Derivation of Einstein's Equation, $E = mc^2$, from the Classical Force Laws

N. Hamdan, A.K. Hariri
Department of Physics,
University of Aleppo, Syria
nhamdan59@hotmail.com , haririak2000@yahoo.com

J. López-Bonilla
SEPI-ESIME-Zacatenco,
Instituto Politécnico Nacional,
Edif. Z-4, 3er. Piso, Col. Lindavista CP 07 738 México DF
joseluis.lopezbonilla@gmail.com

In several recent papers we showed that choosing new sets of postulates, including classical (pre-Einstein) physics laws, within the main body of Einstein's special relativity theory (SRT) and applying the relativity principle, enables us to cancel the Lorentz transformation from the main body of SRT.

In the present paper, and by following the same approach, we derive Einstein's equation $E = mc^2$ from classical physical laws such as the Lorentz force law and Newton's second law. Einstein's equation is obtained without the usual approaches of thought experiment, conservation laws, considering collisions and also without the usual postulates of special relativity.

In this paper we also identify a fundamental conceptual flaw that has persisted for the past 100 years. The flaw is interpreting the formula $E = mc^2$ as the equivalence between inertial mass and any type of energy and in all contexts. It is shown in several recent papers that this is incorrect, that this is a misinterpretation. What Einstein considered to be a central consequence of special relativity is in fact derivable from (pre-Einstein) classical considerations. $E = mc^2$ becomes secondary, not fundamental, and whilst no doubt useful in certain circumstances, need not be valid in all generality.

Keywords: classical force laws, special relativity theory, the Einstein's equation, $E = mc^2$.

Introduction

The SRT [1] has removed the barrier between matter and energy, but it has created a new barrier that cannot be transcended. This barrier separates what is known as non-relativistic from relativistic physics. The physical laws for non-relativistic physics cannot transcend this barrier and hence they form classical physics. The physical laws adequate for relativistic physics can, however, cover the non-relativistic physics domain through the known approximation of the Lorentz transformation (LT): The LT becoming a Galilean transformation where appropriate.

In classical physics, we know that a particle m_0 moving with velocity $\mathbf{v}$ has a momentum $\mathbf{p} = m_0\mathbf{v}$ and a kinetic energy

$$T = \frac{1}{2} m_0 \mathbf{v}^2 ,$$

however, in relativistic physics the momentum and relativistic mass of a particle are

$$\mathbf{p} = \frac{m_0 \mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}} = m \mathbf{v}, \tag{1}$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0, \tag{2}$$

whereas its kinetic energy is

$$T = mc^2 - m_0 c^2, \tag{3}$$

where as we see m_0 represents the rest mass. This m_0 accounts for the inertia of the particle at the moment when its acceleration starts from a state of rest. Relativists also introduce the concept of rest energy $E_0 = m_0 c^2$, and of relativistic energy

$$E = T + E_0 = mc^2. \tag{4}$$

In his paper, Einstein [2] derived equation (4) through a thought experiment. Many further papers have been devoted to the derivation of equation (4) using the popular approaches of conservation laws, consideration of collisions and the postulates of SRT [3-8]. In a previous paper [9] we suggest another way to account for the kinematical effects in relativistic electrodynamics. This method does not use the LT for a charged particle: Instead it involves inserting the Lorentz force within the main body of SRT and applying the principle of relativity (that the laws of physics have the same formulation relative to any inertial system) rather than the principle of special relativity (that the laws of physics are invariant under the LT). We have also presented in another paper [10] our claim that not only relativistic electromagnetism, but also relativistic mechanics can easily be derived using this approach.

As in the previous papers [9,10] we present a derivation of equation (4) starting from classical laws, without calling upon the usual approaches. Further, we clarify that the derivation of $E = mc^2$ can be studied without using the LT or its kinematical effects, i.e. the relation $E = mc^2$ does not need SRT, contrary to Einstein's assertion [11].

Derivation of Einstein's equation $E = mc^2$ from the Lorentz force

Einstein was the first to derive mass-energy equivalence from the principles of SRT in his article titled "Does the Inertia of a Body Depend Upon Its Energy Content?" [2]. Since this derivation was published, it has been the subject of continuing controversy. For instance, the relativistic mass, Eq. (2), is applied in the case of an inertial frame co-moving with the particle, so it is a consequence of the time dilation effect between the two frames i.e. a co-moving reference frame for a particle moving with a uniform velocity where the reference frame is supposed to be at rest at every moment during the motion of the particle.

Therefore, the relativistic mass in this case is a purely kinematical effect. Hence, according to Einstein the relativistic mass is not a physical effect but rather the result of the effect of relative motion on observation. Einstein then derives his equation mathematically and under special conditions, which requires the above speculation. Therefore Eqs. (2) and (4) can appear mysterious and are put in doubt [12,13]. We will show in this paper a purely dynamical derivation of

$E = mc^2$ and the mass increase relation, based solely on the Lorentz force law and Newton's second law (NSL).

Let us consider two inertial systems S and S' with a relative velocity u // ox between them.

Einstein in his SRT assumes the equivalence of all inertial reference frames, but makes use of different assumptions, including: time dilation- length contraction and the relativity of simultaneity. These effects are expressed mathematically by the Lorentz transformation. Our paper makes a distinction between the Lorentz transformation and inertial frames, even though within the context of special relativity they are the same thing. Our definition of inertial reference frame, on the other hand, is in fact the usual one (before Einstein): those reference frames in which Newton's first and second laws of motion are valid.

Our approach in this paper is to consider a single particle that moves along a given direction with common $ox(ox')$ axes. As demonstrated in [9], contrary to what is often claimed in SRT, the relativistic expressions can then be derived starting from the relativity principle and the classical Lorentz's law, i.e.

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})\,. \tag{5}$$

Now assuming that a charged particle q moving with velocity $\mathbf{v}$ in the frame S, subject to an electric field $\mathbf{E}$ and a magnetic flux density $\mathbf{B}$, then the Cartesian components of (5) in frame S are

$$F_x = q(E_x + v_y B_z - v_z B_y) \tag{6a}$$

$$F_y = q(E_y + v_z B_x - v_x B_z) \tag{6b}$$

$$F_z = q(E_z + v_x B_y - v_y B_x) \tag{6c}$$

Applying the relativity principle to Eqs. (6), we have

$$F'_x = q(E'_x + v'_y B'_z - v'_z B'_y) \tag{7a}$$

$$F'_y = q(E'_y + v'_z B'_x - v'_x B'_z) \tag{7b}$$

$$F'_z = q(E'_z + v'_x B'_y - v'_y B'_x) \tag{7c}$$

So following a similar approach to that used in [9], we can obtain the relativistic transformation equation for velocity:

$$v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}} \text{(a)},\quad v'_y = \frac{v_y}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \text{(b)},\quad v'_z = \frac{v_z}{\gamma\left(1 - \frac{u}{c^2}v_x\right)} \text{(c)} \tag{8}$$

where the scalar factor γ was fixed by applying the relativity principle to Eqs. (8):

$$\gamma^2\left(1 - \frac{u^2}{c^2}\right) = 1 \text{ or } \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \tag{9}$$

The conventional way to derive the relativistic transformation equations of the components of the relativistic velocity are obtained from SRT. However, we have already explained that there is an alternative path that does not use the Lorentz transformation in the derivation of the 3-vector relativistic velocity transformations appertaining to a charged particle. With Eqs. (8) this enables us to construct the following relativistic identities:

$$\frac{1}{\sqrt{1 - \frac{v'^2}{c^2}}} = \frac{\left(1 - \frac{uv_x}{c^2}\right)}{\sqrt{1 - \frac{u^2}{c^2}}\sqrt{1 - \frac{v^2}{c^2}}} \text{(a)},$$

$$\frac{v'_x}{\sqrt{1-\frac{v'^2}{c^2}}} = \frac{v_x - u}{\sqrt{1-\frac{u^2}{c^2}}\sqrt{1-\frac{v^2}{c^2}}} \text{ (b)} \tag{10}$$

$$\frac{v'_y}{\sqrt{1-\frac{v'^2}{c^2}}} = \frac{v_y}{\sqrt{1-\frac{v^2}{c^2}}} \text{ (c)}, \quad \frac{v'_z}{\sqrt{1-\frac{v'^2}{c^2}}} = \frac{v_z}{\sqrt{1-\frac{v^2}{c^2}}} \text{ (d)}$$

In spite of the objections by L.B. Okun [14] who states that the proper definition of relativistic momentum is $\mathbf{p} = \gamma m_0 \mathbf{v}$, the traditional definition of momentum, i.e. $\mathbf{p} = m\mathbf{v}$, combined with Eqs. (8) gives all the relativistic expressions and relativistic transformation relations concerning momentum, energy and the relativistic mass as follows: When viewed from S the charged particle q that we have mentioned above has momentum $\mathbf{p} = m\mathbf{v}$ whose components are:

$$p_x = mv_x \quad (a), \quad p_y = mv_y \quad (b), p_z = mv_z \quad (c) \tag{11}$$

Viewed from S' the momentum is $\mathbf{p}' = m\,\mathbf{v}'$ having the components:

$$p'_x = m'v'_x \quad (a), \quad p'_y = m'v'_y \quad (b), \quad p'_z = m'v'_z \quad (c) \tag{12}$$

in accordance with the relativity principle. Combining (8a), (11a) and (12a) we obtain:

$$\frac{p'_x}{m'} = \frac{p_x - um}{m(1-\frac{uv_x}{c^2})},$$

which means that

$$p'_x = k(p_x - um) \quad (a), \qquad m' = mk(1-\frac{uv_x}{c^2}) \quad (b) \tag{13}$$

where k represents an unknown constant and this scalar factor k can be fixed by applying the relativity principle to Eqs. (8a), (11a) and (12a) to get:

$$\frac{p_x}{m} = \frac{p'_x + um'}{m'(1+\frac{uv'_x}{c^2})}$$

Then

$$p_x = k(p'_x + um') \quad (a), \qquad m = m'k(1+\frac{uv'_x}{c^2}) \quad (b) \tag{14}$$

Multiplying (13b) and (14b) side by side we deduce:

$$1 = k^2(1-\frac{uv_x}{c^2})(1+\frac{uv'_x}{c^2})$$

and employing (8a) in the last equation, leads to

$$k^2(1-\frac{u^2}{c^2}) = 1 \quad \text{or} \quad k = \gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \quad \text{(c)} \tag{14}$$

The constant k in Eq.(14c) is equal to the scalar factor γ in Eq.(9).

For simplicity, take the special case that the charged particle is at rest in frame S , thus:

$$v_x = 0, \quad v'_x = -u$$

These results, when substituted into (13b) leads to

$$m' = \gamma m_0 \tag{15a}$$

Observers of frame S measure the rest mass m_0 , observers from S' measure the mass m' given by (15a). We can now assume that the charged particle is at rest in frame S' , so:

$$v'_x = 0 \quad , \qquad v_x = u$$

These results, when substituted into (14b) lead to

$$m = \gamma m_0 \tag{15b}$$

Observers of frame S' measure its rest mass m_0, observers from S measuring mass m given by (15b). We obtain the transformation equation for the $oy(o'y')$ component of the momentum if we combine (11b) and (12b) with (8b). Thus we deduce:

$$p'_y = m'v'_y = mv_y = p_y \tag{13c}$$

In a similar way, we get

$$p'_z = m'v'_z = mv_z = p_z \tag{13d}$$

Now, using (9) in (13a,b) and (15), leads to the transformation equations

$$m' = \gamma m\left(1 - \frac{uv_x}{c^2}\right) \quad , \quad p'_x = \gamma\left(p_x - um\right) \tag{16a,b}$$

$$p'_y = p_y \ , \quad p'_z = p_z \tag{16c,d}$$

and the expression for the relativistic mass in both frames:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \ , \ m' = \frac{m_0}{\sqrt{1 - \frac{v'^2}{c^2}}} \tag{17a,b}$$

If we start with the relativistic identities (10) multiplying both sides with m_0, the result is equations (16) and (17) (see [9]).

To obtain the same relativistic combination between the momentum and energy of the same charged particle q, we start from (14c), and write this equation as:

$$\gamma^2 - \frac{u^2}{c^2}\gamma^2 = 1,$$

Multiplying both its sides with $m_0^2 c^4$ it becomes:

$$c^4\gamma^2 m_0^2 - c^2\gamma^2 m_0^2 u^2 = m_0^2 c^4 \tag{18}$$

We recognize that the term

$$p^2 = \gamma^2 m_0^2 u^2 = m^2 v^2 \ , \qquad u = v$$

represents the square of the momentum in S. Moreover, the root of the first term presented is

$$E = m_0 c^2 (1 - \frac{v^2}{c^2})^{-\frac{1}{2}} = \gamma m_0 c^2 \tag{19}$$
$$= mc^2$$

Eq.(19) is the relativistic energy E, telling us that the change of the mass of a particle is accompanied by a change in its energy and vice versa [15].

With the new notation, (18) becomes:

$$E^2 = c^2 p^2 + m_o^2 c^4 \tag{20}$$

Eq. (20) represents the combination of the momentum and the energy of the same particle. Multiplying both sides of (16a,b) with c^2 we obtain the transformation equation for energy and momentum:

$$E' = \gamma(E - up_x), \quad p'_x = \gamma\left(p_x - \frac{u}{c^2}E\right) \tag{21}$$

The conventional way to derive the relativistic transformation equations of energy-momentum is to consider a collision between two particles from two inertial reference frames and imposing energy-momentum conservation. However, the purpose of our paper is to show that the relativistic dynamics, and especially Einstein's equation $E = mc^2$, could be approached without the popular approaches.

The kinetic energy T should equate the difference between relativistic energy E and rest energy E_0, i.e.

$$\begin{aligned} T &= mc^2 - m_0 c^2 \\ &= m_0 c^2 \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right) \end{aligned} \tag{22}$$

Derivation of Einstein's equation *E* = *mc*^{2} from Newton's second law (NSL).

With the discovery of the SRT, energy was found to be one component of an energy-momentum 4-vector. Each of the four components (one of energy and three of momentum) of this vector is separately conserved in any given inertial reference frame. The vector length is also conserved and is the rest mass. The relativistic energy of a single massive particle contains a term related to its rest mass in addition to its kinetic energy of motion. In the limit of zero kinetic energy or equivalently in the rest frame of the massive particle the energy is related to its rest mass via the famous equation $E = mc^2$. However, many textbooks on SRT often devote great effort to discussing the process of elastic collision between two particles to derive $E = mc^2$ and the relativistic mass $m = \gamma m_0$. Our goal on the other hand is to establish Einstein's formula $E = mc^2$ based solely on the NSL without using the LT or the ideas of Einstein.

As we demonstrated in [16,17], the Lorentz force and the relativity principle, are more natural for describing the physics of relativistic electrodynamics.

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad \text{(a)}, \quad \frac{dE}{dt} = \mathbf{F}\mathbf{v} \, \text{(b)} \tag{23}$$

Moreover, we can now go further to get all of SRT's relations from classical mechanics:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \, \text{(a)}, \; \frac{dE}{dt} = \mathbf{F}\mathbf{v} \, \text{(b)} \tag{24}$$

Let us consider two inertial systems S and S' with a relative velocity u // ox between them and consider from S a particle which has the momentum $\mathbf{p} = m\mathbf{v}$. The Cartesian components of (24) in frame S are:

$$\frac{dp_x}{dt} = F_x \text{ (a)}, \quad \frac{dp_y}{dt} = F_y \text{ (b)}, \quad \frac{dp_z}{dt} = F_z \text{ (c)} \tag{25}$$

and

$$\frac{dE}{dt} = F_x v_x + F_y v_y + F_z v_z \; \text{(d)}$$

Applying the relativity principle to (25), we have:

$$\frac{dp'_x}{dt'} = F'_x \text{ (a)}, \quad \frac{dp'_y}{dt'} = F'_y \text{ (b)}, \quad \frac{dp'_z}{dt'} = F'_z \text{ (c)} \tag{26}$$

and

$$\frac{dE'}{dt'} = F'_x v'_x + F'_y v'_y + F'_z v'_z \qquad \text{(d)}$$

By following the same approach to that used in [21] we can obtain the relativistic transformation equation for momentum, energy and velocity:

$$p'_x = \gamma\left(p_x - \frac{u}{c^2} E \right) \text{(a)}, \; p'_y = p_y \; \text{(b)}, \; p'_z = p_z \; \text{(c)},$$

$$E' = \gamma(E - up_x)\ \text{(d)} \tag{27}$$

$$v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}\ \text{(a)},\quad v'_y = \frac{v_y}{\gamma\left(1 - \frac{uv_x}{c^2}\right)}\ \text{(b)},\quad v'_z = \frac{v_z}{\gamma\left(1 - \frac{uv_x}{c^2}\right)}\ \text{(b)}, \tag{28}$$

We may write Eqs. (28) as:

$$\frac{1}{\sqrt{1 - \frac{v'^2}{c^2}}} = \frac{(1 - \frac{uv_x}{c^2})}{\sqrt{1 - \frac{u^2}{c^2}}\sqrt{1 - \frac{v^2}{c^2}}} \tag{29}$$

and we can put (28b) into (27b) to get

$$m' = \gamma m(1 - \frac{uv_x}{c^2}) \tag{30}$$

Multiplying (29) with m_0, and comparing it with (30), we deduce:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}\ \text{(a)},\quad m' = \frac{m_0}{\sqrt{1 - \frac{v'^2}{c^2}}}\ \text{(b)} \tag{31}$$

From (25d) the total energy is given by:

$$\begin{aligned} dE &= Fvdt = d\left(mv\right)v \\ &= v^2 dm + mvdv \end{aligned} \tag{32}$$

and from (31a) we have:

$$dm = \frac{mvdv}{c^2\left(1 - \frac{v^2}{c^2}\right)} \qquad i.e. \qquad m\,v\,dv = c^2\left(1 - \frac{v^2}{c^2}\right)dm \tag{33}$$

Substituting (33) in (32), we get:

$$dE = c^2 dm$$

and by integration, from v_1 to v_2, we deduce that:

$$E = mc^2 \Big|_1^2 \tag{34}$$

In the particular case when $v_1 = 0$ and $v_2 = v$ then E should equal the kinetic energy T, i.e.

$$T = mc^2 \Big|_1^2 = mc^2 - m_0 c^2$$

So the quantities mc^2 and $m'c^2$ are the total energy E and E' in frames S and S', respectively. It is simple to prove, that Eqs. (27) and (31) lead to

$$E'^2 - c^2(p_x'^2 + p_y'^2 + p_z'^2) = E^2 - c^2(p_x^2 + p_y^2 + p_z^2) = m_0^2 c^4$$

or

$$\begin{aligned} E^2 &= c^2 \boldsymbol{P}^2 + m_0{}^2 c^4 \\ &= m^2 c^4 \end{aligned}, \quad \begin{aligned} E'^2 &= c^2 \boldsymbol{P}'^2 + m_0{}^2 c^4 \\ &= m'^2 c^4 \end{aligned}$$

We show that Einstein's formula $E = mc^2$ can be reached without using conservation laws and avoiding consideration of collisions or Einstein's thought experiment. The derivations are based on the NSL and the principle of relativity and on its direct consequence, the addition law of relativistic velocities. Einstein's original derivation could have been made clearer using methods shown here. Einstein's $E = mc^2$ is derived here using different postulates to those of Einstein and in two different ways. As a result Einstein's equation is shown not to be a core result of Einstein's special relativity but somewhat secondary to the wider context of classical physics. Its fundamental nature is therefore challenged.

The Generalized Mass-Energy Equation $E = Amc^2$

Einstein's 1905 derivation of $E = mc^2$ has been criticized for being circular and for the fact that Einstein's original derivation is not at all clear [18]. It is often said that (inertial) energy E and the so-called relativistic mass m are the same thing, presumably due to the relation $E = mc^2$. However, this argument is demonstrably wrong. Such erroneous conclusions may have come about due to the lack of application of special relativity to particles. When an object is a particle then the relation $E = mc^2$ does not hold: it was demonstrated in Refs. [19,20,21] that Einstein's relation $E = mc^2$ led to serious errors and inexplicable results. As was pointed out in Refs. [17,21] to remove the contradictions with Einstein's relation $E = mc^2$, we considered that Eqs. (19) and (34) could be written as:

$$E = mc^2 = mv^2 + m_0 c^2 \sqrt{1 - \frac{v^2}{c^2}} \tag{35}$$

Specifically, it was shown in paper [17,21] that the new total energy,

$$E_v = mv^2, \tag{36}$$

is an intrinsic energy of a particle. Perhaps this will allow us to reconstruct compatibility between the framework of De Broglie wave theory and SRT without the usual contradictions.

Combining Eq.(36) and Eqs.(19) and (34) gives,

$$E_v = \left(\frac{v}{c}\right)^2 mc^2 = Amc^2, \tag{37}$$

which can be used successfully for a moving particle. We see now that $A = 1$ for a wave travelling with the speed $v = c$, and in this case

Eqs. (35), (36) and (37) are identical to Eqs. (19) and (34); otherwise, the value of A can be equal, less than, or more than unity, depending on the situation.

For this reason some authors argue that Eqs. (19) and (34) are incorrect [18,19]. A. Sharma [18] has considered the total energy as $\Delta E_{total} = A\Delta mc^2$ rather than $E = mc^2$. So that in Eqs. (36) and (37) A becomes a conversion coefficient that can be determined by experiment, and the value of A can be less, more, or equal to unity. E. Bakhoum [19], on the other hand, has taken the approach of defining the total energy according to Eq. (36). A. Sharma has provided experimental evidence that the value of the conversion factor between mass m and energy E is not always c^2.

The integration of special relativity theory with quantum mechanics has yielded many paradoxes that remained unsolved until recent years; like the zitterbewegung problem as well as the fact that the spin prediction of the Dirac equation could only be identified with non-relativistic approximations (Pauli and Foldy-Wouthysen). The most prominent attempt to eliminate such problems was by E. G. Bakhoum and requires a modification in the mass-energy equivalence principle. Bakhoum introduced a new total relativistic energy formula $E = mc^2$ instead of Einstein's $E = mc^2$. This paper carries Bakhoum's work a step further as we have derived Einstein's equation $E = mc^2$ without using special relativity theory, instead starting from the classical physics laws like the Lorentz force law and Newton's second law. Further, the energy formula of a particle $E = mc^2$ allows reconciliation between the de Broglie wave theory and the framework of the relativistic physics without the usual contradictions [17-21].

Einstein has derived the relation $E = mc^2$ under special conditions, i.e.; he derived the equation only for when a body moves with uniform velocity in the relativistic domain. This is the main reason that authors have derived the relation $E = mc^2$ by new methods. Now concurrently with deriving the mass-energy equation by new methods, existing experimental data [18] has been analysed and shows that mass does not in fact have equivalence to energy for a moving particle, i.e.; the value of the conversion factor between mass m and energy E is not always c^2.

Therefore Eqs. (19) and (34) are special cases of Eq.(37).

Conclusion

No one appears to have ever asked the question "what is needed in the most general sense to derive Einstein's formula $E = mc^2$?

Approaching this question ourselves, we started from the classical laws to rebuild the theory of special relativity (SRT) [9,10]. This enables us to derive Einstein's formula $E = mc^2$ from the classical laws without any of the well-known approaches and to show that Einstein's formula $E = mc^2$ can be derived in many different ways, even without the usual methods of thought experiment, conservation laws, considering collisions or the postulates of special relativity. Currently the formula $E = mc^2$ is often interpreted as the equivalence between inertial mass and any type of energy. It is shown in papers [17-21] that this is incorrect, that this is a misinterpretation. What Einstein considered to be central to special relativity is in fact derivable from more classical considerations, rather than as a central consequence only of special relativity. $E = mc^2$ becomes secondary, not fundamental, and whilst no doubt useful in certain circumstances may not even be valid in all generality. The root of this problem is due to an erroneous interpretation of special relativity where the formula $E = mc^2$ is taken to be true outside of the context of relativistic kinematics.

Acknowledgments

It is our pleasure to thank the referees for their support and inspiring remarks concerning the paper. Where their notes completed the paper is gratefully acknowledged.

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