Stoney Scale and Large Number Coincidences

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The Stoney scale, its characteristics and theoretical tendencies are argued to be consistent with Einstein's theory of gravitational ether and with the Stochastic Electrodynamic theory of vacuum-induced gravity. The Stoney scale is shown to be unique in that it posits the non-equivalence of gravitational and inertial mass in an electromagnetic setting. Several large number coincidences provide an interesting background to this study, which includes Dirac's hypothesis, a rationalization of the squared elementary charge, and a derivation of Boltzmann's Constant from the Stoney length.

Keywords: George Stoney, natural scale, Dirac's hypothesis, Boltzmann's Constant, Einstein's gravitational ether, vacuum energy, Stochastic Electrodynamics, electrical charge, equivalence of gravitational and inertial mass

Introduction

Before Max Planck identified the mass that now bears his name, the Irish physicist, George Stoney, identified a similar but smaller mass as a fundamental unit of Nature [1]. Both Planck and Stoney considered a 'natural' standard of measurements a self-evident requirement of science and neither supported his choice with a physical theory. Both scales however are intermediate between microscopic and cosmic processes and it was soon realized that either could be the right scale for a unified theory. The only notable attempt to construct such a theory from the Stoney scale was that of H. Weyl, who associated a gravitational unit of charge with the Stoney length [2][3][4] and who appears to have inspired Dirac's fascination with large number coincidences [5]. However, Weyl's dogmatic adherence to 'the principle of locality' reduced his theory to a mathematical construct with some non-physical implications that were criticized by Einstein [5] [6]. The Stoney scale thereafter fell into such neglect that it has since been re-discovered by others (including by this author and for example [7]). Its rediscovery is easily achieved with today's knowledge. Stoney however discovered his scale at a time when electromagnetic processes were so little understood that he himself was the first to identify and name the electron [8][9].

The Stoney scale accommodates both electromagnetic and gravitational processes in terms of a gravitating Stoney mass. It is ambivalently linear and non-linear in so far as it allows for both the equivalence and non-equivalence of gravitational and inertial mass. This ambivalence could provide theoretical physics with an opportunity to explore differences and similarities in preparation for a unified theory. Weyl was highly selective and his theory took inspiration but very little else from the Stoney scale. A more holistic approach would consider the kind of theory that is implied by the scale itself. That is the approach taken here. The author concludes that the Stoney scale is suited to Stochastic Electrodynamics and to the kind of theory implied by Einstein's notion of 'gravitational ether'.

Mediating between microscopic and cosmic domains, the Stoney scale is sympathetic to coincidences. Dirac's large numbers hypothesis involves just such a coincidence and indeed the first of his large numbers ξ is formally a Stoney number. A new interpretation of this coincidence is presented in this paper. Two other very different large number coincidences are also considered. One of these coincidences involves a 'rationalization' of the dimensions and magnitude of the elementary charge. The other involves a derivation of Boltzmann's Constant from the Stoney length.

The paper is in five sections. The first defines some key quantities and terms and it shows how some electromagnetic and gravitational processes are accommodated within the Stoney scale. The second is a more general study of the formal differences between those processes and it includes a rationalization of the elementary charge. The third section considers electromagnetic theories of gravity and it includes Dirac's hypothesis. The fourth shows how Boltzmann's Constant can be derived from the Stoney length. The last section provides a recapitulation, expansion and summary of the main points.

Definitions

The Stoney energy (E_S), Stoney mass (M_S) and Stoney length (L_S) may be defined in contemporary terms as follows:

$$E_S = \sqrt{\frac{Ke^2c^4}{G}} \tag{1}$$

$$M_S = \sqrt{\frac{Ke^2}{G}} \tag{2}$$

$$L_S = \sqrt{\frac{GKe^2}{c^4}} = \sqrt{\frac{Gm}{c^2} \times \frac{\alpha c\hbar}{mc^2}}, \tag{3}$$

where K is Coulomb's Constant, e is the elementary charge, c is the speed of light in a vacuum, G is the Gravitational Constant, α is the Fine Structure Constant, $\hbar$ is the reduced Planck's Constant, and m is any given mass . The exact values of these Stoney quantities are unknown due to uncertainties in the value of G. However, as shown in (2), M_S is inversely proportional to the square root of G and, moreover, the product of G and the squared Stoney mass is equal to Ke^2, whose value is well known, thus allowing for exact solutions to many equations. As indicated by (3), L_S can be understood as the root mean square of two lengths – one is half the Schwarzschilde radius of any mass and the other is the 'electromagnetic radius' of the same mass. A half Schwarzschilde radius enables the rest energy of a self-gravitating mass to be equated with gravitational potential energy ($Gm^2/r = mc^2$). An 'electromagnetic radius' enables a charged particle's rest energy to be equated with electromagnetic potential energy ($\alpha c\hbar/r = mc^2$). Both lengths are useful mathematical constructs but neither of them has the same physical reality as, for instance, the wavelength of a photon. For the Stoney mass, the two lengths are identical.

This paper features references to 'gravitational speeds' (V_G) and 'electromagnetic speeds' (V_E). In each case, 'speed' is here defined to be the *orbital speed*, 'orbital' being omitted to avoid tedious repetition. A 'gravitational speed' is therefore the speed travelled by any orbiting mass m_1 as a result of the gravitational field established by that mass in association with some other mass m_2:

$$V_G = \sqrt{\frac{Gm_1 m_2}{r^2} \times \frac{r}{m_1}} = \sqrt{\frac{Gm_2}{r}}\,. \tag{4}$$

Here r is the distance between the two masses' centres of gravity (in effect, this is to regard each mass as a point particle), m_1 is a gravitational mass in the numerator and an inertial mass in the denominator. The equivalence of inertial and gravitational mass, which here allows for self-cancellation, is one of the formative principles of the General Theory of Relativity.

The gravitational speed of an orbiting mass is affected by gravitational radiation. In most cases, this is a nugatory affect. Moreover, for an orbiting mass it is associated with a shift in orbit and it can therefore be ignored in so far as it merely posits a different orbit and a different orbital speed.

An 'electromagnetic speed' (V_E) is the speed travelled by a charged, atomic particle in the electromagnetic field established by it in association with another charged, atomic particle. The presence of a magnetic field affects the path of a charged particle but it does not affect the speed– thus for example particle accelerators use magnetic fields to guide the paths of charged particles and only the electric field is used to accelerate them. An 'electromagnetic speed' is in fact the same as an 'electrostatic speed' except the latter is associated with an electrostatic force, in which the magnetic field is disregarded, while the former is usually regarded as a velocity rather than a speed due to the directional influence of the magnetic field. However, the speed is identical and it can validly be termed 'electromagnetic':

$$V_E = \sqrt{\frac{Ke^2}{r^2} \times \frac{r}{m_1}} = \sqrt{\frac{Ke^2}{rm_1}} = \sqrt{\frac{GM_S^2}{r^2} \times \frac{r}{m_1}}\,, \tag{5}$$

where r here is the distance between two charged, point particles and m_1 is the inertial mass of the particle traveling at this speed.

Coulomb's Constant indicates that the 'electrostatic' force is involved. The 'electrostatic' force is here equated with a gravitational force associated with two Stoney masses or with a single, self-gravitating Stoney 'gas'. It is worth noting that, when the electrostatic force is thus equated with a gravitational force, gravitational mass and inertial mass are no longer equivalent ($m_1 \neq M_s$). This is a gravitational force with a difference! However, m_1 traveling at this speed is still subject to gravitational constraints, including relativistic increases.

The speed of a charged particle can be affected by the particle's interaction with radiation, a phenomenon known as the Abraham-Lorentz force. In quantum theory, however, atomic orbits are quantised and radiation is either disallowed or it is associated with a sudden and complete change in orbit and in orbital speed. The effects of this radiation are not considered here.

The paper will also make references to a 'root mean square force' (alternatively 'rms force' or F_{rms}). The rms force is so named because it resembles the square root of the electrostatic force and the gravitational force for any self-gravitating mass, yet it is essentially a gravitational force between that mass and the Stoney mass:

$$F_{rms} = \frac{GM_S m_1}{r^2} = \sqrt{\frac{Gm_1^2}{r^2} \times \frac{Ke^2}{r^2}}\,. \tag{6}$$

The rms force equates the momentum of a charged particle traveling at electromagnetic speed with the momentum of the Stoney mass traveling at gravitational speed:

$$m_1 V_E = m_1 \sqrt{\frac{Ke^2}{m_1 r}} = M_S V_G = M_S \sqrt{\frac{Gm_1}{r}} \tag{7}$$

Here the Stoney mass travels at a gravitational speed derived from the rms force - the Stoney gravitational mass and the Stoney inertial mass have self-cancelled. The rms force thus retains the equivalence of gravitational and inertial mass.

Equating two different forces

Gravity and the electrostatic force for any hydrogenic atom can be equated as follows:

$$\frac{Gm_1m_0}{n_0^4r_0^2}\times\frac{M_S^2}{m_1m_0}=\frac{Ke^2}{n_0^4r_0^2}, \tag{8}$$

$$r_0=\hbar/m_0\alpha c,$$

$$n_0=\sqrt{r/r_0},$$

where m_1 is a charged particle like the proton, m_0 is an oppositely charged particle such as the electron, r_0 is the Bohr orbit associated with m_0, and n_0 is the principal quantum number that equates the Bohr orbit with any radius r. The Bohr orbit is usually associated with the hydrogen electron but it can also be associated with the hydrogen muon. The mass ratio appearing on the LHS indicates that the electrostatic force is a modification of gravity. If the same mass ratio is inverted on the RHS, gravity appears as a modification of the electrostatic force. The Stoney scale is completely impartial and gives no priority to either force.

Contemporary scientific theory seeks to explain gravity in terms of particle physics, positing the ‘graviton’ as a boson specific to gravitational interactions. From this viewpoint, an electromagnetic theory of gravity is a more likely scenario than would be any gravitational theory of the electromagnetic force. However, expressing electromagnetic processes in gravitational terms could still be a useful exercise if it leads to a better understanding of the differences and similarities in the two forces.

Momentum is a key concept in the quantum theory of the atom and yet (7) shows that it can be expressed in gravitational terms: the momentum of a charged particle governed by the electrostatic force is equal to the momentum of the Stoney mass governed by gravity. Similarly, m_1V_Er is not *really* angular momentum in quantum theory, yet it is equal to M_SV_Gr, which can be considered angular momentum in a classical, Newtonian sense.

There are of course significant, formal differences between electromagnetic theory and gravitational theory yet many of these differences can be accommodated within the Stoney scale. The general physical background to electromagnetic theory is a flat, Lorentzian spacetime suited to linear transformations. In Einstein's General Theory of Relativity, spacetime is curved and the transformations are non-linear. To quote Einstein [10] about his own theory however:

"In order to account for the equality of inert and gravitational mass within the theory, it is necessary to admit non-linear transformations of the four co-ordinates. That is, the group of Lorentz transformations and hence the set of 'permissable' co-ordinate systems has to be extended."

In the Stoney scale of things, inertial mass and gravitational mass are equivalent in some contexts but non-equivalent in other contexts. Thus linear transformations and non-linear transformations are equally valid depending on the context.

Another significant difference between electromagnetic theory and gravitational theory lies in the concept of electric charge – electric charges can be either positive or negative and they can either attract or repel, whereas gravity is generally assumed to be universally attractive. The author has not developed a theory of gravitational polarity. However, there is nothing in General Relativity that suggests gravity must always be attractive. In GR, gravity is not so much a force as an effect of space-time geometry [11]. Thus a mass is not forced to take a particular path but simply takes the shortest possible path in curved space-time, from which indeed force is required to dislodge it. Moreover, Einstein introduced into his theory a cosmological constant that can be arbitrarily adjusted to provide for either the attraction or repulsion of the universe's mass. Thus the geometry of space-time could amount to either attractive or repulsive gravity. Einstein's cosmological constant refers to gravity on a cosmic scale and yet particle physicists have since realized that it could have a physical basis in vacuum fluctuations [12]. Vacuum fluctuations are also responsible for microscopic effects in particle physics such as the Lamb Shift and the Cassimir Effect [13]. Indeed, the Cassimir Effect can be either attractive or repulsive, depending on the geometrical configuration of the equipment [14][15]. Repulsive gravity on a microscopic scale might therefore have a mechanical origin in vacuum fluctuations. In fact, the equivalence of gravity and the electrostatic force involves such a complex variety of scenarios that the right conditions for repulsive gravity might be found in this very complexity itself. Expressed in gravitational terms, the electrostatic force could refer to a self-gravitating Stoney cloud enveloping two electromagnetic particles, or two such clouds whose centres of mass are located in electromagnetic particles, or two point particles located in electromagnetic particles, or even a combination of these scenarios. Furthermore, the Stoney mass gravitates not only with itself but also with electromagnetic particles by means of the rms force, and yet these same particles have been shown to be its inertial mass according to the electrostatic force. There is therefore a complex web of interactions that might be used to explain attraction and repulsion in mechanical rather than electrical terms.

It should also be noted that the electric charge, though useful in describing electromagnetic processes, might never the less be considered fictitious both in its dimension and its magnitude. Ampere's Law defines 1 Coulomb of charge in terms of an electrical current and the magnetic force generated by that current:

$$2\times10^{-7}\,N=\frac{\mu_0 LI^2}{2\pi d}, \tag{9}$$

where μ_0 is the permeability of free space, L is a 1 metre long conducting wire, d is a 1 metre distance from a similar wire, and I is a current of 1 Ampere or 1 Coulomb per second. A force of 2 Newtons, though hardly practicable in the context of two conducting wires, would seem more consistent with the other units, 1 meter and 1 second. If the Coulomb were defined in terms of this larger magnetic force, the squared elementary charge would be relatively smaller by 7 orders of magnitude and it would then be numerically equal to the product of any mass and its electromagnetic radius:

$$e_R^2=m\frac{\alpha\hbar}{mc}=2.57\times10^{-45}\,kg.m, \tag{10}$$

where the subscript $_R$ denotes a revised squared charge with the dimension mass x length. These same revised dimensions are obtained from Ampere's Law in (9) if the permeability of free space is understood to be dimensionless. If moreover the permeability of free space is dimensionless, the permittivity of free space then assumes the dimensions of an inverse squared speed. Coulomb's Constant is the inverse of the permittivity of free space and consequently:

$$Ke^2=c^2e_R^2. \tag{11}$$

Rationalization of dimensional quantities is a conventional practice in theoretical physics, especially in the context of natural units, most often Planck units. The squared Planck charge is equal to $\hbar/c$ increased however by 7 orders of magnitude and assigned the units squared Coulombs instead of the more obvious kg.m - these artificial adjustments are needed to bring it into alignment with the squared elementary charge. Often the squared Planck charge is then 'normalized' to 1 while the squared elementary charge is normalized to α. This convention is easily accommodated to the revised squared charge:

$$\frac{\hbar}{c} = \frac{e_R^2}{\alpha}. \tag{12}$$

If charge were defined in the manner suggested here, many quantities that now seem distinctively electromagnetic in dimension would instead seem quite consistent with gravitational relations. Polarity might not then seem a peculiarly electromagnetic phenomenon related to some unique dimension called 'charge' but might instead be thought to have a mechanical origin in the way inertial and gravitational masses interact with each other and possibly with the energy vacuum.

The numerical resemblance between the squared elementary charge and the squared 'revised' charge is the first of the large number coincidences to be dealt with in this paper:

$$e^2 / e_R^2 = \eta 10^7 , \tag{13}$$

$$\eta = 1C^2 kg^{-1} m^{-1} .$$

This coincidence leads to many other coincidences in electromagnetic quantities. For example, the Quantum Hall Effect [16], defining quantum leaps in electrical resistance at very cold temperatures and in strong magnetic fields, features Planck's Constant divided by the product of the squared elementary charge and some integer or some vulgar fraction *i*. In revised units, this is simply a speed:

$$h/ie_R^2 = 2\pi\, c/i\alpha\ . \tag{14}$$

This speed is faster than light for low integer values of *i* and for fractional values. According to the wave/particle duality of quantum theory, these faster than light speeds refer to a wave's phase velocity (c^2/v) and they can be associated with a particle traveling at speeds below light speed (v). The same wave/particle duality extends the significance of the Stoney scale beyond the mere interaction of charges particles, allowing even electromagnetic waves to be explained in terms of a self-gravitating Stoney mass. The speed of a massive photon, traveling with an electromagnetic wave whose group velocity is the speed of light, could for example be calculated as follows:

$$\sqrt{\frac{c^2 e_R^2}{m_1 r}} = \sqrt{\alpha}\, c\ , \tag{15}$$

$$r = \hbar/m_1 c\ ,$$

where m_1 is the mass of the photon associated with the given wavelength. The Stoney mass self-gravitates in a wave traveling at the speed of light in so far as its inertial mass is the photon itself, which is not traveling at the speed of light. The speed calculated in (15) is purely speculative yet it is the sort of equation that follows from an extension of the Stoney scale's significance to cover all electromagnetic processes.

Electromagnetic theories of gravity

The co-incidental resemblance of certain large numbers inspired the English physicist Paul Dirac to imagine a cosmology based on a gravitational constant that changes over time [17][18]. The first of these large numbers, usually denoted ξ, is a factor that equates the electrostatic and gravitational forces for two quantum particles. In the case of the electron (m_e) and proton (m_p):

$$\xi = Ke^2 \big/ Gm_e m_p \approx 10^{40} . \tag{16}$$

The second large number can be interpreted as the ratio of the estimated radius of the universe to the characteristic radius of a fundamental particle:

$$ct \big/ R \approx 10^{40} , \tag{17}$$

where t is the age of the universe and R could for instance be the electron's electromagnetic radius. Equating (16) and (17) then leads to the Dirac inference that the gravitational constant decreases as the universe ages:

$$G = Ke^2 R \big/ m_e m_p ct . \tag{18}$$

Dirac's hypothesis implies that there might be some profound relation between gravity and the electromagnetic force. Science has so far not found any such relation between the two forces yet the symmetries in Dirac's hypothesis suggest that a unified theory could be aesthetically pleasing and it has never lost its charm. There is a considerable body of literature dedicated to it, either reinforcing it (e.g. [19][20]) or subjecting it to scientific restraints based on astrophysical and geophysical observations (e.g. [21] [22]).

The 2002 CODATA value for the Gravitational Constant presents an opportunity for a new interpretation of the Dirac hypothesis:

$$G = 6.6742 \times 10^{-11} kg^{-1} m^3 s^{-2}, \tag{19}$$

$$M_S = 1.859223 \times 10^{-9} kg,$$

$$\xi = M_S^2 / m_e m_p = 2.268695 \times 10^{39} = \nu_p \sigma 10^{16},$$

$$\nu_p = m_p c^2 / h = 2.2687316 \times 10^{23} s^{-1},$$

$$\sigma = 0.999984 s,$$

$$G = Ke^2 / m_e m_p \nu_p \sigma 10^{16}.$$

Thus Dirac's number ξ bares a numerical resemblance to the proton's Compton frequency (ν_p) and G can be expressed entirely in terms of electromagnetic quantities.

The author sent a draft form of this formula for the Gravitational Constant to James Gilson, who subsequently provided it with σ. Gilson has since employed it to develop a quantum theory of gravity based on a Dirac-style cosmology [23]. He has shown that Dirac's number is closely related to a fundamental eigen-quantum number N_G that varies with epoch. While he does not equate $\sigma 10^{16}$ with the age of the universe, he is able to derive an age of the universe from it. The Stoney scale does not feature in Gilson's theory and yet it is clear that ξ, in equating gravity with the electromagnetic force, is formally a Stoney number. The Stoney scale in fact hardly features in any of the scientific literature dedicated to the Dirac hypothesis.

It can be shown that ξ is a specific instance of a general class of numbers that are represented by the product $\nu_E t_G$ where ν_E is an electromagnetic frequency and t_G is a gravitational time:

$$\nu_E t_G = \alpha c \hbar / G m_1 m_2 = M_S^2 / m_1 m_2, \tag{20}$$

$$\nu_E = \alpha c/r \ ,$$

$$t_G = \lambda/V_G = \hbar r/Gm_1 m_2 \ ,$$

$$\lambda = \frac{\hbar}{m_2 \sqrt{Gm_1/r}} \ ,$$

$$V_G = \sqrt{Gm_1/r} \ .$$

The electromagnetic frequency features the classic Bohr orbit speed αc, allowing any mass's Compton frequency to be derived from its electromagnetic radius. The gravitational time t_G is the Compton time associated with gravitational potential energy but perhaps it can better be understood as the time it takes any mass m_2 to travel the de Broglie wavelength derived from the given gravitational speed. In a cosmology where G is invariant, the $\nu_E t_G$ product is invariant for any pair of masses. However, the different times and frequencies that make up the product are always determined by the distance *r*. For the electron and proton $\nu_E = \nu_p$ and $t_G = \sigma 10^{16}$ when *r* is equal to the proton's electromagnetic radius multiplied by 2π:

$$r = 2\pi\, ac\hbar/m_p c^2 \ . \qquad (21)$$

The proton is not an elementary particle since it comprises quarks. Its electromagnetic radius is therefore even less physical than is the electron's electromagnetic radius. The proton's Compton frequency is similarly a mathematical construct and it cannot be associated with a physical wavelength. In Gilson's theory, the proton's rest mass is induced by gravitons, a process that gives its Compton frequency a physical significance it otherwise would not have. This author, though sympathetic to Gilson's theory, takes a broader view – the proton's Compton frequency has heuristic rather than physical significance and other gravitational theories could emerge from its mathematical association with real physical quantities. The same is true of the proton's electromagnetic radius.

The proton's electromagnetic radius emerges in a variety of contexts associated with $\nu_p \sigma 10^{16}$. Consider for example the electromagnetic speed of the hydrogen electron, which can be expressed as follows:

$$\frac{\alpha c}{n_0} = \sqrt{\frac{Gm_1}{n_0^2 r_0} \times \nu_p \sigma 10^{16}} = 2\pi L_S \sqrt{\frac{\nu_1 \nu_p c\sigma 10^{16}}{\alpha 2\pi r}}\,, \tag{22}$$

$$\frac{c\sigma 10^{16}}{\alpha 2\pi} = \frac{\hbar \sigma 10^{16}}{2\pi e_R^2} \approx 6.5 \times 10^{25}\, m \approx R_U\,,$$

where m_1 is the proton, ν_1 is its Compton frequency (equal to ν_p), r_0 is the electron's Bohr radius, n_0 is the electron's principal quantum number, r is any radius that is a multiple of r_0, and R_U is the radius of the universe. It's worth noting that one of the factors comprising R_U is the unit of electrical resistance associated with the Quantum Hall Effect as defined by (14). The proton's electromagnetic radius emerges from these relations when $n_0 = 1$:

$$L_S \sqrt{\frac{c\sigma 10^{16}}{\alpha 2\pi r_0}} = L_S \sqrt{\nu_0 \sigma 10^{16}} = \frac{\alpha \hbar}{m_p c} \approx L_S \sqrt{\frac{R_U}{r_0}}\,, \tag{23}$$

where ν_0 is the electron's Compton frequency. The proton's electromagnetic radius is equated with the Stoney length by means of a Dirac-style length ratio. Moreover, the equation indicates that the Stoney scale mediates between microscopic and macroscopic processes and this is entirely consistent with its electromagnetic and gravitational nature. In a varying G cosmology, changes in G are offset by changes in σ and therefore (22) and (23) both define a constant relation. The Stoney scale however is not necessarily associated with a varying-G cosmology and therefore the Dirac-style ratio can be omitted from the equation even though this might reduce its aesthetic appeal.

A very different electromagnetic theory of gravity that has received wider attention is Stochastic Electrodynamics (SED). According to mainstream physics, gravity and the electromagnetic force may be united only at the very high energy levels typical of the early universe. According to SED, on the other hand, they have always been the same force. The theory began with a surmise by the Russian physicist Andrei Sakharov that gravity might be due to the way that mass interacts with the energy vacuum [24]. Sakharov's ideas were further developed by Harold Puthoff [25] [26]. Puthoff identifies gravity with an attractive, secondary electro-magnetic field produced by zitterbewegung (tiny, rapid movements of elementary particles interacting with zero-point fields).

Puthoff's particular interpretation of SED is generally considered within the scientific community to be flawed and even some of his colleagues have chosen to explore different approaches [27] [28]. Wesson [29] argues that Puthoff's theory implies the existence of Planck-size particles, for whose existence there is no scientific evidence.

The Stoney scale appears naturally suited to Stochastic Electrodynamics even though it hardly features in the relevant literature. It is the natural scale for unification when gravity and the electromagnetic force are united at every time in the universe's history. Moreover the Stoney mass resembles the dynamic SED vacuum as an invisible background to physical effects. Whereas however SED interprets gravity as an electromagnetic effect of the invisible background energy, the Stoney scale is impartial and can also be used to explain electromagnetic processes as the gravitational effect of an invisible background mass. The SED approach would in fact require the Stoney energy to be defined as a quantum of vacuum energy associated with the interactions of elementary particles. A quantum of vacuum energy is already implied in Puthoff's approach, which is the basis for Wesson's objection to an unproven 'Planck-size' particle (in fact there is at least one published paper that claims to have found physical evidence of Planck and Stoney energies at work in the form of dark matter [30]). Wesson also criticizes Puthoff's insistence that a gravity-inducing vacuum does not self-gravitate, an insistence that Wesson attributes to Puthoff's concern about cosmological consequences. Wesson notes that the self-gravitation of the vacuum is feasible so long as it is cancelled out by something like gravitational potential energy or super-symmetry. The same sort of self-cancellation could result from the Stoney scale, depending on the number of Stoney masses associated with all the electromagnetic particles in the cosmos. The mutual gravitation of all these Stoney masses could provide enough gravitational potential energy to cancel out their combined electromagnetic energy, in the same way that the net electrical charge of the universe is zero.

In the Puthoff scheme of things, the electron radiates energy in the classical manner and yet this energy is replaced by energy radiated from the vacuum, thereby resulting in the quantum stability of the Bohr orbit [31]. Equations (22) and (23), which associate the Bohr orbit with Compton frequencies, could be interpreted as an analog of this 'classical' theory of the stability of the ground state orbit. One of Sakharov's criteria for a vacuum-induced theory of gravity was that that the Gravitational Constant should be predicted from within its parameters. Dirac's large number ξ, formally a Stoney number, is well suited to such a requirement.

The Stoney-Boltzmann coincidence

Stoney was a man of indomitable energies and scientific spirit who contributed significantly to many areas of scientific research. One of his achievements was an estimation of the number of molecules in a cubic millimetre of gas using data obtained from the newly emerged kinetic theory of gases. His interest in the theory of gases had nothing to do with his discovery of the Stoney scale. However, it is a curious fact that the Stoney length bares a numerical resemblance to Boltzmann's Gas Constant. Deriving the Stoney length from 2002 CODATA values:

$$L_S = 1.3806681 \times 10^{-36} m, \tag{24}$$

$$k_B = 1.3806505 \times 10^{-23} J.K^{-1},$$

$$k_B / L_S = \zeta 10^{13} J.m^{-1}.K^{-1},$$

$$\zeta = 0.9999872 J.m^{-1}.K^{-1},$$

where k_B is Boltzmann's Constant. The author sent a draft form of these equations to Gilson who subsequently provided ζ. Gilson is particularly interested in the cosmic implications for entropy and dark energy and yet the papers he has subsequently written on the subject feature little or no reference to the Stoney mass e.g.[32].

The difference in magnitude between the Stoney length and Boltzmann's constant is no obstacle to their identification. The Kelvin absolute temperature scale was conceived at a time when ice was considered cold (0^0C = 273.15K) yet scientists today are routinely working with temperatures in the nanoKelvin. A revision of the absolute temperature scale by a dozen orders of magnitude would take the 'nano' out of 'nano-Kelvin'. It would result in changes to some familiar quantities but it would not affect the basic relations defined by those values nor would it affect our understanding of those relations. The large discrepancy in magnitudes is therefore an historical accident without any fundamental significance. It is the dimensional difference between the two quantities that requires some explaining.

Boltzmann's Constant is a constant of proportionality that allows energies to be expressed in terms of absolute temperature. In the context of real gases, however, this constant is not really constant at all and therefore equations relating the energy of a gas to its absolute temperature, such as the van der Waals equation of state, necessarily feature additional terms. Those additional terms compensate for the fact that a real gas isn't just a collection of point particles but comprises atoms or molecules that have intrinsic volumes and which, moreover, are subject to intermolecular forces. Boltzmann's Constant is thus simply an ideal extrapolated from statistical trends that were obtained from the study of gases whose behaviour approximates to an ideal. Typically real gases depart from ideal behaviour at high pressures and low temperatures.

Real gases are subject to a variety of forces, which can be classified as follows:

1. Gravity, which is a significant factor in very large gases such as stars but which exceeds other forces only in the case of gravitational collapse, as in the formation of black holes;
2. intermolecular forces, which may be electrical or quantum mechanical in nature but which are generally much weaker than the electromagnetic force;
3. thermodynamic forces, which are due to the random movements and 'collisions' of gas particles under the constraints of volume and pressure.

Within the simplistic context of the ideal gas law, the only significant force that affects the behaviour of quasi point particles is the thermodynamic force (F_T) exerted by laboratory technicians through adjustments in pressure and area:

$$F_T = PV^{\frac{2}{3}}, \tag{25}$$

where P is pressure and V is volume. An ideal gas is then related to absolute temperature by means of a very simple equation:

$$PV = Nk_B T, \tag{26}$$

where T is absolute temperature and N is the number of point particles. Switching N to form a quotient on the LHS is all that is required to calculate the thermal energy and the volume occupied by a single *ideal* particle (the thermal energy of real molecules involves some additional factors). Hereafter in this paper PV and V will be understood to refer to the energy and volume of a single particle.

The dimensions given to $\zeta 10^{13}$ in (24) indicate that this quotient has an association with force, since Joules per metre can equally well be interpreted as newtons. Moreover, the Stoney length indicates that this associated force must be the rms force:

$$\frac{PV}{L_S} = \frac{GM_S m_{PV}}{L_S^2} = T\zeta 10^{13}, \tag{27}$$

$$m_{PV} = PV/c^2,$$

where m_{PV} is the energy PV expressed as mass. This particular form of the rms force is typically huge for particles approximating to the mass of an atom or molecule and it is of course unrealistic. It is however readily associated with ideal gases by means of a less extreme form of the rms force:

$$\frac{PV}{L_S} = \frac{F_T}{F_{rms}} \times \frac{mc^2}{V^{\frac{1}{3}}} = T\zeta 10^{13}, \tag{28}$$

$$F_T = PV^{\frac{2}{3}},$$

$$F_{rms} = GM_S m / V^{\frac{2}{3}},$$

where m is the mass of a real atom or molecule, typically approximating to the mass of a few protons. The ratio of the energy PV to the Stoney length is here equated with the ratio of the particle's rest energy to the cube root of its volume, multiplied however by a force ratio. The symmetry within the equation suggests that a temperature of fundamental significance should occur when $F_T = F_{rms}$:

$$\frac{PV}{L_S} = \frac{mc^2}{V^{\frac{1}{3}}} = T\zeta 10^{13}. \qquad (29)$$

This temperature T could in fact represent minimum temperature in the context of real gases, as becomes apparent when we equate the gas particle with a real atom and the cube root of V with the van der Waals radius for that atom. The van der Waals radius applies to weakly interacting atoms [33] and it is therefore a good approximation to the radius of a quasi ideal particle that is not subject to intermolecular forces. Noble or inert gases have the weakest intermolecular reactions of all the gases and the best known of them is Helium, with a Van der Waals radius of 1.4 x 10^{-10} meters and an atomic mass of 4 grams per mole or approximately 6.6 x 10^{-27} kilograms:

$$m_{He} c^2 / r_{vdw} \zeta 10^{13} \approx 4.3 \times 10^{-13} K, \qquad (30)$$

where m_{He} is the mass of the Helium atom and r_{vdw} is its van der Waals radius. Applying the same formula to all the noble gases results in an average temperature of approximately 4 x 10^{-12} Kelvin. This average temperature is three orders of magnitude below the temperature achieved by Cornell and Wieman in the first successful production of a Bose-Einstein condensate, using a 'gas' of two thousand ^{87}Rb atoms [34] [35]. It is five orders of magnitude smaller than the temperature achieved by Jin and DeMarco in the first successful production of a degenerate Fermi gas, using about 500 million ^{40}K atoms [36] [37]. Never the less, it is close enough to these temperatures to be considered physically plausible as a candidate for minimum temperature in relation to real gases. Moreover, as Jin observes, the temperature limit is still an issue under consideration [37].

The Ideal Gas Law is known to be inaccurate for real gases at low temperatures, when intermolecular forces begin to dominate over thermodynamic forces. At the extremely low temperatures mentioned above, however, the intermolecular forces themselves are very weak, allowing gases to condense to the extent that the volume occupied by any one atom is hardly more than that atom's intrinsic volume (in the case of a Bose-Einstein condensate many atoms might even occupy the volume of a single atom).

Ideally zero temperature is the temperature of the energy vacuum. It is conventional in theoretical physics to associate the energy vacuum with the Planck scale, from which the Stoney scale differs only by a factor equal to the square root of the fine-structure constant or approximately one order of magnitude. *Ideally* therefore the Planck scale should feature somewhere in a derivation of Boltzmann's constant from theory or, if not the Planck scale, then some other fundamental scale that approximates to it, in this case the Stoney scale. *Ideally* intermolecular forces should not exist at zero Kelvin, particles should not gravitate and the electromagnetic force should cease to function. *Ideally* therefore gravity and the electromagnetic force are united in their negation at zero Kelvin. The Stoney scale is the scale of unification and therefore Boltzmann's Constant could well be associated with the Stoney length *in theory*. The author however is unable to provide an explanation of the physical mechanisms that would turn this theoretical association into scientific fact.

Discussion

This paper has analysed the Stoney scale in relation to three large number coincidences and it is therefore appropriate to consider the nature of these coincidences. Coincidences can be quite varied but a reasonable classification could be as follows:

1. *The accidental coincidence*: this relates things that have no necessary association with each other such that one thing cannot be deduced or predicted from the other;
2. *The non-coincidence*: this relates things that might appear to have no necessary association until it subsequently found that one can in fact be deduced or predicted from the other;
3. *The useful coincidence*: this relates things that have no necessary association but whose association leads to a better understanding of those or other things.

So which kinds of co-incidences are the number coincidences dealt with in this paper? Each shall be considered separately and in the order in which they were presented.

Firstly, e_R^2 and e^2: This is a non-coincidence. The revised squared charge can be deduced from the squared elementary charge when the permeability of free space is considered to be dimensionless. According to SI convention, the permeability of free space is equal to $4\pi \times 10^{-7}$ Kg.m.s^{-2}.A^{-2}. This value however is not consistent with the role played by the permeability of free space as a base unit relative to which the permeability of other media is defined:

$$\mu/\mu_0 = \mu_r, \tag{31}$$

where μ is the permeability of some medium other than free space, and μ_r is the relative permeability of that other medium. The relative permeability of magnetic iron, for instance, is 200. A base unit cannot consistently have a magnitude of 10^{-7} when its own relative value is 1 (in the SI context, it is 1 multiple of 4Pi). A dimensionless form of the permeability of free space conventionally appears in the definition of Coulomb's Constant:

$$K = 1/4\pi\varepsilon_0 , \tag{32}$$

where ε_0 is the permittivity of free space and 4Pi is a dimensionless number. If however the permeability of free space is in fact a dimensionless multiple of 4Pi the significance of (32) changes:

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{e_R^2}{\mu_0\varepsilon_0 r^2} = \frac{c^2 e_R^2}{r^2} . \tag{33}$$

This electrostatic force is quite distinct from the magnetic force. When Maxwell's equations are interpreted in the context of the revised charge, the magnetic force is calculated after some re-arrangement as follows:

$$e\vec{v} \times \vec{B} = \frac{c^2 e_R^2}{r^2} \times \frac{e_R^2}{mr} = \frac{v^2 e_R^2}{r^2} , \tag{34}$$

where $\vec{B}$ is the magnetic field associated with the elementary charge e, and where m is the mass of the charged particle traveling at the electromagnetic speed v. The magnetic force thus expressed is clearly a relativistic variation of the electrostatic force. The Lorentz force is of course obtained as the sum of (33) and (34).

e_R^2 and e^2 could also prove to be a useful co-incidence. It could for instance lead to greater self-consistency in electromagnetic theory. For instance, an electromagnetic field in free space implies that free space is a medium for an electric current – this led Maxwell to define free space as 'the ether', a subtle substance capable of hosting a 'displacement current' [38]. In modern theory, there is no ether and consequently a displacement current in free space is simply a mathematical construct that defines physical effects but which is not an actual current. The revised charge avoids this confusion. A revised electrical current can be associated with the movement of charged particles but it is by definition the square root of the magnetic force. The vacuum of free space might be a barrier to an electrical current but it is no barrier to a force. A vacuum therefore is no barrier to electromagnetic waves that are generated by force rather than by current.

The second coincidence associates Dirac's ξ with the proton's Compton frequency. ξ cannot be deduced from ν_p nor can ν_p be deduced from ξ. Their association is a speculative inference and it would therefore be reasonable to believe that this particular coincidence is of an accidental kind. It could however be a useful coincidence for the development of a unified theory (it was for example a formative influence in Gilson's theory). In fact, $\nu_p \sigma 10^{16}$ is a specific interpretation of the $\nu_E t_G$ product and it could refer to any combination of frequencies and times. The $\nu_E t_G$ product can be deduced from Dirac's ξ.

As previously stated, the viewpoint of mainstream physics is that gravity and the electromagnetic force were the same force at the very high energy levels typical of the early universe. The $\nu_E t_G$ product is one way of phrasing this difference in energy levels and it could be interpreted in a purely historical context and in relation to a wide variety of particles, such as the familiar electron and proton or their historic prototypes. This same difference in energy levels can be expressed in terms of mass ratios featuring the Stoney mass and therefore the Stoney mass could also be understood in a purely historical context. More radically, but consistently with SED, the $v_E t_G$ product and the Stoney mass ratios could be understood to refer to a mediating energy that has always been present and which unites the two forces even today (those who would dismiss such an omnipresent, mediating energy as bizarre should remember that String Theory requires us to believe in multiple hidden dimensions and multiple universes).

The third coincidence associates the Stoney length with Boltzmann's Constant. Such is the difference in magnitudes and dimensions that this coincidence appears at first glance to be nothing but an accident. However, the difference in magnitude has been shown to be irrelevant. The difference in dimensions is inexplicable if the behaviour of ideal gases is described only by the kinetic theory of gases. If however the behaviour of ideal gases can also be explained in terms of forces, Boltzmann's Constant might well have length as an alternative dimension. The Stoney length is well suited to this role by reason of its approximation to the scale of the energy vacuum. Never the less, the coincidence cannot be considered a non-coincidence until an underlying physical mechanism has been identified.

The existence of *three* large number co-incidences associated with a *single* 'natural' scale is itself an interesting coincidence. This accumulation of coincidences could simply reflect the fact that the Stoney scale, like the Planck scale, is conveniently central to macroscopic and microscopic quantities, many of which could be coincidentally related to each other through ingenious interpretations. This however merely underscores the flexibility of the Stoney scale, which indeed makes it a good natural scale for any unified theory.

Einstein at least seems to have had an intuitive understanding of the significance of the Stoney scale, particularly in the context of 'gravitational ether'. The ether of free space is generally considered a discredited concept since it implies an absolute frame of reference, contrary to the Special Theory of Relativity, which implies that there is no absolute frame of reference. However, in an address at the University of Leyden on May 5, 1920 [39], Einstein attributes the discredited notion of ether to Lorentz, while himself giving favourable consideration to 'gravitational ether'. Einstein argues that 'Lorentzian ether' is conditioned by nothing outside itself and it is the same everywhere, whereas 'gravitational ether' is affected by its connections with matter and by its connections with other gravitational ether in neighbouring places. He says he doesn't know if gravitational ether differs from Lorentzian ether only when in the proximity of masses, and he doesn't know if gravitational ether has an essential share in the structure of elementary particles. He is however insistent that relativity theory forbids any kind of motion to be ascribed to gravitational ether. This insistence seems a little difficult to understand – how can gravitational ether be conditioned by its connections with other gravitational ether if no motion can be ascribed to it? The argument becomes more intelligible however if we substitute the Stoney mass for gravitational ether. The Stoney mass moves vicariously through 'charged' particles. Without those particles, it has no location and therefore no motion can be ascribed to it. The fact that Einstein was prepared to believe that gravitational ether might have some fundamental association with elementary particles is a further indication that he was thinking at least intuitively about the Stoney mass. Similarly, the ambivalence of the Stoney scale is consistent with Einstein's own uncertainty about the exact circumstances that distinguish gravitational ether from Lorentzian ether.

However, the strongest clue to the nature of Einstein's thinking about gravitational ether in his Leyden address is his explicit mention of Weyl's approach to a unified theory, which he considers to have doubtful chances of success (though he there does not say why). Einstein's mention of Weyl indicates that he himself must have given some thought to the Stoney scale. The fact that he mentions Weyl in a speech about gravitational ether, which resembles the Stoney mass in some key aspects of its form and behaviour, is surely more than a mere coincidence. The question then is – to what extent was his thinking about gravitational ether influenced by his knowledge of the Stoney scale? How much about the Stoney scale did Einstein actually know and how much did he merely intuit? Did he know, for instance, that the Stoney scale requires the non-equivalence of gravitational and inertial mass? Weyl's approach involves great mathematical sophistication and it avoids the simple kind of analysis that is needed to uncover this aspect. It is possible therefore that Einstein never grasped this fact either. It certainly doesn't enter into his thinking about gravitational ether in the Leyden address. We can only speculate what Einstein might have made of this aspect of the Stoney scale if he had known about it. He abandoned the concept of gravitational ether towards the end of his life.

Einstein's gravitational ether can be understood as a parcelling out of vacuum energy. The physics of vacuum energy is barely known even to contemporary science and it is therefore a convenient prop to speculation. Stochastic Electrodynamics is itself based on speculative assumptions about vacuum physics. The English physicist Edward Tryon speculated that the entire universe might simply be a fluctuation in the vacuum, the self-cancellation of electromagnetic energy and gravitational potential energy providing for zero net energy [40]. It has recently been argued that the vacuum, in addition to uniting forces in the SED manner, might also unite mind and matter [41]. Like all these speculative theories, the Stoney scale's association with vacuum energy or gravitational ether can be dismissed harshly as speculation devoid of real scientific proof, or it can be considered more leniently as one more option in an ongoing enquiry into the requirements of a unified theory. This much however cannot be denied – the opportunities offered by the Stoney scale have hardly been studied let alone exhausted by theoretical physics.

Acknowledgement: I am indebted to James Gilson both for his criticisms and his encouragement over more than two years.

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