# Light Beam Propagation in a Photorefractive Environment: Beam-Fanning Effect and Amplification of Weak Beams

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We investigate the propagation of light in photorefractive media by direct numerical simulation. The two dimensional model is based on solution of nonlinear material equations in steady-state and the equations governing propagation of the optical fields. We present results on fanning effect of a single beam and two-wave amplification in an applied alternating field.

*Keywords*: *BSO, photorefractive, space-charge field, electro-optic, fanning effect.*

## Introduction

Propagation of laser light in photorefractive medium is accompanied by a series of dramatic and fascinating changes of its space structure. Among these are the asymmetric light-induced stimulated scattering (known as the fanning effect) [1-2] (and references therein) and formation of spatial distributions of electromagnetic fields resulting in self-pumped phase conjugation and mutual conjugations of light beams incident on a photorefractive medium, superficial waves and some other effects. Thanks to considerable effort went into development of theoretical models describing this phenomena [3]. Previous analyses resulted in insight into many features of the nonlinear interaction of light with photorefractive media.But, due to the complexity of the processes relied on appropriate conjectures about the spatial structure of electromagnetic radiation inside the medium.

The aim of the present work is partly to report the formation of fanning effect and results on energy transfer in a two-wave mixing configuration in photorefractive media numerically, from first principles with minimum approximations. From the input amplitude of the light beams, we show their distribution of the fields inside the medium.

## Material Rate Equations

The experimental geometry we used is shown in Fig. 1, with a sinusoidal light interference pattern with period $\Lambda$,

$$I = I_0 \left[1 + m\cos(kx)\right], \qquad (1)$$

where $k = 2\pi / \Lambda$, and $m$ are the wave number and interference pattern modulation, respectively; a external electrical field is applied along $x$.Using the band transport model, the response of photorefractive material, that is, the carrier transport from the bright to the dark areas arise from: carrier diffusion, electric field induced drift and the photovoltaic effect. Neglecting the photovoltaic effect and assuming electron transport, the rate equations governing the physical response of the refractive material are [4]:

$$\frac{\partial N^+}{\partial t} = (sI + \beta)(N - N^+) - \gamma n N^+ , \tag{2}$$

$$\frac{\partial n}{\partial t} = \frac{\partial N^+}{\partial t} + \frac{\mu D}{e}\frac{\partial^2 n}{\partial x^2} + \mu(E_{sc} + E_a)\frac{\partial n}{\partial x} + \mu n \frac{\partial E_{sc}}{\partial x} , \tag{3}$$

$$\frac{\partial E_{sc}}{\partial t} = -\frac{\mu D}{\varepsilon\varepsilon_0}\frac{\partial n}{\partial x} - \frac{\mu e}{\varepsilon\varepsilon_0} n(E_{sc} + E_a) + \frac{J_0}{\varepsilon\varepsilon_0} , \tag{4}$$

where the motion of the carriers is along $x$ axis, $N_A$ is the initial number of acceptors, $N^+$ is the concentration of acceptors at instant $t$ and $N$ the total concentration of tramps. The electron concentration is $n$ and their mobility $\mu$ , $D = k_B T$ is the diffusion coefficient, $\gamma$ is the capture coefficient, $\beta$ is the thermal ionization rate, $s$ is the photoionization cross section, $\varepsilon$ the dielectric constant is $\varepsilon_0$ the permittivity in free space. The total electric field $E$ is given by the sum of applied field $E_a$ and space charge field $E_{sc}$ . $J_0$ Is the current density, is a constant independent of the coordinate $x$ , but it is a function of time. This function can be expressed as the spatial average

$$J_0 = \int_0^1 [e\mu n(x',t)E(x',t)]dx' , \tag{5}$$

where $E$ is the total electric field and $x' = x / \Lambda$.

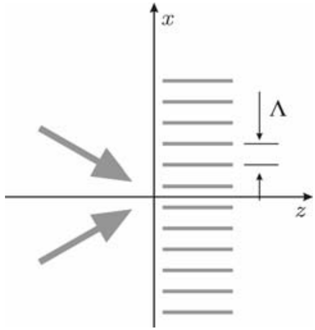

Fig. 1. Diagram showing the direction of grating wave vector in the sample.

The set of nonlinear Eqs. (2)-(4) is solved numerically, with the constriction given by Eq. (5). We have followed the method of lines using a finite element collocation procedure, with time-dependent second degree polynomials for the discretization of the variable $x$. The details are given in Ref. [5]. The solution was obtained from $10^{-10}$ seconds up to $94$ seconds, *i.e.*, until the stationary state was reached. This was done for values of $m$ between $0.01$ and $0.9$.

In the case of $t \to \infty$, that is, in the stationary case, Eqs. (2)-(4) are written as:

$$\frac{\partial n}{\partial x} = -\frac{e}{D} n(E_{sc} + E_a) + \frac{J_0}{\mu D}, \tag{6}$$

$$\frac{\partial E_{sc}}{\partial x} = \frac{e}{\varepsilon\varepsilon_0}(N^+ - N_A - n), \tag{7}$$

$$N^+ = \frac{(sI + \beta)N}{\gamma n + sI + \beta}. \tag{8}$$

We see that (6) permits to deduce $E_{sc}$ if we know $J_0$ and $n(x)$. If we obtain $n(x)$ then from (8) it is possible to find $N^+(x)$. Thus, the solution in this particular steady state means to calculate $J_0$ y $n(x)$.

## Numerical Results

The calculations were performed for BSO with usual physical parameters. An applied electric field of $E_a = 5.0 kV/cm$, an average light intensity of $I_o = 5.0\,\mathrm{mW/cm^2}$; $\varepsilon = 56.0$; $N = 10^{26}\,\mathrm{m^{-3}}$; $\mu = 3.0\times10^{-6}\,\mathrm{m^2/Vs}$; $\beta = 0.0$; $s = 2.0\times10^{-5}\,\mathrm{m^2/J}$; $\gamma = 1.6\times10^{-17}\,\mathrm{m^3/s}$ and $N_A =$ $10^{22}\,\mathrm{m^{-3}}$ for a temperature of $T = 300.0\,\mathrm{K}$.

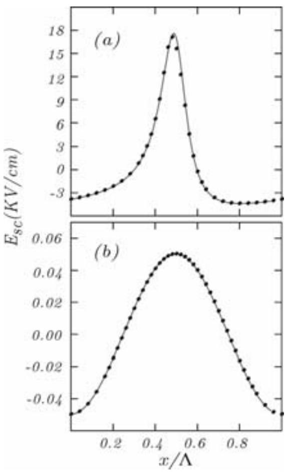


Fig. 2. Space-charge field as function of the modulation with: a) m=0.9, $\wedge = 1.0$ and b) m=0.01, $\wedge = 20.0$. The dotted line is the result of numerical solutions and the continuous line is the result for analytical function.

Under these conditions it is possible to find the space charge field in steady state, via an analytical expression in terms of modulation $m$ and the wavelength number $k$ of the interference pattern. The expression proposed for $E_{sc}$ is:

$$E_{sc} + E_a = \frac{J_0}{\mu e} \frac{m+1}{n(o)(1+m\cos kx)} \exp\left[\frac{K_o mk \sin kx}{(1+m\cos kx)^2}\right] -$$

$$-\frac{D}{e}\left[\frac{K_o mk^2 \cos kx + K_o m^2 k^2 (1+\sin^2 kx) - mk \sin kx (1+m\cos kx)^2}{(1+m\cos kx)^3}\right]$$

with the constants: $K_o = 8.0\times10^{-4}$, $J_0 = 2.998437$, $n(0) = 0.117328\times10^{-1}$. Fig. 2(a)-(b) show a thin line for the calculated analytical values of the space charge field for the extreme values $m = 0.9$, $\Lambda = 1.0$ and $m = 0.01, \Lambda = 20.0$, respectively. The dotted line are the calculated values using the exact numerical method, with times enough big considering an $E_{sc}$ stationary, that is, no more dependence in the time; we have a good concordance between both methods. Notice the scale changes in the space charge field.

## Equations of optical fields

The photorefractive materials are birefringents and optically active. The propagation of light into this media is determined through the index ellipsoid of shown at the Fig. 3 [4]. For the wave propagation in birefringent crystals along the $r$ direction, the two eigenmodes (modes of propagation with orthogonal polarizations) have their electric fields vectors parallel to $Op$, with a refraction index $n_0$, and parallel to $Oq$ with index $n_e(\theta)$.

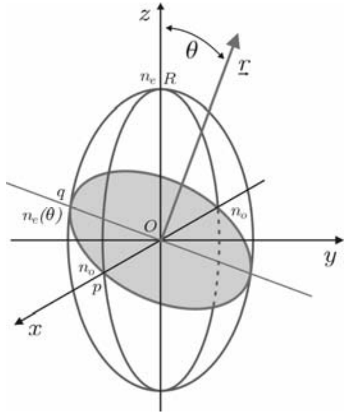


Fig.3. Normal modes determined from the Index ellipsoid.

These two modes of propagation are coupled through an optical activity or by the linear electro-optical effect, which are introduced using the complex number $\kappa + i\Gamma$ (and its conjugate), as elements out of the diagonal into the susceptibility tensor [6]. The real part is related with the optical activity and the imaginary part with the linear electro-optical coefficient.

The electric fields which define the polarization inside of material are coupled:

$$\begin{bmatrix} E_p(r) \\ E_q(r) \end{bmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{bmatrix} E_p(0) \\ E_q(0) \end{bmatrix}, \tag{9}$$

where $E_{p.q}(0)$ are the initial fields and the elements of the matrix G are [6]:

$$G_{11} = \cos Sr + i\left(\frac{\delta}{S}\right)\sin Sr\,,\ G_{12} = \left(\frac{\kappa + i\Gamma}{S}\right)\sin Sr\,,$$

$$G_{21} = \left( \frac{-k\kappa + i\Gamma}{S} \right) \sin Sr \, , \; G_{22} = \cos Sr - i\left( \frac{\delta}{S} \right) \sin Sr \, ,$$

where

$$\kappa = \frac{k_0 a}{2(n_q n_p)^{1/2}} \, , \; \Gamma = \frac{1}{2} k_0 (n_q n_p)^{3/2} r_{qpk} E_k \, , \; \delta = \frac{1}{2}(k_q - k_p) \, ,$$

$$S = (\kappa^2 + \Gamma^2 + \delta^2) \, ,$$

$a$ is the optical activity, $E_k$ is the applied field and $r_{ijk}$ is the electro-optical coefficient.

## The beam propagation method

The beam propagation method (BPM) is at present the tool most widely used in the study of light propagation in longitudinally varying waveguides. The standard version of BPM, based on the fast Fourier transform, splits the actual wave propagation into propagation in a homogeneous medium, followed by a phase correction corresponding to the inhomogeneous index distribution of the waveguide structure. However, limitations of the FFT-BPM should be noted. The probe beam remains essentially collimated, this occurs whenever the grating frequencies are sufficiently small, with low concomitant diffraction angles. Another constraint is that the birefringence of the grating medium, most no cause a strong walk off of orthogonally polarized laser beams. In this approximate analysis the boundary conditions become trivial to satisfy, and the second order wave propagation equation can be well approximated by a first order equation in which backscattered orders are neglected. All higher diffraction orders with any measurable optical power are assumed to be propagating, not evanescent. Such assumptions are accurate only when the optical inhomogeneities defined the grating are sufficiently weak, as is the case of the device modeling reported in this work.

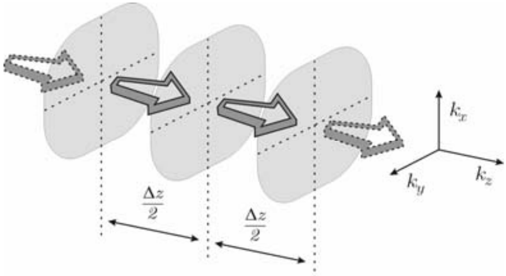

Fig. 4. The stepwise BPM.

The BPM is a stepwise algorithm, consisting essentially of replacing the optical beam propagation in an inhomogeneous and / or nonlinear medium by beam propagation through a sequence of homogeneous thin layers with a phase / polarization corrections after each layer. These corrections introduce the entire information about optical nonuniformity and nonlinearity of a layer by a single phase and polarization corrections. Fig. 4, shows the basic scheme of computation based on the ordinary BPM. The calculation begins with two input waves, $E_p(0)$ and $E_q(0)$ (see Eq. (9)), with mutually orthogonal polarization. These fields are propagate on a half of the interval of $\Delta z / 2$ in a uniform linear media, after that, the phase and polarization are corrected by multiplication by the matrix G, where we use the expression proposed for $E_{sc}$, in order to get the electro optical beam coupling. Finally the fields are propagated in the second half-step of $\Delta z / 2$. The calculation is repeated stepwise until get the length $z = L$.

The algorithm was implemented in [7], but in this work was done an approximation in the calculation of space charge field, once that the obtained field is extrapolated for small modulations (see Fig. 2(b) and compare the orders of magnitude with the Fig. 2(a)), to high modulation depths using a saturation function, where $E_{sc}$ can not be bigger than the applied field, this is not a valid situation as indicated in Fig, 2(a). As a difference with [7], we employ an analytical expression to calculate $E_{sc}$, and we consider the wavelength $\Lambda$ of the diffraction pattern which permit to determine $E_{sc}$ for different angles in according with the Bragg relation [4], where, if $\Lambda$ is small then the angles are bigger. Thus we use the ingenious algorithm proposed in [7], with the complements founded.

The Fig. 5 depicts the optical scheme of our experimental in accordance with [7], we carried out our experiment on BSO sample that were cut from bulk crystals along the $<110>$ crystallographic axis (along the axis $z$ in Fig. 5). A thin layer of silver paint covered the two faces of each sample to apply the square-wave external field along the $<\bar{1}11>$ axis (axis $x$ in Fig. 5. The axis $y$ is the $<1\bar{1}2>$ axis). The sample dimensions were $1.0\times 3.0\times 20.0mm^3$. The electric field was applied across the smallest side of the sample, where a He-Ne laser ($\lambda = 0.6328\mu m$) enters the crystal along the axis $z$ with a waist diameter of $75\mu m$. The following parameters of the BSO crystal were used: $a = 21^o/mm$, $r_{41} = 5.0\times 10^{-6}\mu m/V$, $n_o = 2.54$.

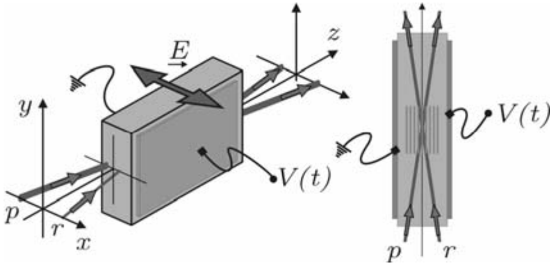


Fig. 5. Schematic of the experiment for two-wave mixing.

Fig. 6 shows the distribution of light intensity inside the crystal. Is a typical picture of asymmetric incoherent stimulated photorefractive scattering (fanning) of a single light beam. This picture is the result of the two dimensional numerical simulation (plane $xz$ in Fig. 5) of the Gaussian beam propagation inside BSO crystal, with an angle of $5^o$, under an alternating electric field of $12.0 kV/cm$. We observe that as the beam is being propagated into the crystal, the fanning effect begins to grow up. We note the formation of a "compression" in the beam, and after it is divided in many secondary beams making the fanning look.

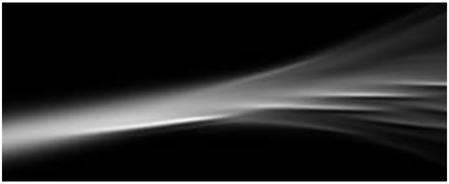

Fig. 6. Fanning of beam in a photorefractive medium

The recording and reading of optical gratings in photorefractive materials have been widely investigated. A related effect, namely the transfer of energy from one optical beam to another (also refereed to as beam coupling or parametric amplification) was first demonstrated in LiNbO, and was later found in a number of other materials including BSO, BTO, GaAs, KNbO. BSO is a promising material on account of its high speed but its rather small electro-optic coefficient was thought to preclude its application to amplification. In fact, the photorefractive effect may be increased by a simple technique which involves apply the square-wave external electric field [7]. The aim of the next application is report simulate results on energy transfer in two-beam coupling. Fig. 7 shows the distribution of light intensity inside the crystal of the two-dimensional numerical simulation of beam coupling in BSO crystal. The input beam denoted by $r$ has a low intensity (less than 5% of intensity $I_o$ of the input beam denoted by $p$) not seen in Fig.7. While two beams are being propagated they interact into the crystal making an interference pattern perfectly visible, after these beams interchange energy in according with Eq. (9), thus the weak beam goes up amplified, now visible in Fig.7.

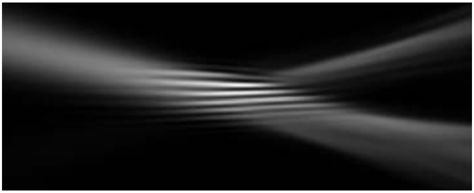

Fig.7. Two-wave mixing

We see that the process of beam coupling can be understand with the help of Fig. 8, in where we have considered that applied voltage is zero and the optical activity is now $a = 18^o / mm$ in order to rotate $360^o$ the plane of polarization of the emerging light. Fig. 8(a) are the progressive sections and its graph in gray tones pertinent to the distribution of light intensity inside the crystal for the mode parallel to electrical field, and the Fig. 8(b) exhibits the corresponding light intensity graphs for the mode perpendicular to such field. Fig. 8(c) indicates the total intensity of the electrical field, it is clear that in a simple look we could not distinguish this energy interchange. Fig. 8(d) shows the evolution of the plane of polarization of light passing through the medium. It is clear that this energy interchange is being manifested in a circularly polarized light.

The effect of the applied field can be exhibited changing the emergent plane polarization, as it is shown in Fig. 9(a)-(d), where were applied alternating voltages of 50V, 200V, 400V and 600V, respectively. We do not use higher voltages in order to eliminate the fanning effect. It is evident that we are simulating a phase modulator with a simple application of the beams propagation into active crystals.

## Conclusion

We have shown the possibility to find an analytical solution to the Kuktharev equations. Here we study a particular case of parameters, but with this equation is possible to investigate the structure formations inside photorefractive materials. We are interested in the analysis of phase conjugators. To do this it is important make propagations in both directions along the crystal, a situation that enables us to make our analytical solution.

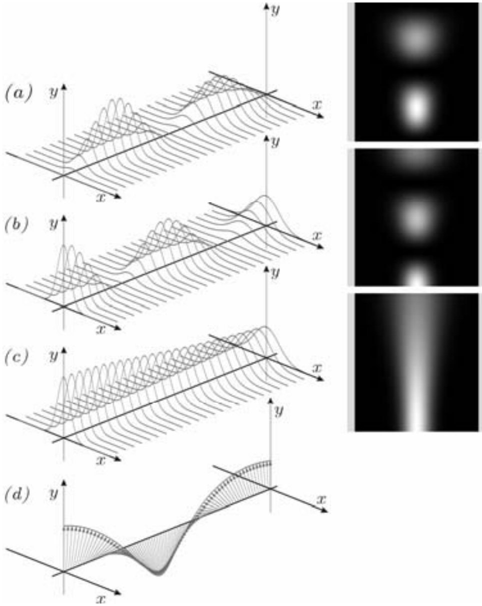


Fig.8. Evolution of the distribution of light intensity inside the BSO crystal with optical activity of 18.0o/mm for the mode: a) Parallel to applied field. b) Perpendicular to applied field. c) Total intensity. d) Evolution of polarization. In all the cases the applied electric field is zero.

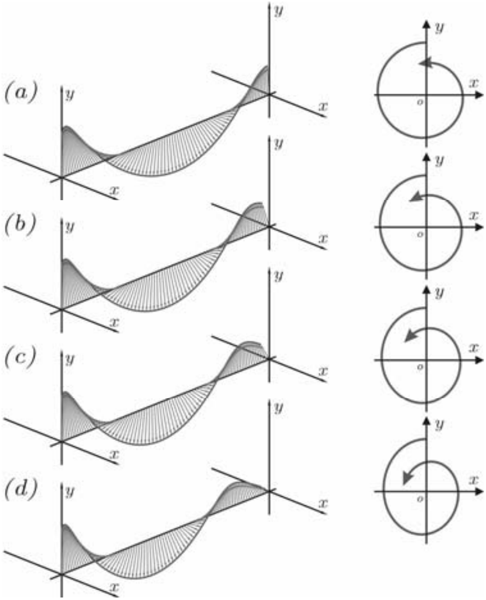


Fig.9. Evolution of the plane of polarization as function of the applied field for: a) 50V. b)200V. c)400V and d)600V.

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