plane always remains undecided. This orientation is, again, out of our control, *but can be measured*. It can be accounted for by an angle easy to measure in case we have a reference direction in the octahedral plane at our disposal. Assume indeed, that we have such a reference, as given by a particular tensor of the form given in equation (11) with the characteristic vector $\vec{\xi}$ say. Then, for this tensor we have, with obvious notations,

$$\langle \xi | \vec{n} \rangle = -\frac{1}{\sqrt{3}} \xi^2 ; | \xi_t \rangle = \frac{2}{3} \xi^2 \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} \tag{27}$$

If the vector $\vec{\xi}$ is perpendicular on both $\vec{u}$ and $\vec{v}$ then the tensors **w** and **ξ** commute. Thus they have a common reference frame and it can be arranged that their octahedral planes coincide. It is in this case that the direction of the vector from equation (27), which is fixed, can be correctly chosen as a reference direction in the octahedral plane. Then the angle ψ of the vector (26) with respect to this fixed direction can be calculated from the obvious geometrical formula

$$\cos\psi = -\frac{e}{\sqrt{4e^2 - 3\vec{g}^2}} \tag{28}$$

This shows that, under the specified conditions, the angle ψ is independent of the reference vector. With a proper choice of sign for the square root, the origin ψ = 0 of this angle occurs for e = g. This condition means in turn that the angle θ between the vectors $\vec{u}$ and $\vec{v}$ is given by equation

$$\left|\sin\theta\right| = \frac{1}{2}\frac{\lambda u^2 + \mu v^2}{\sqrt{\lambda\mu u v}} \tag{29}$$

As the quantity from the right hand side here is always greater than, or equal to 1, the angle between vectors $\vec{u}$ and $\vec{v}$ cannot be but 90°. Thus, the initial condition for the characteristic angle of tensor **w** in the octahedral plane takes place when the vector $\vec{u}$ is perpendicular to $\vec{v}$ and their plane is perpendicular to vector $\vec{\xi}$. If this last vector is given by a ray for instance, we have the classical image of the propagation according to Fresnel, without using however *neither the electromagnetic theory nor the concept of frequency for describing the light*. One has to notice though, that the price paid here for avoiding the classical kinematics in describing the vibratory motion, is accepting from the very beginning the planar description of the wave by two vectors whose physical meaning may be a challenge.

Regarding the problem of measurement, one can notice that it refers actually to just two quantities and an angle: anything else seems to be inference from these three quantities. The redundancy is due, as always in Physics, to our geometrical models of reality: vectors and tensors. Mention should be made of the important fact that the perpendicularity of the vectors $\vec{u}$ and $\vec{v}$ is not a purely geometrical property, but the consequence of *some underlying statistics*.

Conclusions

It is our opinion that the Ether can be understood, in all its contradictory features, if we eliminate the concept of force from the considerations regarding it. Indeed this vector concept is strictly based upon our natural capability of sensual perception, while the Ether is elusive to this kind of observation. It is not a pure chance that the wave theory of light has appeared along the history the way it did. The analysis based on a constitutive description as presented in this essay, shows that the space form of the light propagation as first proposed by Huygens is a necessary consequence of the essential quality of Ether to be *totally or partially uncontrollable*. Then the message which, according to Fresnel's philosophy, follows from this geometrical description can be phrased by asserting that the *Maxwell theory of electromagnetic light is a natural reaction of intellect to the uncontrollability of Ether*.

This last sentence may require a little elaboration in order to be properly understood. The theory presented here follows only the geometric side of the problem; it is something that, we think, the "learned geometers" of the times past would have enjoyed. In this geometrical theory the planar wave is not a natural consequence of the periodicity of the light vector, as classically inferred by Fresnel from his experiments, but simply the consequence of the existence of two vectors describing the generally uncontrollable Ether. The periodicity properties of light are however something soundly established by experiments: we cannot avoid their explanation but, nevertheless, this cannot be offered only based on Kinematics.

Here comes the electromagnetic theory of light which, as well-known, is based on the property of variable electric and magnetic fields to act at a distance by inducing their counterparts. Within this phenomenology it then comes as only natural the fact that, in order to satisfy the periodicity, the two geometrical vectors are to be conceived as periodically variable and of different nature – one electric and one magnetic – and they "entertain" each other so to speak all along the propagation of light. This is the classical image of electromagnetic wave propagation, showing first and foremost that the Geometry alone is not sufficient to describe the Ether. However, the electromagnetism might not be sufficient either, inasmuch as it is formally limited to vectors. For instance we showed here that the perpendicularity of electric and magnetic fields in a vacuum electromagnetic wave *is not a geometrical property but a straight statistical one*. The key to this problem seems to be, as usually in any theory for that matter, somewhere between Geometry and Electromagnetism. For it seems that the Electrodynamics stepped too soon into the arguments regarding the light and the Ether. And, to answer our main question addressed by this essay, no matter how we characterize it, *the Ether is here to stay*!

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