Does the Three Wave Hypothesis Imply a Hidden Structure?

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The three wave hypothesis (TWH) is an attempt to relate the particle to an associated wave phenomenon. This hypothesis regards that the particle is associated with three waves: one transformed and two dispersive waves. Since the two dispersive waves are associated with a single particle, in this work we try to get a single representation for these two dispersions. The single representation exhibits similarities with those of a classical gear consisting of two perpendicular wheels. According to this similarity, the parameters of dispersive waves correspond to those of the wheels, and the transformed Compton wave corresponds to the system parameters (the combination). This similarity between the two models may possibly point to a hidden structure.

Keywords: Three wave hypothesis, Matter wave, De Broglie wave, gear model.

1. Introduction

One of the controversial problems in quantum mechanics is the problem of the relationship between the de Broglie wave and its associated particle. There have been many suggestions to clarify this problem. One of these is the Three-Wave Hypothesis (TWH) that was proposed by Horodecki [1, 2]. This hypothesis implies that a massive particle is an intrinsically spatially as well as temporally extended non-linear wave phenomenon [2]. In addition to TWH, Elbaz [3, 4] proposed an amplitude wave to be in association with the Compton wave of a massive particle. This concept is somehow implied in TWH as a dual wave [2].

The TWH is based on an assumption that, in a Lorentz frame where the particle is at rest it can be associated with an intrinsic non-dispersive Compton wave. When the particle moves with velocity v (relative to the lab frame), it will be associated with the three waves: the superluminal de Broglie wave (of wavelength λ_B), a subluminal dual wave (of wavelength λ_D), and a transformed Compton wave (of wavelength λ_C):

$$\lambda_C{}^2 = \lambda_B \, \lambda_D \qquad (1\text{-}1).$$

It should be noted that the properties of the amplitude wave [3] are similar to those of dual wave (of TWH). The dispersion relations of the de Broglie wave (λ_B) and dual wave (λ_D) are [1]:

$$\omega_C = \pm\left(c^2 \left| \frac{2\pi}{\vec{\lambda}_B} \right|^2 + m_\circ{}^2 c^4 \hbar^{-2} \right)^{\frac{1}{2}} \qquad (1\text{-}2\text{-}a),$$

and

$$\omega_C = \pm\left(c^2 \left| \frac{2\pi}{\vec{\lambda}_D} \right|^2 - m_\circ^{\,2} c^4 \hbar^{-2} \beta^{-2} \right)^{\frac{1}{2}} \qquad \text{(1-2-b).}$$

where ω_C, m_o, c, $\hbar, \beta$ are Compton angular frequency, the rest mass of the particle, the velocity of light, Planck's constant, and the ratio of particle velocity to the velocity of light ($\beta = \frac{v}{c}$) respectively. In addition, the ratios of wavelength are:

$$\frac{\lambda_B}{\lambda_C} = \frac{c}{v_D} = \frac{1}{\beta} \qquad \text{(1-3),}$$

$$\frac{\lambda_B}{\lambda_D} = \frac{v_B}{v_D} = \mu_T = \left(\frac{1}{\beta} \right)^2 \qquad \text{(1-4).}$$

Here μ_T is the ratio of the wave parameters and its limit is $1 < \mu_T \le \infty$. v_D and v_B are:

$$v_D = \frac{\omega_C \lambda_D}{2\pi}, \; v_B = \frac{\omega_C \lambda_B}{2\pi}.$$

v_D corresponds to the group velocity (of the de Broglie wave of phase velocity v_B) and equals the particle velocity (v). v_D can be considered as the phase velocity of the dual wave (or amplitude wave), where [1, 3]:

$$c^2 = v_D v_B \qquad \text{(1-5),}$$

and $v_D < c < v_B$.

In order to explain these results, Horodecki assumed that the wave-particle duality is due to the existence of the particle-aether hidden interaction [1].

It is obvious that:

1- TWH considers the dispersion in a manner similar to that of light dispersion in a medium, where the refractive index is a function of wavelength and there is no change in the frequency. There is no justification for this consideration.

2- TWH proposes two dispersion relationships, as though there were two separate waves or two separate media. At the same time they are supposed to be in association with a single particle.

In this present attempt we try to overcome these two points, and look for a single relationship. This relationship implies a structure similar to a classical gear model, which can be accepted through the existence of a sub-quantum medium.

2. The Dispersion and wave parameters

Eqs. (1-2-a) and (1-2-b) are formulas of normal and anomalous dispersion respectively. They can be rewritten in terms of wave parameters as:

$$\omega_C = \pm\left(c^2\left|\frac{2\pi}{\vec{\lambda}_B}\right|^2 + {\omega_{Co}}^2\right)^{\frac{1}{2}} \qquad \text{(2-1-a),}$$

and

$$\omega_C = \pm\left(c^2\left|\frac{2\pi}{\vec{\lambda}_D}\right|^2 - {\omega_{Co}}^2\beta^{-2}\right)^{\frac{1}{2}} \qquad \text{(2-1-b),}$$

where $\omega_{C\circ} = \dfrac{m_o c^2}{\hbar} = \omega_C \left(1-\beta^2\right)^{\frac{1}{2}}$.

It is obvious that there are explicit inclusions of the three wavelengths (λ_C , λ_B and λ_D), and there is only one frequency ω_C (ω in the paper [1]). ω_{Co} and λ_{Co} are the parameters of the nondispersive wave, and λ_B and λ_D describe dispersive waves. The amplitude wave of ELBAZ [3] is of angular frequency $\Omega = 2\pi N$, and:

$$N\lambda_B = c \tag{2-2}$$

This frequency is the same as that in first term of Eq. (1-2-a), ($c\left|\dfrac{2\pi}{\vec{\lambda}_B}\right|$). Accordingly, the effect of dispersion on the wavelength may be generalized to include the frequency. This consideration will have no effect on the formulation of TWH. It is implicit. So there will be three frequencies (ν_C , ν_B and ν_D) with the three wavelengths. The N frequency of Elbaz may be called the de Broglie frequency ($\nu_B \equiv N$ or $\omega_B \equiv \Omega$). Nothing changes in using these representations: $\dfrac{2\pi c}{\lambda_D} = \omega_D$ and $\dfrac{2\pi c}{\lambda_B} = \omega_B$. So Eqs. (1-2) can be rewritten in terms of frequencies rather than the wavelength (of dispersive waves) as:

$$\omega_C = \pm\left(\omega_B{}^2 + c^2\left|\frac{2\pi}{\vec{\lambda}_{Co}}\right|^2\right)^{\frac{1}{2}} \tag{2-3-a}$$

$$\omega_C = \pm\left(\omega_D{}^2 - c^2\left|\frac{2\pi}{\vec{\lambda}_{Co}}\right|^2 \beta^{-2}\right)^{\frac{1}{2}} \qquad \text{(2-3-b).}$$

Or as:

$$\omega_C = \pm\left(\omega_B{}^2 + \omega_{C\circ}{}^2\right)^{\frac{1}{2}} \qquad \text{(2-4-a)}$$

$$\omega_C = \pm\left(\omega_D{}^2 - \omega_{C\circ}{}^2\beta^{-2}\right)^{\frac{1}{2}} \qquad \text{(2-4b).}$$

It is now possible to obtain a formula similar to that of wavelengths (Eq. (1-1)):

$$\omega_C{}^2 = \omega_D\omega_B \qquad \text{(2-5).}$$

Then Eq. (1-4) can be presented in terms of the dispersive parameter ratio (μ_T) as:

$$\frac{a_B}{a_D} = \frac{\omega_D}{\omega_B} = \mu_T = \left(\frac{1}{\beta}\right)^2 \qquad \text{(2-6)}$$

where $a = \frac{\lambda}{2\pi}$, and:

$$\omega_D a_D = \omega_B a_B = \omega_C a_C = c \qquad \text{(2-7).}$$

The wave parameters may be divided in two groups: those of dispersive (a_B, a_D, ω_D, and ω_B) and nondispersive (a_{Co}, ω_{C0}) waves. The relationships ((2-5), (2-6), and (2-7)) show a symmetry between the wavelength and angular frequency forms.

From both sets of formulations, in frequencies and wavelengths, it is worth noting that:

1- The parameters of the dispersive waves do not form four-vectors (ω_B, a_B^{-1}) and (ω_D, a_D^{-1}).

2- The only possible four- vector (positive interval or time-like) is for ω_C, a_B^{-1}, which is originally related to ω_C a_C^{-1} (transformed Compton). These quantities (ω_C & a_C^{-1}) are represented by the product of the dispersion quantities (Eqs.(1-1) and (2-5)).

2.1. The three wave system

The frequency ω_C is common to the two types of dispersion, and Eqs. (2-4-a) & (2-4-b) are equivalent. Then, one finds that:

$$\left(\omega_D{}^2 - \omega_B{}^2\right)^{\frac{1}{2}} = \pm\omega_{C\circ}\left(1+\frac{1}{\beta^2}\right)^{\frac{1}{2}} \tag{2-8-a}$$

or

$$\left(\omega_D{}^2 - \omega_B{}^2\right)^{\frac{1}{2}} = \pm\omega_C\left(\frac{1}{\beta^2}-\beta^2\right)^{\frac{1}{2}} \tag{2-8-b}$$

In terms of μ_T (Eq. (2-6)), this form can be rewritten as:

$$\left(\omega_D{}^2 - \omega_B{}^2\right)^{\frac{1}{2}} = \pm\omega_C\left(\mu_T-\frac{1}{\mu_T}\right)^{\frac{1}{2}} \tag{2-8-c}$$

Eqs.(2-8) are the single representations of the three waves in terms of the dispersive parameters ratio.

The proportionality (between the wavelengths and angular frequencies) and the single representation (of ω_D and ω_B) then shows that there is:

1- Similarity between Eq. (2-6) and the ratio for a gear train of two wheels.
2- Similarity between Eq. (2-7) and the velocity of the two wheels of the gear train.
3- The forms of Eqs. (2-8) are the same as that of a gear train of two perpendicular wheels.

3. The gear system

A simple gear system is assumed and will be considered in a classical frame. This system consists of two perpendicular, touching, circular units (e.g. a bevel gear) of radii a_1 and a_2 (where $a_1 \le a_2$) as in Fig.(3-1).

The wheel of large radius is the *guiding* wheel. The ratios of the angular velocities and the radii are:

$$\frac{a_2}{a_1} = \frac{\omega_1}{\omega_2} = \frac{\tau_2}{\tau_1} = -\mu \qquad (3\text{-}1),$$

where $\omega = \frac{2\pi}{\tau}$. The limit of μ is $1 \le \mu < \infty$. The negative sign is related to opposite rotation of wheels; in this work the absolute ratio will be considered. μ is the characteristic-coupling constant of the two wheels. The linear velocity is:

$$\omega_1 a_1 = \omega_2 a_2 = v_R \qquad (3\text{-}2).$$

The absolute angular velocity (ω_R) of the orbiting wheel (relative to an inside observer) is:

$$\left(\omega_1^{\ 2} + \omega_2^{\ 2}\right)^{\frac{1}{2}} = \pm\omega_R \qquad (3\text{-}3\text{-}a).$$

We also have

$$\left(a_1{}^2 + a_2{}^2\right)^{\frac{1}{2}} = \pm a_R \qquad \text{(3-3-b).}$$

To demonstrate the similarity, it is possible to reformulate Eq. (3-3-a) as:

$$\omega_1{}^2 - \omega_2{}^2 = \omega_R{}^2 - 2\omega_2{}^2 \qquad \text{(3-4)}$$

Let α be the angle between the resultant (ω_R) and the component ω_1 . With aid of some trigonometric relations and simple algebra, then:

$$\sin\alpha = \frac{1}{\sqrt{1+\mu^2}} \quad , \cos\alpha = \frac{\mu}{\sqrt{1+\mu^2}} \quad ,$$

and

$$\omega_R{}^2 = \frac{\omega_1\omega_2}{\sin\alpha\cos\alpha} \ , \ a_R{}^2 = \frac{a_1 a_2}{\sin\alpha\cos\alpha} \qquad \text{(3-5).}$$

The linear velocity v_R is:

$$v_R = a_R\omega_R = \frac{\sqrt{a_1 a_2 \omega_1 \omega_2}}{\mu}\left(1+\mu^2\right),$$

$$v_R = a_R\omega_R = \sqrt{\frac{a_1 a_2 \omega_R{}^2\left(1+\mu^2\right)}{\mu}} \ ,$$

and

$$v_R = \sqrt{\frac{\left(v_{1R}v_{2R}\right)\left(1+\mu^2\right)}{\mu}} \qquad \text{(3-6).}$$

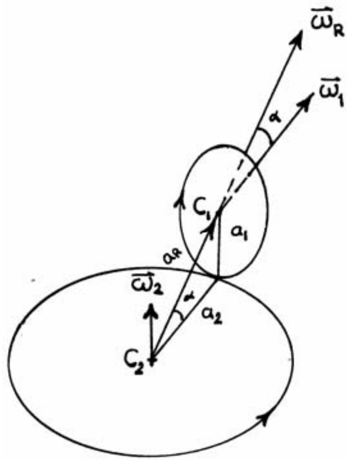

Fig. 3-1 The Gear System

Where $v_{1R} = a_1\omega_R$ and $v_{2R} = a_2\omega_R$.

Eq. (3-4) becomes:

$$\left(\omega_1^{\,2} - \omega_2^{\,2}\right)^{\frac{1}{2}} = \pm\left(\omega_1\omega_2\right)^{\frac{1}{2}}\left(\mu - \frac{1}{\mu}\right)^{\frac{1}{2}} \qquad \text{(3-7-a)},$$

and

$$\left(a_2^{\,2} - a_1^{\,2}\right)^{\frac{1}{2}} = \pm\left(a_1 a_2\right)^{\frac{1}{2}}\left(\mu - \frac{1}{\mu}\right)^{\frac{1}{2}} \qquad \text{(3-7-b)}.$$

There are two types of parameters; the first is related to units (ω_1, ω_2, a_1, and a_2) and the second to the system (ω_R and a_R). With the aid of the trigonometric functions, one can represent the parameters of units in terms of system parameters as:

$$\omega_1 = \omega_R \mu \left(\frac{1}{\sqrt{1+\mu^2}} \right),$$

$$\omega_2 = \omega_R \left(\frac{1}{\sqrt{1+\mu^2}} \right),$$

$$a_1 = a_R \left(\frac{1}{\sqrt{1+\mu^2}} \right),$$

and

$$a_2 = a_R \mu \left(\frac{1}{\sqrt{1+\mu^2}} \right) \quad (3\text{-}8).$$

This gear system is a macroscopic and classical system.

4. The similarity and consequences

It is clear that there are similarities in the structure of the equations between the following two sets:

1. Eqs. (2-6) and (3-1).
2. Eqs. (2-7) and (3-2).
3. Eqs. (2-8-c) and (3-7-a).

In addition to that, the left side in both equations ((2-8-c) and (3-7-a)) refers to a spinning element (either the particle or the first wheel).

The differences are: 1- that the first set of equations is for TWH, which is a relativistic phenomenon, whereas the second set of equations is for a gear which is a classical system. 2-A wave phenomenon is described by the first set whereas the second describes angular motions. 3- From Eqs. (2-8-c) and (3-7-a), the two dispersive waves correspond to the two wheels, whereas the nondispersive wave corresponds to the system or gyroscope ($\omega_1\omega_2$).

Now, can one compare the particle to the gear model? Probably that is possible, if we assume that 'for a lab observer the system appears as a relativistic particle'. That means 'the classical structure is virtual and unobservable, or hidden'. The concept of a hidden medium is not new. It has been proposed by Bohm and Vigier [5] as the level of physical reality much deeper than the quantum physical level. It was proposed in order to explain the probability (rather than the complex wave function) of quantum mechanics in same way as that of the classical approach. The probability appears as a result of our ignorance of the correct variables that are used in describing a system of large number of units. That concept of hidden structure became the base of the statistical consideration, and it has been adopted by de Broglie in his attempt of double solution theory as in "Hidden thermodynamics of the particles" [6].

However, the *virtual classical realm* differs from the ordinary classical realm. The classical realm is an observable existence due to its interaction with the detecting field (it is an approximated case, where the effect of the field is negligible), whereas the virtual classical realm is considered to be beyond observation (hidden). It is a purely geometrical consideration. It may be deduced from the behaviour of the observable particle (hypothetical system). It is a postulated existence but not an observed existence.

In the present work there is no statistical consideration. The hidden structure concept may lead to 'hidden mechanics' of the particle.

Within this consideration, the model can give explanations for the complex wave and spin phenomena. This part of the work is in preparation.

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