A Method Of Triangulating Point Sources Using Omnidirectional Sensors

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A method of triangulating point sources is described that uses the placement of four omnidirectional sensors in a triangular pattern. With this method, the derivation of the Cartesian coordinates of the source relative to the sensor array is described, as well as the source's intensity.

Keywords: beamforming, triangulation, point sources, sensor array

1. Introduction

The use of phased arrays for finding the bearing angle of electromagnetic and acoustic sources is well known [1]. Such methods use the interference pattern created between two or more sensors. The time-delay between sensors of known distance is also employed to provide a calculated difference in the number of waves received between the two sensors from a source [2]. One of the limitations inherent in these methods is the reliance upon a single or small range of sampling frequencies which requires the sensors to be spaced at an integer number of wavelengths apart. This often limits interferometer arrays based on size, for example in ELF or gamma-ray observatories where the wavelengths involved limit the physical construction or performance of the interferometer [3]. The additional requirement to filter at certain frequencies based on a fixed sampling frequency requires additional computational memory and processing. Interference from outside sources at the same sampling frequency also causes artifacts and false predictions of the source angle in phased arrays [4].

To alleviate these problems, an energy detection method is proposed that does not require an integer number of wavelengths of a particular sampling frequency for interferometer construction. This method also has the advantage of predicting the Cartesian coordinates of the source relative to the sensors and the intensity of the source. The proposed method uses knowledge of the radiative transfer of energy from the source in conjunction with the spherical pick-up pattern of omnidirectional sensors to perform triangulation. A beamformer can be developed based on the coordinates of the source that improves gain and signal-to-noise ratio in the direction of the source.

2. Sensor Architecture

The architecture of the system consists of four omnidirectional sensors situated in a plane, which are equally spaced from an origin (see Figure 1). This method provides Cartesian coordinates of the source relative to the plane, as well as an estimate of the source's intrinsic energy.

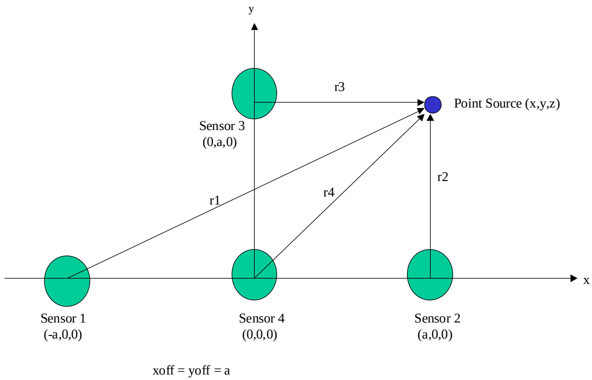

Figure 1 Sensor Placement In The Array

The tracking of point sources with the four omnidirectional sensors in Figure 1 is accomplished by utilizing a general equation of radiative transfer for point sources as follows:

$$V_{\det} = \frac{kI}{r^2} \tag{1}$$

where $V_{\det}$ is the voltage measured on each sensor, I is the intensity of the source, r is the distance from the source to the sensor, and k is a constant of proportionality that is assumed to be known or the same for each sensor. This is the general inverse-square law for propagation of energy in a spherical shell from a point source. The relation in (1) is applied to each sensor in Figure 1 as follows lows:

$$V_{\det_j} = \frac{kI}{r_j^2} \qquad j = 1,\ 2,\ 3,\ 4 \tag{2}$$

The radial distance from each sensor to the source is then determined from Figure 1:

$$\begin{aligned} r_1^2 &= (x+x_{off})^2+y^2+z^2, \quad r_2^2 = (x-x_{off})^2+y^2+z^2 \\ r_3^2 &= x^2+(y-y_{off})^2+z^2, \quad r_4^2 = x^2+y^2+z^2 \end{aligned} \tag{3}$$

where x_{off} and y_{off} are the offsets of each sensor from the x-axis and y-axis, respectively, and are assumed to be the same. Also, (3) will provide the solutions to points of the source's location in a sphere of radius r encompassing each sensor. Combining (2) and (3) allows us to write the following simultaneous set of equations in the variables x, y, z and r_1:

$$\begin{aligned} ar_1^2 &= (x+x_{off})^2+y^2+z^2, \quad br_1^2 = (x-x_{off})^2+y^2+z^2 \\ cr_1^2 &= x^2+(y-y_{off})^2+z^2, \quad dr_1^2 = x^2+y^2+z^2 \end{aligned} \tag{4}$$

where the constants a, b, c, and d formed the matrix:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & \frac{V_{\det 1}}{V_{\det 2}} \\ \frac{V_{\det 1}}{V_{\det 3}} & \frac{V_{\det 1}}{V_{\det 4}} \end{bmatrix} \tag{5}$$

It is in fact the constant matrix that incorporates the ratios of measured voltages from each sensor as a result of combing (2) and (3).

Note that the constants of proportionality k from (2) in this particular case are the same for each sensor and they cancel in (5) along with the source intensity I, eliminating the need to find their value. In the case of different values of k for each sensor, the values must be known and substituted into (5).

The simultaneous solutions for the variables x, y, z, and r_1 found by solving (4) and (5) (assuming $x_{off} = y_{off} = e$, which is the known distance from the sensors to the origin of the array) are:

$$x = \frac{e(b-1)}{2(b+1-2d)}, \quad y = \frac{e(b+1-2c)}{2(b+1-2d)}$$
$$z = \frac{e(8db-16d^2+8d-2-4c^2-2b^2+4bc+4c)^{1/2}}{2(b-2d+b+1)}, \quad r_1^2 = \frac{2e^2}{(b-2d+1)} \tag{6}$$

Once r_1 is known, the intensity of the source can be found from:

$$I = \frac{r_1^2 V_{\det 1}}{k} \tag{7}$$

where the constant k is known and V_{det1} is the measured voltage of sensor 1.

For beamforming applications that utilize this method, the magnitude of the radial vectors r_1, r_2, r_3 and r_4 are first found from the equations above. Then the speed of propagation of energy from the point source must be known. A summer is then applied to the time delay of the signal along each radial vector as shown in Figure 2. This allows for summation of in-phase signals from the point source to

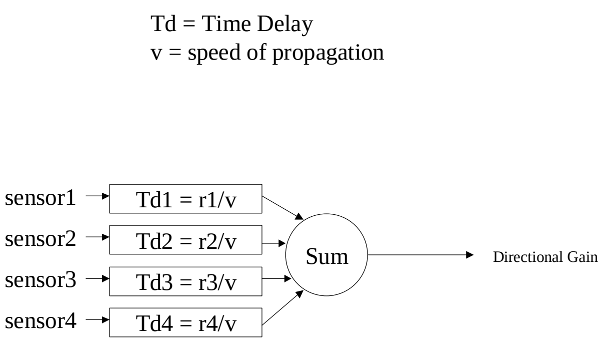

Figure 2. Beamforming From Triangulation

obtain directional gain. In contrast to phased-array beamforming, this approach is independent of a particular sampling frequency in order to form a summer where each sensor is in phase.

3. Conclusions

A method of triangulating point sources has been proposed which uses four omnidirectional sensors in a triangular array where the construction of the array does not require an integer number of source wavelengths. The outputs of the algorithm are the Cartesian coordinates of a single point source relative the origin of the sensors, as well as the intensity of the source if the constants of the sensors have been calibrated. The radial vectors from each sensor to the source can then be determined from the Cartesian coordinates, allowing for a beamformer to be developed which provides directional gain to the source based on the propagation speed of the signals. The beamforming is then independent of the frequency of the source, allowing for tracking of very long wavelength (ELF waves) or very short wavelength (gamma ray) sources without concern for interference at the sampling frequency or error due to short baseline considerations.

Applications of this method also include geophysics where the triangulation of s and p waves allows for three-dimensional determination of epicenters (fault location), and radiation detection where the location of radioactive materials can be determined from the inverse-square law of propagation.

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