

Electromagnetic Waves, Inertial Transformations and Compton Effect

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The Inertial Transformations (IT) are a new set of transformations of the space and time variables providing an alternative (but empirically equivalent) approach to the Theory of Special Relativity (TSR). With the IT, the one way velocity of light is isotropic only in a privileged reference frame, S_0 . In this new theory only a weak form of relativity principle holds. We apply the IT to the collision of an energetic photon with an electron (Compton effect). A theoretical description of the dual quantum mechanical photon having both wave and particle properties is required. From the undulatory point of view we use the IT to deduce the e.m. wave equations in an inertial reference frame S moving with respect to S_0 with (absolute) velocity \vec{V} . Using the Maxwell equations in the form suitable to S , we show that the e.m. plane waves in S have the same properties as in S_0 : the fields are perpendicular to one another and perpendicular to the propagation direction of the field energy. The latter direction, however, does not coincide, in S , with the propagation direction of the e.m. plane wave. From the corpuscular point of view, we show that in the

framework of the IT the usual equations relating the photon energy and momentum to frequency also hold. The result of this research on the Compton effect is a complete empirical equivalence between the TSR and the IT approach.

1. The inertial transformations

The Theory of Special Relativity (TSR) and the Lorentz Transformations (LT) are fundamentally based on the well known Einstein synchronization of clocks. Mansouri and Sexl [1] showed that the Lorentz transformations contain a purely conventional term devoid of any empirical basis: the coefficient of x in the transformation of time. The task of this coefficient is to ensure that the one way velocity of light has the invariant value c .

A more detailed examination of the problem shows that there are many methods of synchronizing clocks, the most used ones being essentially four: Einstein's method, slow transport method, absolute method and method of symmetrical velocities. Selleri [2] succeeded in obtaining a set of transformations of space and time in which a suitable free parameter appears, e_1 (*synchronization parameter*), from which the TSR is obtained by considering a particular nonzero value of e_1 : in this way it is possible to formulate an infinite set of theories empirically equivalent to the TSR, that are theories compatible with the following assumptions:

1. An inertial reference system S_0 exists in which the velocity of light is c in every point and direction;
2. the two way velocity of light is the same in all directions and in all inertial reference frames;
3. the pace of clocks in motion with respect to S_0 with velocity V slows down by the usual factor $R = \sqrt{1 - \beta^2}$, where $\beta = V/c$.

One reasonably assumes also that

- i. space is homogeneous and isotropic, and time is homogeneous, at least for observers at rest in S_0 ;
- ii. The axes of S and S_0 coincide for $t = t_0 = 0$;
- iii. The origin of S , seen from S_0 , moves with velocity $V < c$ parallel to the $+x_0$ axis, that is according to the equation $x_0 = V t_0$;
- iv. Maxwell's equations in S_0 hold in the usual form;

The equivalent transformations (ET) from S_0 to S obtained by Selleri are:

$$\left\{ \begin{array}{l} x = \frac{x_0 - Vt_0}{R} \\ y = y_0 \qquad \qquad \qquad z = z_0 \\ t = Rt_0 + e_1(x_0 - Vt_0) \end{array} \right. \quad (1)$$

The TSR is recovered for $e_1 = -\beta/Rc$, value introducing a well known symmetry between the space and time variables, forcing the latter to a geometrical role in the fourdimensional Minkowski space. Different values of e_1 correspond to different theories of space and time.

Remarkable consequences of the ET (1) are:

A1. Relatively to the system S , the one way velocity of light propagating in a direction forming an angle \mathcal{G} with the velocity V of S relative to S_0 (*absolute velocity*) is:

$$c_1(\mathcal{G}) = \frac{c}{1 + \Gamma(\beta)\cos \mathcal{G}} \quad (2)$$

with $\Gamma(\beta) = \beta + ce_1 R$. Notice that, for all theories satisfying (i) and (ii) and having $\Gamma \neq 0$, S is a *necessarily anisotropic frame*, so that S_0 , assumed isotropic, is unique.

A2. Lorentz-Fitzgerald contraction. A rod at rest in S , whose extremes coincide with the coordinates x_1 and x_2 , is seen in motion from S_0 where it appears shortened along the x_0 direction according to the equation: $x_{02} - x_{01} = R(x_2 - x_1)$. A rod at rest in S , whose extremes have coordinates y_1 and y_2 , has the same length in S_0 as in S .

A3. Larmor retardation. A clock at rest in S marking time t , is seen in motion from S_0 where it appears retarded according to the equation: $t_{02} - t_{01} = (t_2 - t_1)/R$.

A4. Michelson and Fizeau type experiments, the aberration of starlight, the occultations of Jupiter satellites, the radar distances of planets and the determination of the International Atomic Time do not depend on the value of e_1 as it was to be expected.

Rizzi and collaborators [3] proved the following theorem: Selleri's assumptions:

1. At least one Inertial Reference Frame (IRF) S_0 exists in which the velocity of light is c in all points and in all directions (S_0 is optically isotropic).
2. The two ways velocity of light is the same in all IRFs and in all directions.
3. The pace of clocks in motion with respect to S_0 with velocity V slows down by the usual factor $\sqrt{1 - \beta^2}$, with $\beta = V/c$.

are *equivalent* to Einstein's basic assumptions of the TSR.

According to Rizzi *et al.*, the important consequences of this theorem are:

- a) No experiment can discriminate between different values of e_1 ;
- b) It is impossible to detect the privileged IRF S_0 .

Therefore the role of privileged IRF played by S_0 is only artificial as S_0 is the IRF in which, by convention, the Einstein synchronization procedure was adopted. Any IRF S can then play the role of S_0 .

- c) The transformation:

$$\begin{cases} t = \hat{t} + \frac{\Gamma(\beta)}{c} \hat{x} \\ x = \hat{x}; \quad y = \hat{y}; \quad z = \hat{z} \end{cases} \quad (3)$$

allows one to pass, within any given IRF S , from the Einstein synchronization to Selleri's generalized synchronization. In this way one also passes from the Lorentz to the equivalent transformations.

Point a), it is our conviction, is not generally true. The hypothetical indifference of the objective reality with respect to clock synchronization holds only so far as Weakly Accelerated Reference Frames (WARFs) are excluded [4]. In fact the physical continuity with such frames chooses the transformations specified by the condition $e_1 = 0$ (called Inertial Transformations).

Point b) states the impossibility of detecting the privileged IRF S_0 . In fact there is a well defined way to resynchronize clocks [5] in all IRFs allowing one to pass from a given privileged IRF S_0 to another privileged IRF S , arbitrarily chosen, where the "privilege" is the isotropy of space, e.g. with respect to the propagation of light. The equivalent theories (1) possess the following properties:

1. It is impossible to detect experimentally the existence of an absolute motion of the Earth with respect to any eventually existent privileged IRF. This is known as Weak Relativity Principle (WRP).

2. The two way velocity of light is invariant.

Due to the WRP the predictions of the equivalent theories for the fundamental experiments (Bradley, Römer, Fizeau, Foucault, Michelson, ecc.), are identical to the predictions of the TSR.

The inertial transformations, obtained from (1) by assuming $e_1 = 0$ are:

$$\left\{ \begin{array}{l} x = \frac{x_0 - Vt_0}{R} \\ y = y_0 \quad ; \quad z = z_0 \\ t = Rt_0 \end{array} \right. \quad (4)$$

Remarkable consequences of the inertial transformations (4) are all those already seen for the ET (A1, A2, A3, A4).

The origin of S , observed from S_0 , is seen to move with velocity $V < c$, while the origin of S_0 , observed from S , is seen to move with velocity $-\beta c/R^2$. The latter velocity can be larger than c , but cannot be superluminal. In fact a luminous pulse travelling in the direction $-x$ has a velocity given by (2) with $\vartheta = \pi$, that is $\tilde{c}(\pi) = c/(1-\beta)$ which is certainly larger than $\beta c/R^2$, if $0 \leq \beta < 1$. The relative velocities, in any direction ϑ , can grow without limit, but they always remain smaller than $c_1(\vartheta)$. The absolute velocities can never be larger than c .

Absolute simultaneity. Two spatially separated events happening at the same time in the IRF S_0 , are simultaneous also with respect to S .

The remarkable successes marked by the theory of the IT push us to extend their applications to "dynamical" physical phenomena, recalling that up to the present time the published applications had

mainly a kinematical nature. We will therefore consider the Compton effect, that is a phenomenon in which the relativistic treatment of electrons and photons is considered essential. In spite of this we will show that the theory based on the IT is perfectly able to explain the empirical evidence if the dual nature of the photon (particle and wave) is taken into account.

2. Propagation of energy

We consider the propagation of a light particle P (in practice a very small light pulse) in the privileged frame S_0 , relative to which the velocity of light is c in all directions. The position of P is given by two coordinates x_0 and y_0 satisfying, at time t_0 :

$$x_0 = c \cos \mathcal{G}_0 t_0 \quad ; \quad y_0 = c \diamond\diamond\diamond \mathcal{G}_0 t_0 \quad (5)$$

Relative to the moving inertial frame S , which superimposes with S_0 at time $t_0 = t = 0$, we describe P with the coordinates x and y satisfying, at time t

$$x = c_E \cos \mathcal{G}_E t \quad ; \quad y = c_E \sin \mathcal{G}_E t \quad (6)$$

Given the smallness of the pulse, the energy transported by it is clearly propagating with the same velocity and in the same direction. This is the reason why the index E was appended to velocity c and angle \mathcal{G} .

Our first task is to use the inertial transformations to determine c_E and \mathcal{G}_E in terms of the S_0 quantities c and \mathcal{G}_0 , which are considered known.

The ("inverse") inertial transformations from the moving frame S to the privileged frame S_0 are:

$$x_0 = R x + \frac{V}{R} t \quad ; \quad y_0 = y \quad ; \quad t_0 = \frac{t}{R} \quad (7)$$

Substituting (7) into (5) we find

$$x = \frac{c}{R^2} (\cos \mathcal{G}_0 - \beta) t \quad ; \quad y = \frac{c}{R} \diamond\diamond\diamond\diamond \mathcal{G}_0 t \quad (8)$$

with $\beta = V/c$. A comparison of (6) with (8) gives

$$c_E \cos \mathcal{G}_E = \frac{c}{R^2} (\cos \mathcal{G}_0 - \beta) \quad ; \quad c_E \diamond\diamond\diamond\diamond \mathcal{G}_E = \frac{c}{R} \diamond\diamond\diamond\diamond \mathcal{G}_0$$

By dividing side by side these two equations one gets

$$\tan \mathcal{G}_E \frac{R \sin \mathcal{G}_0}{\cos \mathcal{G}_0 - \beta} \quad (10)$$

By squaring and adding the two equations (9) one can also obtain

$$c_E = \frac{c}{R^2} (1 - \beta \cos \mathcal{G}_0) \quad (11)$$

Eq. (10) coincides with the well known relativistic aberration formula [6], [7]. All quantities appearing in the right hand side of (10) are relative to the system S_0 where Lorentz and inertial theories agree on the numerical values of \mathcal{G}_0 and β . Therefore the aberration angle is predicted by the two sets of transformations to have exactly the same value, and this for arbitrary S . One obtains thus a complete explanation of the aberration effect within the approach based on the inertial transformations.

Substitution of Eq. (11) into (9) gives

$$\cos \mathcal{G}_E = \frac{\cos \mathcal{G}_0 - \beta}{1 - \beta \cos \mathcal{G}_0} \quad ; \quad \sin \mathcal{G}_E = \frac{R \sin \mathcal{G}_0}{1 - \beta \cos \mathcal{G}_0} \quad (12)$$

Thus we see that the propagation direction of the light pulse in S is specified by the unit vector with components:

$$\hat{n}_E = \left(\frac{\cos \mathcal{G}_0 - \beta}{1 - \beta \cos \mathcal{G}_0}, \frac{R \sin \mathcal{G}_0}{1 - \beta \cos \mathcal{G}_0} \right) \quad (13)$$

The pulse velocity can be expressed in terms of the angle \mathcal{G}_E in S . Comparing (13) with $\hat{n}_E = (\cos \theta_E, \sin \theta_E)$ one can easily obtain

$$1 + \beta \cos \mathcal{G}_E = \frac{R^2}{1 - \beta \cos \mathcal{G}_0} \quad (14)$$

which, substituted in (11), finally gives

$$c_E = \frac{c}{1 + \beta \cos \mathcal{G}_E} \quad (15)$$

As expected, c_E coincides with the one-way velocity of light relative to S [8] obtained in the framework of the inertial transformations.

3. Wave equations relative to the moving system

We start from the transformation equations for the space and time derivatives. From the inertial transformations (7) one can easily obtain:

$$\frac{\partial}{\partial x_0} = \frac{1}{R} \frac{\partial}{\partial x} \quad ; \quad \frac{\partial}{\partial y_0} = \frac{\partial}{\partial y} \quad ; \quad \frac{\partial}{\partial z_0} = \frac{\partial}{\partial z} \quad (16)$$

$$\frac{\partial}{\partial t_0} = -\frac{V}{R} \frac{\partial}{\partial x} + R \frac{\partial}{\partial t}$$

From (16) one gets:

$$\frac{\partial^2}{\partial x_0^2} = \frac{1}{R^2} \frac{\partial^2}{\partial x^2} \quad ; \quad \frac{\partial^2}{\partial y_0^2} = \frac{\partial^2}{\partial y^2} \quad ; \quad \frac{\partial^2}{\partial z_0^2} = \frac{\partial^2}{\partial z^2} \quad (17)$$

$$\frac{1}{c^2} \frac{\partial^2}{\partial t_0^2} = \frac{1-\beta^2}{c^2} \frac{\partial^2}{\partial t^2} - 2 \frac{\beta}{c} \frac{\partial^2}{\partial x \partial t} + \frac{\beta^2}{1-\beta^2} \frac{\partial^2}{\partial x^2}$$

Therefore the Laplace operator in S_0 satisfies:

$$\nabla_0^2 = \nabla^2 + \frac{\beta^2}{1-\beta^2} \frac{\partial^2}{\partial x^2} \quad (18)$$

As it is well known, from Maxwell's equations in S_0 follow the wave equations:

$$\nabla_0^2 \vec{E}_0 = \frac{1}{c^2} \frac{\partial^2 \vec{E}_0}{\partial t_0^2} \quad ; \quad \nabla_0^2 \vec{H}_0 = \frac{1}{c^2} \frac{\partial^2 \vec{H}_0}{\partial t_0^2} \quad (19)$$

We wish to see how they must be written in the S frame. By applying (18) and (17):

$$\nabla^2 \vec{E}_0 = \frac{1-\beta^2}{c^2} \frac{\partial^2 \vec{E}_0}{\partial t^2} - \frac{2\beta}{c} \frac{\partial^2 \vec{E}_0}{\partial x \partial t} \quad ; \quad \nabla^2 \vec{H}_0 = \frac{1-\beta^2}{c^2} \frac{\partial^2 \vec{H}_0}{\partial t^2} - \frac{2\beta}{c} \frac{\partial^2 \vec{H}_0}{\partial x \partial t} \quad (20)$$

Consider now the transformations of the fields established in [9]:

$$\begin{aligned}
 E_{0x} &= E_x \quad ; \quad E_{0y} = \frac{1}{R}(E_y + \beta H_z) \quad ; \quad E_{0z} = \frac{1}{R}(E_z - \beta H_y) \\
 H_{0x} &= H_x \quad ; \quad H_{0y} = \frac{1}{R}(H_y - \beta E_z) \quad ; \quad H_{0z} = \frac{1}{R}(H_z + \beta E_y)
 \end{aligned}
 \tag{21}$$

Owing to (21), the equations (20) can respectively be written:

$$\left\{ \begin{aligned}
 \nabla^2 E_x - \frac{1-\beta^2}{c^2} \frac{\partial^2 E_x}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 E_x}{\partial x \partial t} &= 0 \\
 \nabla^2 E_y - \frac{1-\beta^2}{c^2} \frac{\partial^2 E_y}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 E_y}{\partial x \partial t} &= -\beta \left(\nabla^2 H_z - \frac{1-\beta^2}{c^2} \frac{\partial^2 H_z}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 H_z}{\partial x \partial t} \right) \\
 \nabla^2 E_z - \frac{1-\beta^2}{c^2} \frac{\partial^2 E_z}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 E_z}{\partial x \partial t} &= \beta \left(\nabla^2 H_y - \frac{1-\beta^2}{c^2} \frac{\partial^2 H_y}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 H_y}{\partial x \partial t} \right)
 \end{aligned} \right.$$

$$\left\{ \begin{aligned}
 \nabla^2 H_x - \frac{1-\beta^2}{c^2} \frac{\partial^2 H_x}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 H_x}{\partial x \partial t} &= 0 \\
 \nabla^2 H_y - \frac{1-\beta^2}{c^2} \frac{\partial^2 H_y}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 H_y}{\partial x \partial t} &= -\beta \left(\nabla^2 E_z - \frac{1-\beta^2}{c^2} \frac{\partial^2 E_z}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 E_z}{\partial x \partial t} \right) \\
 \nabla^2 H_z - \frac{1-\beta^2}{c^2} \frac{\partial^2 H_z}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 H_z}{\partial x \partial t} &= \beta \left(\nabla^2 E_y - \frac{1-\beta^2}{c^2} \frac{\partial^2 E_y}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 E_y}{\partial x \partial t} \right)
 \end{aligned} \right.$$

Notice that the second of (22) and the third of (23) have the structure:

$$\{F = -\beta G \oplus G = -\beta F\} \Rightarrow F = \beta^2 F \Rightarrow F = G = 0 \tag{24}$$

Also the third of (22) and the second of (23) have a similar structure and lead to a similar consequence. Therefore, repeating also the first equations (22) and (23) we write:

$$\left\{ \begin{array}{l} \nabla^2 E_x - \frac{1-\beta^2}{c^2} \frac{\partial^2 E_x}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 E_x}{\partial x \partial t} = 0 \\ \nabla^2 E_y - \frac{1-\beta^2}{c^2} \frac{\partial^2 E_y}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 E_y}{\partial x \partial t} = 0 \\ \nabla^2 E_z - \frac{1-\beta^2}{c^2} \frac{\partial^2 E_z}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 E_z}{\partial x \partial t} = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} \nabla^2 H_x - \frac{1-\beta^2}{c^2} \frac{\partial^2 H_x}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 H_x}{\partial x \partial t} = 0 \\ \nabla^2 H_y - \frac{1-\beta^2}{c^2} \frac{\partial^2 H_y}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 H_y}{\partial x \partial t} = 0 \\ \nabla^2 H_z - \frac{1-\beta^2}{c^2} \frac{\partial^2 H_z}{\partial t^2} + \frac{2\beta}{c} \frac{\partial^2 H_z}{\partial x \partial t} = 0 \end{array} \right. \quad (25)$$

Eq.s (25) are the wave equations in S . In vectorial form they become:

$$\nabla^2 \vec{E} = \frac{1-\beta^2}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} - \frac{2\beta}{c} \frac{\partial^2 \vec{E}}{\partial x \partial t} \quad ; \quad \nabla^2 \vec{H} = \frac{1-\beta^2}{c^2} \frac{\partial^2 \vec{H}}{\partial t^2} - \frac{2\beta}{c} \frac{\partial^2 \vec{H}}{\partial x \partial t} \quad (26)$$

We see that also in S the wave equations have the same form for \vec{E} and \vec{H} . As the strong form of the relativity principle does not hold with the inertial transformations, eq.s (26) are different from eq.s (19) valid in S_0 .

4. Properties of the plane waves

It is well known that the Eq.s (19) have plane wave solutions of the form

$$\vec{E}_0 = \vec{\varepsilon}_0 e^{2\pi i(\vec{k}_0 \cdot \vec{r}_0 - \nu_0 t_0)} \quad (27)$$

where $\vec{\varepsilon}_0$ is the constant amplitude, ν_0 is the frequency, \vec{k}_0 is the wave vector giving the propagation direction. Relative to S_0 the velocity of light is c , therefore $k_0 = \nu_0 / c$.

A linear transformation (Lorentz, inertial, ...) of the space and time variables and of the field components maintains the general structure of the plane wave. Therefore we can assume that a solution of the first eq. (26) is of the type:

$$\vec{E} = \vec{\varepsilon} e^{2\pi i(\vec{k} \cdot \vec{r} - \nu t)} \quad (28)$$

with $\vec{\varepsilon}$ constant amplitude, ν frequency, \vec{k} wave vector, all relative to S . Inserting (28) into (26) and noting that:

$$\nabla^2 \vec{E} = -4\pi^2 k^2 \vec{E} ; \quad \frac{\partial^2 \vec{E}}{\partial t^2} = -4\pi^2 \nu^2 \vec{E} ; \quad \frac{\partial^2 \vec{E}}{\partial x \partial t} = 4\pi^2 \nu k_x \vec{E}$$

we obtain the second degree equation in k :

$$k^2 - \frac{2\beta}{c} \nu \cos \mathcal{G} k - \frac{1-\beta^2}{c^2} \nu^2 = 0 \quad (29)$$

with $k_x = k \cos \mathcal{G}$. The $k > 0$ solution of (29) is:

$$k = \frac{\nu}{c} \left[\beta \cos \mathcal{G} + \sqrt{1 - \beta^2 \sin^2 \mathcal{G}} \right] \quad (30)$$

This holds for plane waves satisfying (26). In the system S it is analogous to the relationship $k_0 = \nu_0 / c$ holding in the privileged

frame. Eq. (30) implies that the seeming velocity of the wave, c_ϕ , is the same for all frequencies ν (therefore the waves do not disperse in the vacuum) but depends on the propagation direction according to:

$$c_\phi = \frac{\nu}{k} = \frac{c}{\beta \cos \vartheta + \sqrt{1 - \beta^2 \sin^2 \vartheta}} \quad (31)$$

We will later see, however, that the direction of \vec{k} does not coincide with the propagation direction of the energy transported by the wave [10] and that (31) does not represent the true velocity of the wave in S .

Naturally we could have deduced the previous results from the magnetic equation (26) by studying its plane wave solutions similar to (28). Actually the electric and magnetic equations lead to the same properties of the plane waves.

Next consider again eq. (28), for which:

$$\frac{\partial \vec{E}}{\partial x} = 2\pi i k_x \vec{E} \quad \dots \quad \frac{\partial \vec{E}}{\partial t} = -2\pi i \nu \vec{E} \quad (32)$$

the dots standing for y and z derivatives similar to the x derivative. Thus to the vectorial operator ∇ corresponds the imaginary vector $2\pi i \vec{k}$ and the time derivative operator to the imaginary scalar $-2\pi i \nu$. In the same sense one can see from (27) that in S_0 to ∇_0 corresponds $2\pi i \vec{k}_0$ and to the time derivative corresponds $-2\pi i \nu_0$. That is:

$$\begin{aligned} \nabla &\rightarrow 2\pi i \vec{k} \quad ; \quad \frac{\partial}{\partial t} \rightarrow -2\pi i \nu \\ \nabla_0 &\rightarrow 2\pi i \vec{k}_0 \quad ; \quad \frac{\partial}{\partial t_0} \rightarrow -2\pi i \nu_0 \end{aligned} \quad (33)$$

We can now easily find the transformations of wave vector and frequency between the two reference frames S and S_0 . The phase invariance of the plane wave (27) and (28) can be written

$$2\pi i \left(k_{0x}x_0 + k_{0y}y_0 + k_{0z}z_0 - \nu_0 t_0 \right) = 2\pi i \left(k_x x + k_y y + k_z z - \nu t \right)$$

Substituting the inverse inertial transformations (7) and noting that the previous equation holds for arbitrary values of the space and time variables one can easily obtain:

$$k_{0x} = \frac{1}{R}k_x ; k_{0y} = k_y ; k_{0z} = k_z ; \nu_0 = R\nu + \frac{V}{R}k_x \quad (34)$$

whence one gets:

$$k_x = Rk_{0x} ; k_y = k_{0y} ; k_z = k_{0z} ; \nu = \frac{\nu_0 - V k_{0x}}{R} \quad (35)$$

Let us define the unit vector \vec{n}_0 by writing $\vec{k}_0 = k_0 \vec{n}_0 = (\nu_0 / c) \vec{n}_0$.

One has then $Vk_{0x} = \vec{V} \cdot \vec{n}_0 (\nu_0 / c)$ and the last eq. (35) becomes:

$$\nu = \frac{\nu_0}{R} \left(1 - \frac{\vec{V} \cdot \vec{n}_0}{c} \right) \quad (36)$$

The Doppler effect formula (36) was obtained in Ref. [10]. Consequences of (35) are:

$$k = k_0 \sqrt{1 - \beta^2 \cos^2 \mathcal{G}_0} ; \cos \mathcal{G} = \frac{k_x}{k} = \frac{R \cos \mathcal{G}_0}{\sqrt{1 - \beta^2 \cos^2 \mathcal{G}_0}} \quad (37)$$

where $\cos \mathcal{G}_0 = k_{0x} / k_0$. Eq.s (37) are the transformations of the wave vector modulus and direction. From the second of (37) one gets easily:

$$\sin \mathcal{G} = \frac{\sin \mathcal{G}_0}{\sqrt{1 - \beta^2 \cos^2 \mathcal{G}_0}} \quad (38)$$

Thus the unit vector normal to the wave front in S : $\vec{n} = (\cos \mathcal{G}; \sin \mathcal{G})$, can be written:

$$\vec{n} = \left(\frac{R \cos \mathcal{G}_0}{\sqrt{1 - \beta^2 \cos^2 \mathcal{G}_0}}, \frac{\sin \mathcal{G}_0}{\sqrt{1 - \beta^2 \cos^2 \mathcal{G}_0}} \right) \quad (39)$$

This is to be compared with the energy propagation direction given by (12):

$$\hat{n}_E = \left(\frac{\cos \theta_0 - \beta}{1 - \beta \cos \theta_0}, \frac{R \sin \theta_0}{1 - \beta \cos \theta_0} \right) \quad (40)$$

Substituting in (39) the relationship

$$\beta^2 \cos^2 \mathcal{G}_0 = (\vec{n}_0 \cdot \vec{V})^2 / c^2 \quad (41)$$

we get:

$$\vec{n} = \frac{1}{\sqrt{1 - (\vec{n}_0 \cdot \vec{V})^2 / c^2}} \left[\vec{n}_0 + (R - 1) \frac{\vec{V} \cdot \vec{n}_0 \vec{V}}{V^2} \right] \quad (42)$$

Eq (42) coincides with the Puccini-Selleri formula [10]. For the seeming phase velocity c_ϕ (already found in (31)) by using the transformations (36)-(37) as well as (40) one has:

$$c_\phi = \frac{v}{k} = \frac{v_0}{R} \frac{1 - \vec{V} \cdot \vec{n}_0 / c}{k_0 \sqrt{1 - \beta^2 \cos^2 \mathcal{G}_0}} = \frac{c}{R} \frac{1 - \vec{V} \cdot \vec{n}_0 / c}{\sqrt{1 - (\vec{n}_0 \cdot \vec{V})^2 / c^2}} \quad (43)$$

Also (43) coincides with a result found by Puccini-Selleri [10], and furthermore it coincides with (31).

Further conditions are found by assuming that the plane waves satisfy the Maxwell equations in the form established in [9]. The Maxwell equations in S in the vacuum in regions where there are no charges and currents have the form:

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{H}'}{\partial t} ; \quad \nabla \times \vec{H} = \frac{1}{c} \frac{\partial \vec{E}'}{\partial t} ; \quad \nabla \cdot \vec{H}' = 0 ; \quad \nabla \cdot \vec{E}' = 0 \quad (44)$$

where

$$\vec{H}' = \vec{H} + \vec{\beta} \times \vec{E} \quad ; \quad \vec{E}' = \vec{E} - \vec{\beta} \times \vec{H} \quad (45)$$

with $\vec{\beta} = \vec{V}/c$. Using the correspondences (33) we can write for $\nabla \times \vec{E}$ and $\nabla \times \vec{H}$ as given by (44) and (45):

$$\vec{k} \times \vec{\varepsilon} = \frac{V}{c} (\vec{h} + \vec{\beta} \times \vec{\varepsilon}) \quad ; \quad \vec{k} \times \vec{h} = -\frac{V}{c} (\vec{\varepsilon} - \vec{\beta} \times \vec{h}) \quad (46)$$

Multiplying the first and second Eq. (46) respectively by $\vec{\varepsilon}$ and \vec{h} we get:

$$\vec{\varepsilon} \cdot \vec{h} = 0 \quad (47)$$

Thus the solutions of the wave equations (26) respecting the Maxwell equations in S have, just as in S_0 , perpendicular electric and magnetic fields. Using again the correspondences (43) we can write for $\nabla \cdot \vec{E}'$ and $\nabla \cdot \vec{H}'$ as given by (44) and (45):

$$\vec{k} \cdot (\vec{\varepsilon} - \vec{\beta} \times \vec{h}) = 0 \quad ; \quad \vec{k} \cdot (\vec{h} + \vec{\beta} \times \vec{\varepsilon}) = 0 \quad (48)$$

From (48) another important result follows trivially:

$$\vec{k} \cdot \vec{\varepsilon} = \vec{k} \cdot \vec{\beta} \times \vec{h} \quad ; \quad \vec{k} \cdot \vec{h} = -\vec{k} \cdot \vec{\beta} \times \vec{\varepsilon} \quad (49)$$

Eq.s (49) show that the plane wave solutions of the wave equations, differently than in S_0 , in general are not transverse in S . That is, the electric and magnetic fields, even if perpendicular to one another, are not orthogonal to the seeming propagation direction \vec{k} . Furthermore the fields obey the Lorentz transformations (21), from which one easily obtains:

$$\varepsilon^2 - h^2 = \varepsilon_0^2 - h_0^2 ; \quad \vec{\varepsilon} \cdot \vec{h} = \vec{\varepsilon}_0 \cdot \vec{h}_0 \quad (50)$$

Given that $\varepsilon_0 = h_0$ one sees that:

$$\varepsilon = h \quad (51)$$

In conclusion in S the wave fields are perpendicular to one another and have equal moduli.

5. The physical wave vector

In S_0 the propagation direction of the electromagnetic energy is perpendicular to the electric and magnetic fields $\vec{\varepsilon}_0$ and \vec{h}_0 . The theory of the inertial transformations should show empirical equivalence to the TSR. According to this expectation also *in S the propagation direction of energy should be perpendicular to the electric and magnetic fields $\vec{\varepsilon}$ and \vec{h}* . We can show that this is indeed the case by writing (48) as:

$$\vec{k} \cdot \vec{\varepsilon} = \vec{\beta} \cdot (\vec{h} \times \vec{k}) ; \quad \vec{k} \cdot \vec{h} = -\vec{\beta} \cdot (\vec{\varepsilon} \times \vec{k}) \quad (52)$$

Inserting now Eq.s (46) in the right hand sides of (52) we get:

$$\left(\vec{k} - \frac{v}{c} \vec{\beta} \right) \cdot \vec{\varepsilon} = 0 ; \quad \left(\vec{k} - \frac{v}{c} \vec{\beta} \right) \cdot \vec{h} = 0 \quad (53)$$

Eq.s (53) imply that the vector:

$$\vec{k}_E \equiv \rho \left(\vec{k} - \frac{v}{c} \vec{\beta} \right) \quad (54)$$

is perpendicular both to $\vec{\varepsilon}$ and to \vec{h} (ρ is a normalizing factor). Notice that

$$k_E \equiv \rho \sqrt{\left(\vec{k} - \frac{v}{c} \vec{\beta} \right) \cdot \left(\vec{k} - \frac{v}{c} \vec{\beta} \right)} = \rho \sqrt{k^2 - 2 \frac{v}{c} \beta k \cos \vartheta + \frac{v^2}{c^2} \beta^2} \quad (55)$$

By using (29) one can also write:

$$k_E = \rho v / c \quad (56)$$

We can now show that the propagation direction of the electromagnetic energy transported by the wave, as described in the frame S , is given by \vec{k}_E . Representing \vec{k}_E with its components we have:

$$\vec{k}_E \equiv \rho \left(k_x - \frac{v}{c} \beta, k_y \right) \quad (57)$$

whence, using (35) and (36)

$$\vec{k}_E = \rho \left(Rk_0 \cos \vartheta_0 - \frac{v_0}{cR} (1 - \beta \cos \vartheta_0) \beta, k_0 \sin \vartheta_0 \right) \quad (58)$$

which is the same as:

$$\vec{k}_E = \rho \frac{k_0}{R} (\cos \vartheta_0 - \beta, R \sin \vartheta_0) \quad (59)$$

Therefore \vec{k}_E is parallel to \vec{n}_E (see (40)) and one has necessarily:

$$\vec{n}_E \equiv \frac{\vec{K}}{K} = \frac{c}{v} \left(\vec{k} - \frac{v}{c} \vec{\beta} \right) \quad (60)$$

This shows that the vector $\vec{k}_E = k_E \vec{n}_E$ gives the propagation direction of the electromagnetic energy transported by the wave relatively to the frame S . Therefore \vec{k}_E as defined by (54) will be called effective wave vector. It is useful to define also the effective wave length L :

$$L = \frac{c_E}{\nu} \quad (61)$$

which due to (15) and (36) becomes:

$$L = \frac{1}{R} \frac{c}{\nu_0} \quad (62)$$

elongated by a factor $1/R$, as expected, with respect to the wavelength $\lambda_0 = c/\nu_0$ measured in S_0 .

6. Photonic energy and momentum

Suppose that relative to the inertial frame S_0 a photon has energy and momentum respectively given by:

$$E_0 = h\nu_0 \quad ; \quad \vec{p}_0 = \frac{h\nu_0}{c} \vec{n}_0 \quad (63)$$

The photon is the quantum of the electromagnetic field. Its energy relative to the moving frame S [11] is:

$$E = \frac{1}{R} (E_0 - V p_{0x}) = \frac{h\nu_0}{R} (1 - \beta \cos \vartheta_0) \quad (64)$$

where ϑ_0 is the angle between the photon propagation direction and the x_0 axis of S_0 . The frequency is related to ν_0 by eq. (35). Therefore, substituting (35) in (64) one gets:

$$E = h\nu \quad (65)$$

Using the first of the inertial transformations for momentum [11] one gets:

$$p_x = \frac{1}{R} \left[p_{0x} - \frac{\beta}{c} E_0 \right] = \frac{h\nu_0}{Rc} [\cos \mathcal{G}_0 - \beta] \quad (66)$$

whence, introducing once more eq. (35),

$$p_x = \frac{h\nu \cos \mathcal{G}_0 - \beta}{c 1 - \beta \cos \mathcal{G}_0} \quad (67)$$

Therefore, thanks to (12)

$$p_x = \frac{h\nu}{c} \cos \mathcal{G}_E \quad (68)$$

Supposing now for simplicity that the photon has $p_{0z} = 0$, one can write in S

$$p_y = p_{0y} = \frac{h\nu_0}{c} \sin \mathcal{G}_0 \quad (69)$$

Using once more eq. (35) one gets:

$$p_y = R \frac{h\nu}{c} \frac{\sin \mathcal{G}_0}{1 - \beta \cos \mathcal{G}_0} \quad (70)$$

or also, introducing (12)

$$p_y = \frac{h\nu}{c} \sin \mathcal{G}_E \quad (71)$$

From (68) and (71) one sees that the modulus of the photon momentum in the frame S is

$$p = \frac{h\nu}{c} \quad (72)$$

In conclusion, given that relative to the frame S the photon has energy and momentum respectively given by (65) and (72), these quantities can be written as functions of the frequency in formally identical ways in S_0 and S . This is of course the standard result in the TSR, but it needed to be proven anew in the approach based on the IT.

7. The Compton effect

We now apply the found results to the Compton effect [12]. Suppose that in the system S the photon has initially the frequency ν and travels in the +x direction and the electron is at rest.

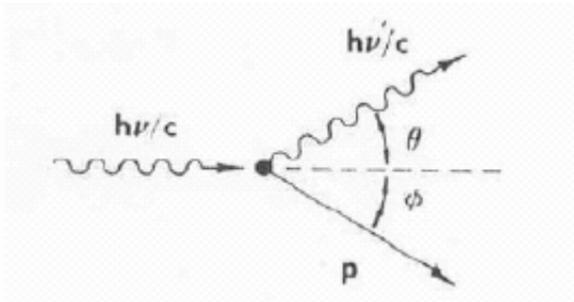


Figure 1. The Compton effect.

From the conservation of energy and momentum in the frame S one gets the equations:

$$\begin{aligned}
 h\nu + mc^2 &= h\nu' + \tilde{E}'_e \\
 \frac{h\nu}{c} &= \frac{h\nu'}{c} \cos \vartheta + p' \cos \phi ; \quad 0 = \frac{h\nu'}{c} \sin \vartheta - p' \sin \phi
 \end{aligned}
 \tag{73}$$

In order to get rid of the angle ϕ we rewrite the second and third eq.s (73) as:

$$p' \cos \phi = \frac{h\nu}{c} - \frac{h\nu'}{c} \cos \vartheta \quad ; \quad p' \sin \phi = \frac{h\nu'}{c} \sin \vartheta \quad (74)$$

By squaring, summing, and multiplying by c^2 we get:

$$p'^2 c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu') \cos \vartheta \quad (75)$$

From the first equation (73):

$$\tilde{E}_e'^2 = (h\nu - h\nu' + mc^2)^2 \quad (76)$$

By subtracting side by side (75) from (76) and using the electron mass condition we get after a few steps:

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{mc^2} (1 - \cos \vartheta) \quad (77)$$

This is the famous Compton formula expressed in terms of the synchronization independent frequencies. Thus also the inertial transformations predict essentially the same formula that Compton found in 1923 by using the theory of relativity. Rewriting the Compton formula in terms of wavelengths is not trivial in the present approach, given that the velocity of light relative to S is given by Eq. (15). It will be done in the next section.

8. The wavelength determination

In Compton's experiment (see the setup in Fig. 2) x-rays of wavelength 0,0709 nm produced in an x-ray tube struck a carbon target. A series of slits allowed only those scattered x-rays which left the target in a direction forming an angle θ with the direction of the incident x-rays to enter the spectrometer. The value of θ could be varied by moving the x-ray source. The spectrometer consisted of a rotating framework with a calcite crystal to diffract the x-rays and an ionization chamber acting as a detector. As the glancing angle ϕ at

which the primary beam of x-rays struck the crystal was varied, the angle between the ionization chamber and the primary beam was kept at 2ϕ , in order to receive the secondary beam reflected from the crystal. Since the spacing of the crystal planes in calcite is known, the wavelength of the scattered x-rays could be accurately determined from the angle ϕ at which they were diffracted with maximum intensity (Bragg's law, a well known interference effect). Thus the output of the spectrometer, for any chosen value of θ , was essentially the intensity of the scattered x-rays as a function of wavelength.

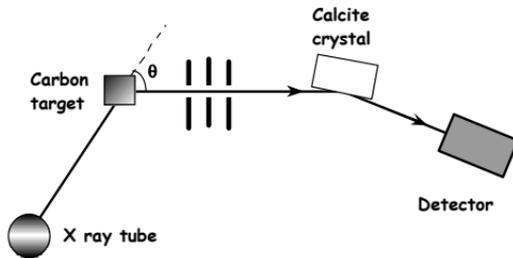


Figure 2. In Compton's experiment x-rays produced in an x-ray tube struck a carbon target. A series of slits allowed to enter the spectrometer only those scattered x-rays which left the target at an angle θ with respect to the incident x-rays. The value of θ could be varied by moving the x-ray tube. The spectrometer consisted of a calcite crystal to diffract the x-rays and an ionization chamber acting as a detector. An application of Bragg's law determined the wavelength of the scattered x-rays.

A laboratory is at rest in the inertial system S moving with *absolute velocity* \vec{v} and in it a Compton experiment is performed with a beam of x-rays. Given the quantum mechanical properties of photons a correct description of the propagation must take into account both particle and wave aspects of the radiation. From the undulatory point of view one should consider that in such an experiment two different photons are incoherent and do not interfere. Therefore the interference taking place in the crystal arises from a

very large number of repetitions of interference of an individual photon with itself.

A photon born in the point P propagates to the point Q where it is detected. The extension of the quantum mechanical wave describing the photon implies that its propagation can be represented by infinitely many Feynman trajectories[13], two of which are shown as (a) and (b) in Fig. 3. Actually the curved part of the trajectory is limited to the inside of the Calcite crystal of Fig. 2, outside of which the propagation is rectilinear. Notice that the big grey arrow of Fig. 3 represents the laboratory "absolute" velocity (velocity of the inertial reference frame S in which the laboratory is at rest with respect to the privileged isotropic frame S_0). The interference in Q (a point of the detector) is determined by the time delay ΔT between the two paths. According to the TSR light propagates in all directions with the same speed c also with respect to S and one has:

$$\Delta T = T_b - T_a = \frac{L_b - L_a}{c} \quad (78)$$

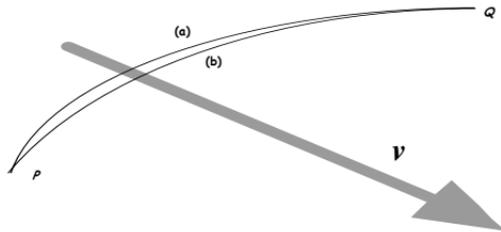


Figure 3. A photon born in the point P propagates to the point Q where it is detected. From the undulatory point of view the propagation can take place along infinitely many trajectories, two of which are shown as (a) and (b). The big grey arrow represents the laboratory absolute velocity.

where L_a and L_b are the P to Q lengths of the curves (a) and (b):

$$L_a = \int_{(a)} d\ell_a \quad ; \quad L_b = \int_{(b)} d\ell_b \quad (79)$$

Next we calculate ΔT from the equivalent transformations, according to which the inverse velocity of light relative to S is given by (9). One has:

$$\Delta T = \int_{(b)} \frac{d\ell_b}{c_1(\theta_b)} - \int_{(a)} \frac{d\ell_a}{c_1(\theta_a)} \quad (80)$$

where $\theta_a(\theta_b)$ is the angle between $d\vec{\ell}_a$ and \vec{v} ($d\vec{\ell}_b$ and \vec{v}). By inserting (A8) in (79):

$$\begin{aligned} \Delta T &= \frac{L_b - L_a}{c} + \frac{\Gamma}{c} \int_{(b)} d\ell_b \cos \theta_b - \frac{\Gamma}{c} \int_{(a)} d\ell_a \cos \theta_a \\ &= \frac{L_b - L_a}{c} + \frac{\Gamma}{c} \left\{ \int_{(b)} d\vec{\ell}_b - \int_{(a)} d\vec{\ell}_a \right\} \cdot \frac{\vec{v}}{v} \end{aligned} \quad (81)$$

The term with curly brackets vanishes because the two integrals separately equal the vector joining the points P and Q . Thus (78) and (81) are the same. Therefore Γ (containing e_1) disappears from the result and there is a unique ΔT predicted by all theories of the set based on the equivalent transformations. This ΔT leads to a unique numerical prediction for the angle θ at which there is constructive interference of the photonic waves accompanying a photon after interaction with the atomic lattice of the crystal. At this point it is possible to apply an arbitrary theory of the set, e.g. the STR assuming that the velocity of the x-rays is c also relative to S . Given the result (77) of the previous section and assuming $\lambda\nu = c$ one clearly gets the Compton formula for wavelengths:

$$\lambda - \lambda_0 = \lambda_c (1 - \cos \theta) \quad (82)$$

A different result is possible if one starts from a different theory of the set, for which $\lambda \nu = c_1(\theta)$, but the difference is in any case only formal and not substantial, as we have shown above.

Appendix

In this appendix we will present a simple method to obtain, consistently with the equivalent transformations (1), the velocity of a flash of light propagating in a medium at rest in the generic inertial frame S . Consider the triangle ABC of fig. 4, at rest in the inertial frame S , with sides having lengths l_{AB} , l_{BC} and l_{CA} and with suitably oriented mirrors placed in B and C (not shown).

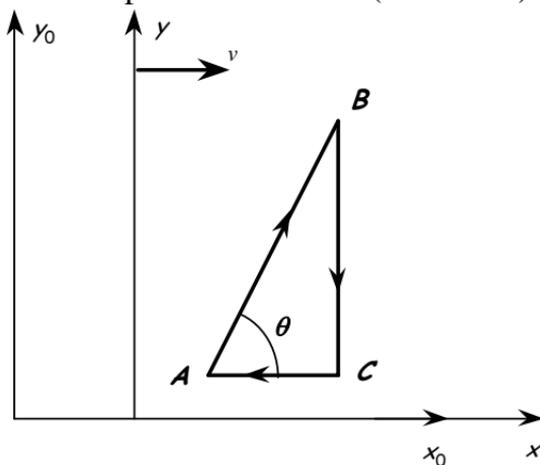


Figure 4. The triangle ABC is at rest in the inertial frame S . Suitably oriented mirrors in B and C (not shown) force a flash of light emitted in A to propagate on the closed path ABC. Along AB light propagates in a medium, while the sides BC and CA are in the vacuum.

The time t_{ABC} required by light to propagate on the closed path ABC can be measured with a single clock in A independently of clock synchronization and is given by

$$t_{ABC} = \frac{\ell_{AB}}{c_{AB}} + \frac{\ell_{BC}}{c_{BC}} + \frac{\ell_{CA}}{c_{CA}} \quad (A1)$$

where c_{AB} is the velocity of light from A to B, and so on.

The sides BC and CA are intended to be in the vacuum, while we assume that the path AB is inside a medium (refraction index n) at rest in S . In the TSR the velocity of light in such a medium is given by:

$$c_{AB}^{\text{TSR}} = \frac{c}{n} \quad (A2)$$

In fig. 4 BC is perpendicular and CA antiparallel to the absolute velocity v of S . Therefore, by using eq. (2) for light velocity in the vacuum we have

$$t_{ABC} = \frac{\ell_{AB}}{c_{AB}} + \frac{\ell_{BC}}{c} + \frac{\ell_{CA}}{c}(1-\Gamma) \quad (A3)$$

The prediction of the TSR according to (A2) is instead

$$t_{ABC} = \ell_{AB} \frac{n}{c} + \frac{\ell_{BC}}{c} + \frac{\ell_{CA}}{c} \quad (A4)$$

The time t_{ABC} , measurable with a single clock, is independent of the synchronisation parameter e_1 . Therefore (A3) and (A4) must be equal and it follows

$$\ell_{AB} \left[\frac{1}{c_{AB}} - \frac{n}{c} \right] = \frac{\ell_{CA}}{c} \Gamma \quad (\text{A5})$$

whence, considering that $\ell_{CA} = \ell_{AB} \cos \theta$:

$$\frac{1}{c_{AB}} = \frac{n}{c} + \frac{\Gamma \cos \theta}{c} \quad (\text{A6})$$

This result shares with eq. (2) a property that can be written for any two points X and Y connected by light propagation in the vacuum or in a medium as follows :

$$\frac{1}{c_{XY}} = \frac{1}{c_{TSR}} + \frac{\Gamma \cos \theta}{c} \quad (\text{A7})$$

where Γ is independent of the medium. Clearly, the propagation time dt over the very small distance $d\ell$ is

$$dt = \frac{d\ell}{c^{TSR}} + \frac{\Gamma}{c} \frac{d\vec{\ell} \cdot \vec{v}}{v} \quad (\text{A8})$$

where $d\vec{\ell}$ is the vector of length $d\ell$ oriented in the direction of light propagation. The latter result is used in the text for deducing the last expression (81) of ΔT .

References

- [1] Mansouri R. and Sexel R., *Gen. Relat. Gravit.*, **8** (1977) 497, 515, 809.
- [2] Selleri F., - *Bell's spaceships and special relativity*, in *Quantum [Un]speakables, from Bell to Quantum Information*, edited by R.A. Bertlmann & A. Zeilinger, (Springer, Berlin) 2002, pp. 413-428.
- [3] Rizzi G., Ruggiero M.L. and Serafini A., *Synchronization Gauges and the Principles of Special Relativity*, *Found. Phys.* **34** (2005) 1885.
- [4] Selleri F., *Noninvariant One-Way Speed of Light And Locally Equivalent Reference Frames*, *Found. Phys. Lett.* **10** (1997) 73.

- [5] Selleri F., *The Inertial Transformations and the Relativity Principle*, *Found. Phys. Lett.* **18** (2005) 325.
- [6] Einstein A., *Ann. Physik*, **17** (1905) 891. English translation in: Lorentz H.A., Einstein A., Minkowski H. and Weyl H., *The Principle of Relativity* (Dover, New York) 1952, p. 37.
- [7] Møller C., *The Theory of Relativity*, Oxford Press (1952), p. 25.
- [8] Selleri F., *Space, Time, and Their Transformations*, *Chinese Jour. Syst. Eng. Electr.* **6** (1995) 25;
- [9] Buonaura B., *Maxwell Equations and Inertial Transformations*, *Found. Phys. Lett.* **17** (2004) 627.
- [10] Puccini G. D., Selleri F., *Doppler effect and aberration of light from the point of view of absolute motion*, *Nuovo Cim. B*, **117** (2002) 283.
- [11] Selleri F., *Noninvariant One-Way Velocity Of Light And Particle Collisions*, *Found. Phys. Lett.* **9** (1996) 43.
- [12] Compton A.H., *A Quantum Theory of the Scattering of X-rays by Light Elements*, *Phys. Rev.* **21** (1923) 483.
- [13] Feynman R.P., *Space–Time Approach to Non-Relativistic Quantum Mechanics*” *Rev. of Mod. Phys.* **20**, (1948) 367.