Universes with constant total energy: Do they solve important cosmological problems?

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We study cosmological consequences of total energy conservation strictly valid for the whole universe. As prime consequence of ergodically behaving universes very specific scaling laws for relevant cosmic quantities with the diameter R of the universe are derived. Especially the R^{-2}-scaling of mass - and vacuum energy - density directly leads to a vanishing cosmic curvature parameter $k = 0$ and abolishes for such universes the horizon problem. The longstanding problem of observationally indicated very low cosmic vacuum energies in contrast to the very large quantumfield estimates now is easily solved when the vacuum energy density decay with R^{-2} is taken into account.

Keywords: Horizon and flatness problem, vacuum energy

Introduction

The most recently celebrated cosmological implications of the cosmic microwave background studies with WMAP [28], though fascinating by themselves, do, however, create some extremely hard conceptual challenges for the present-day cosmology. These recent extremely refined WMAP observations seem to reflect a universe which was extremely homogeneous at the recombination age and thus is obviously causally closed at the time of the cosmic recombination era. From the very tiny fluctuations apparent at this early epoch the presently observable nonlinear cosmic density structures can, however, only have grown up, if in addition to a mysteriously high percentage of dark matter an even higher percentage of dark energy is admitted as responsible drivers of the cosmic evolution. The required dark energy density, on the other hand, is nevertheless 120 orders of magnitude smaller then the theoretically calculated value. These are only some of the outstanding problems of present-day cosmology which we are facing here under new auspices. In the following we shall investigate up to what degree a universe abolishes all these exposed outstanding problems, if it simply behaves as a universe with constant total energy.

As we shall show basic questions like: How at all could the gigantic mass of the universe of about 10^{80} proton masses at all become created? - Why is the presently recognized and obviously indispensable cosmic vacuum energy density so terribly much smaller than is expected from quantumtheoretical considerations, but nevertheless terribly important for the cosmic evolution? -Why is the universe within its world horizon a causally closed system? - , can perhaps simply be answered, when only the assumption is made, that the actually prevailing universe is one with a minimal and constant total energy, a so-called economical universe. As we can show in this paper, the strict validity of total energy conservation for the whole universe offers very attractive solutions for the above mentioned problems. For instance, the R^{-2}-scaling of mass - and vacuum energy - density which here is derived for a universe with vanishing total energy leads to a vanishing cosmic curvature parameter, i.e. $k = 0$, and abolishes the horizon problem. Due to the scaling there exists no problem anymore with the cosmic vacuum energy density, since it now can easily be reconciled with its theoretical expectation values. We also indicate why the mass of the universe can increase and in fact can even grow up from a Planck mass as a primary vacuum fluctutation.

It should be pointed out that the problems of cosmology addressed in this paper are very prominent, however, there exist many other with equal importance. A good overview of the present-day unsolved problems in astrophysics is given by Wesson [27] or by Bahcall and Ostriker [2]. The spectrum of open questions covers topics like the "Hierarchy Problem", "Quantum Gravity", "Cosmological Parameters" or "Large-Scale Structures", just to name a few. Another challenging unsolved problem is the "Redshift Quantization" of galaxies as first claimed by Tifft [23]) and as strongly supported by investigations by Guthrie and Napier [10].

Introduction to the ergodic universe

This paper has its focus on the horizon and flatness problem and on the problem of the vacuum energy. Several new speculative ideas have been recently proposed which could help to overcome some of these problems. Besides the theory of cosmic inflation which goes back to Guth and Steinhardt [11], Linde [15], Guth [12], or Liddle and Lyth [14], some authors (e.g. Albrecht and Magueijo [1], Fritsch [9] or Barrow [5]) have postulated a speed of light variable with cosmic scale or time which would solve the flatness problem and the horizon problem for cosmology. The problem connected with the cosmologically required dark energy, however, remains completely unsolved so far.

Already at earlier occasions the concept of a zero-energy or economical universe which can be created from nothing but a quantum fluctuation, since as a whole representing a vanishing total energy, was introduced and discussed in first steps by Brout et al. [6], Vilenkin [25] or Tryon [24], and it may even go back to Jordan [13]. More recently, however, we have revisited their ideas and have rederived a new form of this economical universe (Overduin and Fahr [18], Overduin and Fahr [19], Fahr [7]). In these latter publications it was formulated as a crucial requirement for an economical universe, that its total energy $L = L(R)$ be minimal und equal to a constant L_0, or in other words requiring that the change of $L = L(R)$ with the world radius R, or with the cosmic evolution time t, vanishes at all times of the cosmic evolution.

This requirement is expressed by the following relation:

$$L = E + U = L_0 \tag{1}$$

where the quantity E is expressed by

$$E = \frac{4\pi}{3} R^3 (\rho c^2 + 3p) \tag{2}$$

and represents the accumulated energy in a cosmic volume of radius R constituted by the rest mass energy density ρc^2 of baryonic and dark matter, of the mass equivalent of vacuum energy density $\rho_{vac} c^2$, and of the pressure energies of matter, p, and of vacuum, p_{vac}. The quantity U is given by

$$U = -\frac{8\pi^2 G}{15} (\rho + \frac{3p}{c^2})^2 R^5 \tag{3}$$

and represents the negative potential gravitational binding energy which was calculated by Fahr [7] with the help of the well-known Poisson- equation for the cosmic gravitational potential Φ written for a homogeneous universe. This potential in the neighborhood of an arbitrary space point is obtained from the equation:

$$\Delta\Phi = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial\Phi}{\partial r}) = -4\pi G (\rho + \frac{3p}{c^2}) \tag{4}$$

In the above equations c is the light velocity, G is Newton's gravitational constant, ρ is the density of all contributing masses in the universe (i.e. baryonic , dark and vacuum), and p is the cosmic pressure connected with these mass representers. The pressure contributions depend on the associated types of mass density (i.e. density of baryonic, dark or vacuum-related equivalent masses).

In the following we assume that according to standard modern views the total density ρ is composed of ρ_{mat} due to baryonic and dark matter, and of ρ_{vac} due to the mass equivalent of the energy density of the cosmic vacuum, thus yielding the expression $\rho = \rho_{mat} + \rho_{vac}$.

To further evaluate the equations (1) through (3), we have to consider the cosmic pressures and their relation to densities. Here we base ourselves on the well known thermodynamic relation used everywhere in cosmology and given by:

$$\frac{d(\rho c^2 R^3)}{dt} = -\frac{d(\varepsilon R^3)}{dt} - p\frac{d(R^3)}{dt} \tag{5}$$

The above equation describes the change of the energy density with the expansion of the cosmic scale R (Term 1) which is connected with the corresponding change of the inner energy density ϵ or enthalpy (Term 2) and of the work that is done by the cosmic pressure p at the expansion of the cosmic volume (Term 3). Hereby one may remind that the effective total pressure is constituted by $p = p_{mat} + p_{vac}$. In the present epoch of the cosmic evolution it is justified to assume that due to adiabatic temperature decrease the inner energy density ϵ can be neglected with respect to the rest mass energy density converting equation (5) into its simpler form:

$$\frac{d(\rho c^2 R^3)}{dR} = -p\frac{dR^3}{dR} \tag{6}$$

In the above relation we have converted derivatives with respect to t into those with respect to R by setting $d/dt = (dR/dt)d/dR$.

Representing all cosmic densities which in a homogeneous unverse can only depend on R, by a general form of an R- dependence according to R^{-n}, one then obtains for the pressure p from equation (6):

$$p = -\frac{3-n}{3}\rho c^2 \tag{7}$$

This result contains the well known relations of state relating the thermodynamic pressure p and the density ρ within the expanding universe and reveals the fact that both pressure and density are described by the same dependence on the cosmic scale R. This allows to regain the relations well known from general literature, namely for the case of pure matter universe with: $p_{mat} = 0$ for $\rho_{mat} \sim R^{-3}$, for the case of pure photon pressure with: $p_\gamma = 1/3\rho_\gamma c^2$ for $\rho_\gamma \sim R^{-4}$, und for the case of a pure cosmic vacuum with $p_{vac} = -\rho_{vac}c^2$ for $\rho_{vac} = const.$, which can easily be checked by insertion of the respective pressures into equation (6).

While a pressure $p_{mat} = 0$ for adiabatically cooling matter, and a pressure $p_\gamma = 1/3\rho_\gamma c^2$ for cosmologically redshifted photons seem to be in accordance with our physical intuition, the negative pressure $p_{vac} = -\rho_{vac}c^2$ for the vacuum with constant vacuum energy density, however, appears to be counterintuitive at first glance. One should, however, not forget that such a result arises only in line with the assumption that according to (6) the mass density of the cosmic vacuum is constant at the expansion of the universe. If, however, vacuum energy density is constant then the expanding universe, creating new cosmic volume, though doing work against the vacuum pressure also permanently has to create new amounts of vacuum energy connected with the increased volume. This in fact is only thermodynamically permissible, if the vacuum pressure is negative according to equation (6).

Based on these knowledges on the behaviour of cosmic density ρ in an expanding universe and of the associated cosmic pressure p we may now come back to the equations (1) through (3). To fulfill the requirement $L = E + U = L_0$ for the economical universe, namely:

$$\frac{dL_0}{dR} = \frac{d}{dR}[\frac{4\pi}{3}R^3c^2(\rho + \frac{3p}{c^2}) - \frac{8\pi^2 G}{15}(\rho + \frac{3p}{c^2})^2 R^5] = 0 \qquad (8)$$

exactly two solutions are existing. First, for the case $(\rho + \frac{3p}{c^2}) \neq 0$, one can obtain from (8) the following condition:

$$(\rho + \frac{3p}{c^2}) = \frac{5c^2}{2\pi G R^2} \qquad (9)$$

Second, for the case $(\rho + \frac{3p}{c^2}) = 0$, one obtains

$$p = -\frac{1}{3}\rho c^2 \qquad (10)$$

Interestingly enough, the requirements of equations (9) and (10) are fulfilled simultaneously, as can easily be confirmed. Equation (9) states, that the density ρ has to scale according to R^{-2}, while equation (10) yields a pressure $p = -\frac{1}{3}\rho c^2$, from where one derives with equation (7), that on the basis of the formal law describing the R-dependence according to R^{-n}, the necessary exponent has to be $n = 2$. This, however, also implies that density as well as pressure should scale with R^{-2}.

This permits the recognition that obviously a universe is possible which over the whole epoch of its evolution and existence represents a vanishing total energy with $L_0 = 0$. In the following we shall also show that such a universe can in fact be created from nothing, i.e. from nothing more than a vacuum fluctuation, if , and only if, both the total density and the total pressure in the universe scale with R^{-2}. From this latter behaviour of the mass density with R one must, however, conclude that in a curvature-less universe the total mass of the universe, i.e. $M = \frac{4\pi}{3}\rho R^3$, does linearly scale with the scale R yielding

$$M \sim R \tag{11}$$

With the above result for an $L_0 = 0$ - universe one obtains a nonclassical connection of the world mass M and the size, or scale, R, of the universe. This quasi-Machian form of a scaling of masses with the dimension of the universe has, in fact already often been claimed for (see e.g. Mach [16], Barbour [3], Whitrow [29], Barbour and Pfister [4], Thirring [22], or Hoyle 1990, 1992). The cosmological consequences of this surprising scaling law for the world mass M shall now be further investigated in the following part of the paper. The $M \sim R$ relation and the $\rho \sim R^{-2}$ relation surprisingly enough have just most recently again been supported by new formulations of logically and physically rigorous conceptions of an instantaneous mass and the effective diameter of an observer-related universe (Fahr and Heyl [6]).

The radius of the universe

Starting from a total mass of the universe given in the form $M_{tot} = M_{mat} + M_{vac}$ one can calculate an associated Schwarzschild radius R_S as given by the following expression:

$$R_S = \frac{2GM_{tot}}{c^2} \tag{12}$$

Replacing now masses by densities according to the relation $M_{tot} = \frac{4\pi}{3}R^3\rho_{tot}$ permits to solve equation (12) with respect to the density ρ_{tot} which then yields:

$$\rho_{tot} = \frac{3c^2}{8\pi G}\frac{R_S}{R^3} \tag{13}$$

The above result can only be reconciled with the earlier result that the density of the universe scales with R^{-2}, if the extent or the radius of the universe just is the above defined Schwarzschildradius, i.e. if $R = R_S$, leading to the following relation:

$$\rho_{tot} = \frac{3c^2}{8\pi GR^2} = \frac{3c^2}{8\pi GR_S^2} \tag{14}$$

From this above relation one can draw the conclusion that an economical universe has a world radius R which is identical with its Schwarzschildradius R_S defined by its total mass M_{tot}.

The expansion velocity of the universe

Now we want to investigate the characteristic expansion velocity for an economical universe. This velocity is determined as the rate by which the radius of the universe changes in units of cosmic time t , i.e. is given by $dR/dt = \dot{R}$, and also delivers the Hubble parameter H for this universe by the well known relation $H = \dot{R}/R$. To obtain this velocity one has to start from the second of the Friedmann-type cosmological equations yielding the acceleration of the cosmic expansion and given in the following form (see e.g. Stephani [21]):

$$\frac{1}{R}\frac{d^2R}{dt^2} = \frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho + \frac{3p}{c^2}) \tag{15}$$

For an economical universe we have shown that equation (10) has be fulfilled requiring $\rho + 3p/c^2 = 0$. With this condition equation (15) now immediately shows as a consequence that the acceleration has to vanish, i.e. $d^2R/dt^2 = \ddot{R} = 0$, meaning that the expansion velocity of this universe $dR/dt = \dot{R}$ always has to be constant. According to the cosmological principle that the universe from all cosmic space points has to look completely the same this means that from whatever standpoint is taken the expansion rate of the universe is always the same. According to the theory of special and general relativity there is only one velocity that is constant and identical at all reference systems, namely the velocity of light c. This at least strongly suggests that the required constant expansion velocity should be identical with the velocity of light, i.e. $dR/dt = \dot{R} = c$.

The conclusion from the above reasonings therefore should be that in an economical universe the cosmic expansion velocity $\dot{R}$ always has to amount to the velocity of light c, a result already claimed for from completely different reasons in a very early publication by Milne [30].

Density, mass of the universe and the cosmic flatness problem

Based on the fact that the economical universe permanently expands with the velocity of light one can also draw some new insights from a view to equation (14) concerning the cosmic mass density. With $H = \dot{R}/R = c/R$ one then writes:

$$\rho_{tot} = \frac{3H^2}{8\pi G} = \rho_{crit} \tag{16}$$

meaning that the density not only once during the evolution of the universe, but permanently equals the so-called critical density ρ_{crit}. The latter, however, is defined from the first of the Friedmann equations given in the form:

$$\left(\frac{\dot{R}}{R}\right)^2 = H^2 = \frac{8\pi G}{3}\rho_{tot} - \frac{kc^2}{R^2} \tag{17}$$

For $\dot{R} = c$ this requires that the curvature parameter k , characterizing the curvature of cosmic spacetime, vanishes, i.e. $k = 0$. This then in addition means that an economical universe, not only permanently preserves its critical density ρ_{crit}, but at the same time it also permanently stays curvatureless, or to say it in other words: it is topologically flat all over its evolution. This flatness which is a necessary prerequisite of an economical universe in fact, interestingly enough, also seems to be an observational fact of the actually present universe (see WMAP [28]).

Replacing further on the density by the mass of the universe using the relation $\rho_{tot} = M_{tot}/(\frac{4\pi}{3}R^3)$, one then is lead to the result:

$$M_{tot} = \frac{c^2}{2G}R \tag{18}$$

which once again from a different side states that, astonishingly enough, in an economical universe the total mass of the universe scales with its cosmic radius R.

This finally challenges to put the question of how large the mass of the universe under these auspices might have been at the very early cosmic time, when the radius of the universe only amounted to one Planck scale $R_{pl} = \sqrt{hG/c^3}$. Then with equation (18) one obtains the following surprising answer to this question:

$$M_{tot}(R_{Pl}) = \frac{1}{2}m_{Pl} \tag{19}$$

This interestingly enough means that an economical universe, even of the size of the present universe with its gigantic mass, can possibly have evolved at its expansion from half a Planck mass which according to quantummechanical results can appear at any time on the Planck scale as a virtual mass fluctuation, i.e. as a quantum fluctuation. In view of the required scaling of the cosmic mass with the scale R the present world mass amounts to:

$$M_{tot} = \frac{c^2}{2G}R = \frac{c^3}{2G}\frac{R}{c} = \frac{c^3}{2GH} \approx 10^{53}kg \approx 10^{80}m_{prot} \tag{20}$$

if one adopts a Hubble parameter of $H = 72km/s/Mpc$ for the present universe or synonymous, with a present radius of the universe R amounting to 4167 Mpc (m_{prot} is the mass of a proton). The above values are consistent with standard estimations, e.g. what concerns the visible universe to consist of about 10^{11} galaxies with 10^{11} solar-type stars each.

A small rearrangement of equation (20) in addition also leads to a useful formula for the Hubble parameter of an economical universe given by:

$$H = \frac{c^3}{2GM_{tot}} \tag{21}$$

This then finally permits as a further conclusion that our universe, if it is an economical universe, can easily evolve from a quantummechanical fluctuation which is allowed on the Planck scale $R_{Pl} = \sqrt{hG/c^3}$. Its density permanently represents the critical density ρ_{crit} and thus it has a vanishing curvature with $k = 0$.

The horizon problem

Another big cosmological problem consists in the so-called "horizon problem" that arises from the question, whether or not a photon released at the very beginning of the universe, "t=0", i.e. the Big Bang, can meanwhile have reached any potential observer or cosmic space point. This question leads one to the introduction of the term "light horizon", reflecting the maximum distance from any observer's space point up to which this observer can be informed about physical events in the universe, since photons are assumed to communicate such past events to the observer till today. On the other hand, regions of the universe which are located behind this light horizon are thus causally decoupled from the observer.

The distance of the light horizon for a universe with curvature $k = 0$ can be calculated as (see e.g. Stephani [21]):

$$r_{hor} = c \int_0^{t_{heute}} \frac{d\tilde{t}}{R(\tilde{t})} \tag{22}$$

with $R(\tilde{t})$ denoting the time dependent extension of the universe. Cosmic expansions with $R \sim t^{\alpha}$ and $\alpha < 1$ yield a finite r_{hor} and therefore yield a causally unclosed universe - the sword of Damocles of the todays cosmology, since to the contrast for instance the cosmic microwave background with its origin about 150 kiloyears after the Big-Bang is clearly indicating causal closure. In contrast to standard cosmological solutions with $\alpha < 1$, the expansion of an economic universe simply follows the law $R = ct$, i.e. requiring $\alpha = 1$! , and hence the light horizon r_{hor} in this case is given by:

$$r_{hor} = c \int_0^{t_{heute}} \frac{d\tilde{t}}{R(\tilde{t})} = \int_0^{t_{heute}} \frac{d\tilde{t}}{\tilde{t}} = \infty \tag{23}$$

Therefore we obtain the interesting result that an economical universe is causally closed over the whole period of its expansion - or with other words: All space points of an economical universe could have undergone a physical interaction with each other, since at any time following the start of the expansion the causal light horizon was infinite.

Hence we can draw the conclusion, that an economical universe does not encounter a "horizon problem" and that no inflation is required to explain the highly pronounced isotropy of the 2.735 K cosmic microwave background (CMB) observed by WMAP.

The mystery of the vacuum energy

According to the present observations (WMAP [21]; Perlmutter et al. [20]) the mass equivalent of the cosmic vacuum energy is expected to contribute by about 70% to the total mass of the universe. However, the vacuum energy as it is calculated by quantumfield theoreticians confronts us with the shocking problem that it amounts to values higher by a factor $\approx 10^{120}$ compared to the value it should have to be compliant with the nowadays observational results of the universe. This terribly confusing situation can now, in view of the above results, be looked at in a more relaxed manner. Namely, when the R^{-2} - scaling of the mass density and the vacuum energy density valid in the economical universe is taken into account. A scaling of the vacuum energy density with R^{-2} by the way has already been speculated on at many other places in the literature (for a review see Overduin and Cooperstock [17]).

Let us start with a look on the vacuum energy which is generally interpreted as the overall sum of the energy zero-point oscillations with proper frequencies ω_j of the vacuum. This sum can be expressed by the following equation:

$$E_{vac} = \frac{1}{2}\sum_j \hbar w_j \tag{24}$$

The corresponding density of the vacuum energy is then calculated to yield (Weinberg [26]):

$$\rho_{vac}c^2 = \frac{c\hbar k^4_{\max}}{16\pi^2} \tag{25}$$

with $\hbar$ the Planck constant and k_{max} the so-called "cut-off" wave number of the oscillations. Such a "cut-off" is needed to prevent divergencies when calculating the energy density in the frame of QED. Since the wave number k_{max} can be expressed by the Planck mass $m_{Pl} = \sqrt{\hbar c/G}$ as $k_{max} = m_{Pl}c^2/\hbar$, we therefore obtain the vacuum energy density in the form:

$$\rho_{vac}c^2 = \frac{(2\pi)^4 c^7}{16\pi^2 \hbar G^2} \tag{26}$$

We now apply the R^{-2} scaling and choose the Planck length λ_{Pl} as a reference scale for the beginning of the expansion of the universe at the Big Bang. This then leads us to:

$$\rho_{vac}c^2 = \frac{(2\pi)^4 c^7}{16\pi^2 \hbar G^2} \frac{\lambda_{Pl}^2}{R^2} = \frac{\pi^2 c^4}{GR^2} \tag{27}$$

The ratio of this vacuum density and the critical density given in equation (16) finally yields:

$$\frac{\rho_{vac}}{\rho_{crit}} = \frac{\varepsilon_{vac}}{\varepsilon_{crit}} = \frac{\pi^2 c^4}{GR^2} \left(\frac{3c^4}{8\pi GR^2} \right)^{-1} = \frac{8}{3}\pi^3 \simeq 83 \tag{28}$$

The above result still exceeds the presently favoured ratio of 0.7 by about a factor ≈ 100, however, the factor 10^{120} has disappeared and the remaining discrepancy is not at all alarming, since equation (25) only is an approximation which only considers QED fields and does not take into account effects from e.g. quantum chromodynamics (QCD).

Again at this point the above findings for the economical universe can help further to extend the understanding and the meaning of cosmic vacuum energy. Since we know that ρ_{vac}

should not exceed ρ_{crit} we are able to provide a formula for the upper limit of the vacuum energy which is given by:

$$\rho_{vac}c^2 \approx \rho_{crit}c^2 = \frac{3H^2}{8\pi G}c^2 = \frac{3c^4}{8\pi G R^2} \tag{29}$$

Now, we again apply this R^{-2}- scaling and calculate the vacuum energy density at a phase of the evolution when $R = R_{Pl} = \sqrt{\hbar G/c^3}$ and we obtain instead of equation (25):

$$\rho_{vac}c^2 = \frac{3c^7}{8\pi\hbar G^2} = \frac{3c\hbar k_{\max}^4}{128\pi^5} \tag{30}$$

which, using the Planck mass $m_{Pl} = \sqrt{\hbar c/G}$ and the Planck volume $V_{Pl} = \frac{4}{3}\pi R_{Pl}^3$, gives the following upper limit for the density of the vacuum energy:

$$\rho_{vac}c^2 = \frac{1}{2}\frac{m_{Pl}c^2}{\left(\frac{4}{3}\pi R_{Pl}^3\right)} \tag{31}$$

From this it can be concluded that the maximum density of the vacuum energy at the very beginning of the universe obviously seems to be nothing else but the energy density of half the Planck rest mass, or in terms of energy: $E_{vac} = \rho_{vac}c^2\frac{4}{3}\pi R_{Pl}^3 = \frac{1}{2}m_{Pl}c^2$. A comparison with equation (21) then yields the surprising result:

$$\frac{1}{2}m_{Pl}c^2 = \frac{1}{2}\sum_j \hbar\omega_j \Leftrightarrow m_{Pl}c^2 = \sum_j \hbar\omega_j \tag{32}$$

The above finding hence suggests the conclusion - that had already been expressed as a presumption very often in the literature: The vacuum energy and the equivalent energy of the

Planck rest mass are in fact identical. In the frame of the economical universe it now turns out, that the vacuum energy (= Planck mass) can in fact be the source of all energy and matter in the universe.

Then, if one further on believes in the correctness of the presently favoured fraction $\rho_{vac}/\rho_{mat} \simeq 0.7/0.3$ of the vacuum energy and matter density, respectively, one can also conclude that for some reason about 70% of the vacuum energy permanently remains in the vacuum during the expansion of the universe while about 30% manifest itself as matter. This ratio must be constant during the whole evolution of the universe because both, vacuum energy and matter density, follow the derived R^{-2} scaling of an economical universe.

Interpretation of the mass increase

One of the very surprising results of this paper is the scaling of the mass of the universe with its extension R which can be easily explained, however, in the frame of quantum mechanics. A look at the uncertainty principle $\hbar/2 \approx \Delta E \Delta t = \Delta m c^2 \Delta t$ shows the possibility of the virtual apperarance of half a Planck mass within a time interval $\Delta t = t_{Pl}$, i.e. Planck time:

$$\frac{\hbar}{2} \approx \Delta m c^2 t_{Pl} = \Delta m c^2 \sqrt{\frac{G\hbar}{c^5}} \Rightarrow \Delta m = \frac{1}{2}\sqrt{\frac{\hbar c}{G}} = \frac{1}{2} m_{Pl} \quad (33)$$

The virtual Planck mass may stay in the real world of the economical universe if its rest mass energy is compensated to zero. This is, however, always guaranteed by the negative gravitational binding energy which leads - as shown in this paper

- to a vanishing total energy. Thus, the mass increase of the universe is due to virtual Planck masses which become real and which contribute over the lifetime t_{univ} of the universe to the total mass with a "production rate" of half a Planck mass each time interval t_{Pl}:

$$M_{tot} = \frac{1}{2}\frac{m_{Pl}}{t_{Pl}}t_{univ} = \frac{1}{2}\frac{c^3}{G}\frac{R}{c} = \frac{c^2}{2G}R \qquad (34)$$

Here we have used again the result that the extension R of an economical universe is simply given by $R = ct_{univ}$. It is amazing to recognize an identical scaling law for the mass as given in eq. (18). Obviously, quantum mechanical considerations on one hand and independently retrieved results for the economical universe on the other hand are in reconciliation. Furthermore, if we calculate the time derivative of the Schwarzschild radius in eq. (12) we retrieve:

$$\dot{R}_S = \frac{\mathrm{d}R_S}{\mathrm{d}t} = \frac{2G}{c^2}\frac{\mathrm{d}M_{\mathrm{tot}}}{\mathrm{d}t} = \frac{2G}{c^2}\frac{\left(\frac{1}{2}m_{\mathrm{Pl}}\right)}{t_{\mathrm{Pl}}} = c. \qquad (35)$$

Since the Schwarzschild radius, as shown earlier, represents the extension of the investigated economical universe the above result again indicates an expansion velocity c.

Summary

The trust in the validity of one of the most fundamental laws of physics - the conservation of energy - combined with the belief that this law also holds true for the universe offers very attractive solutions for the cosmological problems investigated in this paper. The derived R^{-2} scaling of matter and vacuum energy densities in the frame of the discussed "economy" lead to a universe with curvature $k = 0$ which does not face a horizon problem any longer and thus does not require a cosmic inflation at the beginning. Furthermore, the theoretically calculated and unexplainable high amount of vacuum energy - with a value about 10^{120} higher than observed - one perfectly fits into the idea of an economical universe if the R^{-2} scaling is consequently also applied to the vacuum energy density. In addition, it has been shown, that the present universe might have its origin in a quantum mechanical fluctuation that took place in the Planck era and that the vacuum energy is nothing else but the scaling rest energy associated with the Planck mass within a Planck volume V_{Pl}. Finally, the whole mass of the universe can be explained by the accumulation of Planck masses up to the present time which are generated as virtual quantum mass releases per Planck time and permitted to become real in the expanding economical universe.

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