Cross antenna: An experimental and numerical analysis

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The cross antenna is a medium gain and circular polarization structure made of a conductor or strip line over a ground plane, following a cross contour of four or more branches. One end is feed by a generator and the other one is charged with a load impedance. This paper presents a theoretical and experimental analysis of an eight arms cross antenna, loaded with four different impedances. The theoretical study is made via the computational solution of Pocklington's equation applied to the structure; experimental results are obtained over an antenna of a 12 AWG wire over a ground plane, working in 3.2 GHz. We present radiation efficiency, gain, field pattern, and axial rate results.

Keywords: Pocklington equation; Cross antenna; Method of Moments

Introduction

The cross structure belongs to the family of the traveling wave current distribution antenna, it was first presented by Antoine Roederer [1] but it seems that no other article has been written about the subject; considering its important potential applications, due its small size and weight, we have been studying its performance, and results are presented in this paper. The antenna can be used in mobile communication or as primary radiator of parabolic reflectors, when circular polarization is needed. The construction repeatability is very easy as well the facility to obtain 14 dB gain in a very small antenna.

The Method of Moments (MoM) solution of the generalized Pocklington equation [2] is used to obtain the current distribution and the main parameters of the antenna. Our technique employs the Pocklington equation for thin wires with arbitrary geometry [3], and it requires to specify antenna geometry by means of the vectorial equations for the wire and the equivalent current filament axis. For the experimental results we construct several antennas, to compare its repeatability, modifying their positions with respect to ground plane and load impedance.

General Formulation for the Pocklington Equation

As is well- known, Pocklington's equation establishes a relationship between the current distribution in a straight wire and the impressed electric field on its surface. The model considers an electrically small wire's radius of an antenna made of a perfect conductor. Pocklington's approach supposes that the current density in the conductor can be simulated by a current filament parallel to the antenna axis, leaving the rest of the conductor as part of the free space. For general geometries, as in figure 1, the model takes into account the bent characteristics. Formally the general Pocklington equation [4], is obtained from Maxwell equations, magnetic and electric potentials in the Lorenz gauge [5] as:

$$E^{i}_{\tan} = -\frac{j}{\omega\varepsilon}\int_{s'} I_s(s')\left[k^2\mathbf{s}\bullet\mathbf{s}' + \frac{\partial^2}{\partial s\partial s'}\right]\frac{e^{-jkR}}{4\pi R}ds'. \quad (1)$$

According to figure 1, a small wire's segment defines a local co-ordinate system, rotated respect to the reference co-ordinate system, rotation is represented by $\mathbf{s}\bullet\mathbf{s}'$, while antenna geometry is defined by s(s), r(s), s'(s') and r'(s').

The general Pocklington equation can be simplified, for a more efficient solution, expanding the integral kernel by means of the vector equation representing the wire's axis, r(s), and the equivalent current filament equation, r'(s) :

$$\begin{aligned} \mathbf{r}(s) &= x(s)\mathbf{i} + y(s)\mathbf{j} + z(s)\mathbf{k}, \\ \mathbf{r}'(s') &= \mathbf{r}(s') + a\mathbf{n}(s') \end{aligned} \quad (2)$$

where n(s) is the unit normal vector for the wire's axis and a the wire's radius; as the parallel curve is separated a distance a from the axis curve, analysis points never coincide and there is no possibility of any singularities.

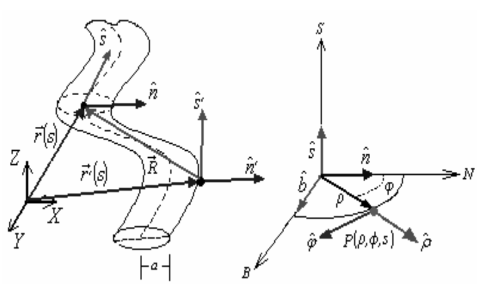


Figure 1. Arbitrary bent wire and the relations between its vectors and geometry.

We see that the curve representing the current filament is a parallel curve to the one representing the wire's axis. For any particular geometry it is possible to define tangential unit vectors to obtain the dot product s · s', expressed by:

$$\mathbf{s}(s) = \frac{dx(s)}{ds}\mathbf{i} + \frac{dy(s)}{ds}\mathbf{j} + \frac{dz(s)}{ds}\mathbf{k}\,,$$
$$\mathbf{s}'(s') = \frac{dx'(s')}{ds'}\mathbf{i} + \frac{dy'(s')}{ds'}\mathbf{j} + \frac{dz'(s')}{ds'}\mathbf{k}\,. \tag{3}$$

Once the vector relationship has been found it is possible to simplify the kernel of (1) expanding the operator $\partial^2 / \partial s \partial s'$ over Green function, transforming it into a pure integral equation:

$$E_{\tan}^{i} = -\frac{j}{\omega\varepsilon}\int_{s'} I_s(s')\left[\begin{array}{l} R^2\left(k^2R^2 - 1 - jkR\right)\mathbf{s}\bullet\mathbf{s}' + \\ \left(3 + 3jkR - k^2R^2\right)\left(\mathbf{R}\bullet\mathbf{s}\right)\left(\mathbf{R}\bullet\mathbf{s}'\right)\end{array}\right]\frac{e^{-jkR}}{4\pi R^5}\,ds'\,. \tag{4}$$

R is the distance between observation and source points given by:

$$R = |\mathbf{R}| = |\mathbf{r} - \mathbf{r}'| = \sqrt{\left[x(s) - x'(s')\right]^2 + \left[y(s) - y'(s')\right]^2 + \left[z(s) - z'(s')\right]^2} \tag{5}$$

The MoM solution employs the simple point-matching technique, using pulse functions as basis functions and Dirac's delta as weight functions.

$$i_n(s') = \begin{cases} 1 & \text{if} \quad (n-1)\Delta s' \le s' < n\Delta s' \\ 0 & \text{elsewhere} \end{cases},$$
$$w_m = \delta(s - s_m)\,, \quad m, n = 1, 2, \cdots, N\,. \tag{6}$$

The general MoM for the antenna matrix equation is given by:

$$\left[Z_{mn}\right]\left(I_n\right)=\left(V_m\right),$$

where [Z_{mn}] is known as the impedance matrix, given for equation (4) as:

$$Z_{mn}=-\frac{j}{4\pi\omega\varepsilon}\int_{(n-1)\Delta s'}^{n\Delta s'}\left[\begin{array}{l}R^2\left(k^2R^2-1-jkR\right)\mathbf{s}\bullet\mathbf{s}'+\\ \left(3+3jkR-k^2R^2\right)\left(\mathbf{R}\bullet\mathbf{s}\right)\left(\mathbf{R}\bullet\mathbf{s}'\right)\end{array}\right]\frac{e^{-jkR}}{R^5}ds'\Bigg|_{s=s_m}$$

(7)

While (V_m) matrix, known as voltage matrix, is defined by:

$$V_m=\int_s w_m E^i_{\tan} ds \tag{8}$$

and current matrix (In) is:

$$\left(I_n\right)=\left[Z_{mn}\right]^{-1}\left(V_m\right) \tag{9}$$

Cross Antenna Algorithm

The cross antenna that we analyze is an eight arms geometry as shown in figure 2, the MoM solution gives the current distribution from which is possible to obtain other parameters as gain, field pattern, impedance, etc. As Roederer proposes, the antenna characteristics as function of effective wavelength λe, are given in table 1.

Vectorial equations describing geometry of each segment of the antenna are:

$$\mathbf{r}\left(s\right)=x\left(s\right)\mathbf{i}+y\left(s\right)\mathbf{j}+z\left(s\right)\mathbf{k} \tag{10}$$

$$\mathbf{r}_j\left(s'\right)=s\,\hat{t}+\mathbf{r}_j\left(s\right) \tag{11}$$

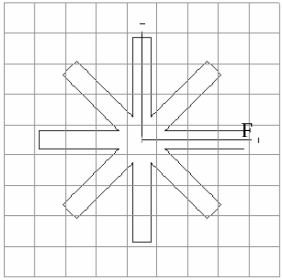


Figure 2. Cross antenna

Arm Length	0.543 λe
Arm Width	0.136 λe
Cross Diameter	1.42 λe
Wire Diameter	0.02 λe
Height over ground plane	(0.0625 a 0.1) λe

Table 1. Cross antenna characteristics

Equation (10) represents the wire axis, and equation (11) describes each segment of the antenna, where:

$\hat{t}$ = unit vector of current propagation direction

s validates the segment equation

Current filament is represented by:

$$\mathbf{r}'(s') = \mathbf{r}_j(s') + a\,\hat{\mathbf{n}}(s') \tag{12}$$

$$\vec{r}_n(s) = \left(s_n - \sum_{j=0}^{n-1} s_j \right) \hat{t}_n + \vec{r}_n$$
$$s_n \in \left[\sum_{j=0}^{n-1} s_j, \quad \sum_{j=0}^{n} s_j \right] \tag{13}$$

$\vec{r}_n$ is the vector pointing to the beginning of each straight segment; n = 1, 2,......, 23

$\vec{r}_1$ is the reference vector (1.2A + B) **i** + 0.5A **j** (point F in figure 2).

S_j is A=0.136λ or B=0.543λ (arm width or arm length, respectively).

$S_0 = 0$ is the feed point (initial condition)

Numerical Results

Using the parametric equations and dimensions shown in table 1 for 3.2 GHz, it is possible to obtain current distribution as seen in figure 3; there is an exponential falling with segment position, which

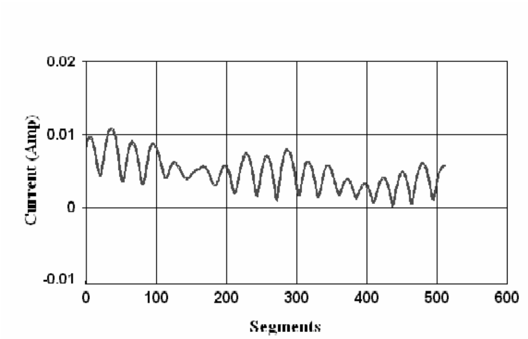


Figure 3. Current distribution

represents a traveling wave distribution current. Although the graph is constructed for a short circuit load, there are similar results for the other load impedances. We suppose that the undulations are present due reflexions in the antenna corners.

Using current distribution we obtain results for radiation efficiency (figure 4), and gain (figure 5).

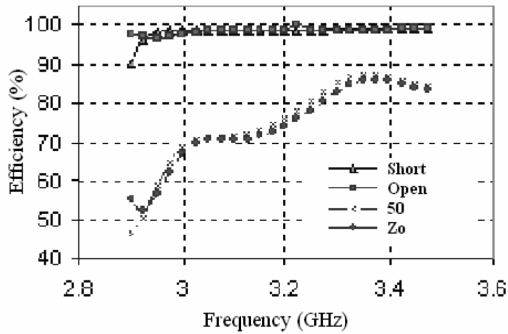


Figure 4. Radiation efficiency

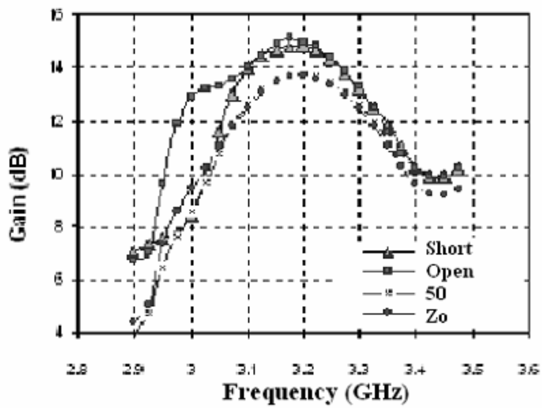


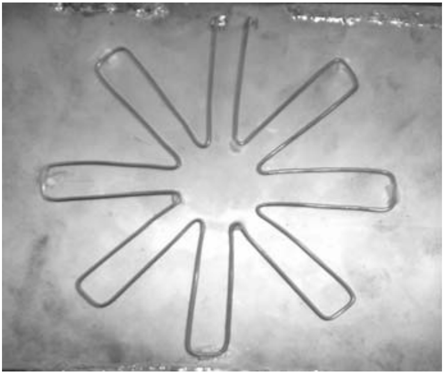
Figure 5. Antenna gain

As seen in figures 4 and 5 the best results are obtained with open and short circuit load impedances, with a 15 dB gain for both.

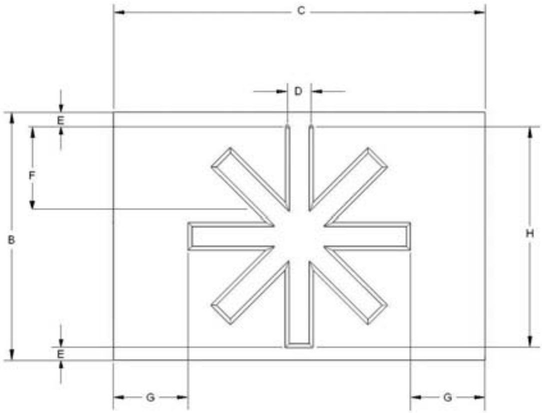
Experimental Results

There were constructed several antennas, both wire and strip line, one of them is shown in figure 6; the wire antenna uses Teflon supports over the ground plane, giving flexibility to change the height over it. We present the results for the height in table 1 to have a comparative analysis with numerical results.

The constructed antennas were measured in an anechoic chamber, using a HP8510 network analyzer for impedance and VSWR; for gain and field pattern we use a E8254A Agilent signal generator and a E4407B Agilent spectrum analyzer.



a)



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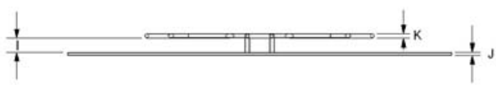

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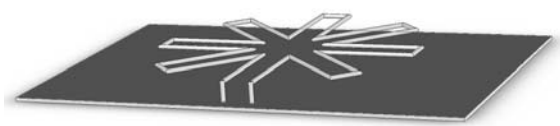

d)

Figure 6. Wire antenna over a ground plane

B	1.6λ
C	2.132λ
D	0.136λ
E	0.09λ
F	0.543λ
G	0.356λ

H	1.420λ
I	.1λ
J	0.016λ
K	0.02λ

Table 2. Cross antenna dimensions of fig. 6

Here we exhibit the results for a load impedance of short circuit; the impedance is shown in figure 7, around 3.2 GHz the input impedance is: Z = 48 - j40

Figure 8 shows the VSWR of 2.9 to 3.5 GHz

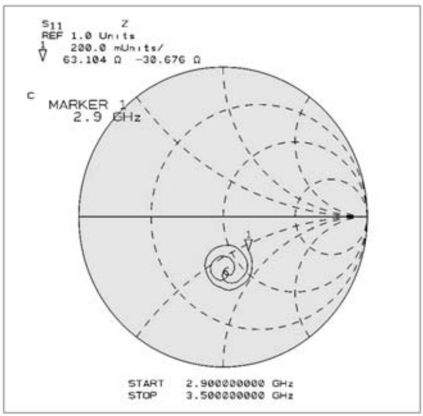

Figure 7. Input impedance

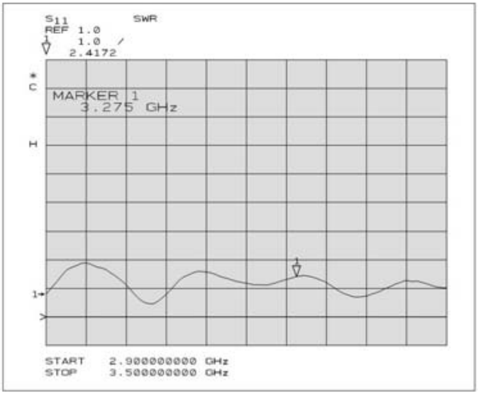

Figure 8.VSWR

The short circuit load for both, horizontal (ϕ=0°) and vertical polarization (ϕ= 90°), is shown in figure 9. Both graphics can be used to obtain axial ratio, which is about 1.5 dB at maximum gain around 3.2 GHz.

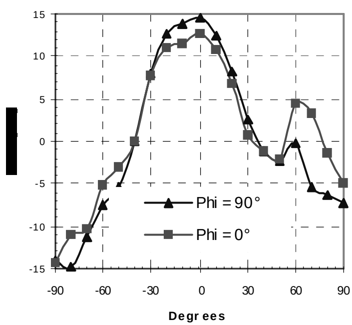

Figure 9. Horizontal and vertical polarizations gain

Table 3 shows maximum experimental gain for different load impedances, for horizontal and vertical polarizations:

Pol/Z_L	Short	Open	50	Zo
Hor	13.5	14	13	13
Ver	15	15	13.5	13.5

Table 3. Gain for different load impedances

Figure 10 exhibits the field pattern comparison between experimental and numerical results, we see that both curves are almost the same, the main difference is around 60°. According to Roederer the peak in such angle is produced due the feed end.

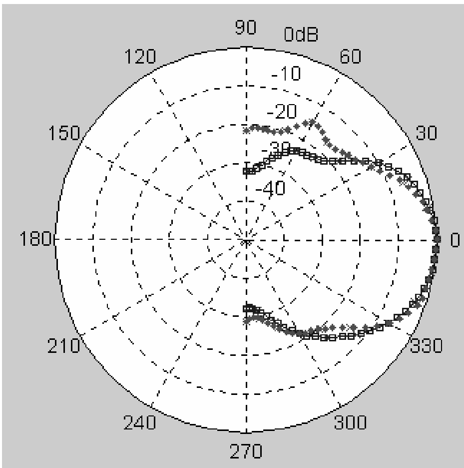

Figure 10 Comparison between experimental (red) and numerical (blue) results for field pattern.

Conclusion

We have presented experimental and numerical results for a cross antenna to 3.2 GHz, with a very good coincidence between the digital simulation and the experiment.

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