

On the Origin of Inertial Force

C. Johan Masreliez
Redmond, WA
jmasreliez@estfound.org

According to general relativity inertial mass and gravitational mass are equivalent. Therefore, inertial acceleration might, like gravitational acceleration, be associated with changing metrical coefficients in the line element of general relativity. This possibility is investigated. If a cosmological reference frame exists, the Minkowskian line element may be modified by a velocity dependent scale factor for which all accelerating motion becomes motion on spacetime geodesics. This might model inertia as a gravitational-type phenomenon.

PAC: 01.55.+b

1. Introduction

With his paper on special relativity in 1905 Einstein abolished the ether, which until then had defined an absolute cosmological reference frame. By special relativity all coordinate frames moving with constant relative velocities (inertial frames) are considered physically equivalent and the speed of light is constant and equal in all these frames. This concept is difficult to understand if photons are particles that move at the speed of light. Special relativity also

predicts that clocks run more slowly in moving coordinate systems, which implies that observers in two systems in relative motion would see the clock in the other system moving at a slower pace. The infamous Twins Paradox is the result, where twins traveling in relative motion each find that the other twin ages slower. The Twins Paradox has been heatedly discussed at great length ever since the introduction of special relativity and today this discussion is still very much alive.

There is an aspect of the cosmos that suggests the existence of a specific reference frame and that is the phenomenon of inertia. Inertia is the tendency of a particle to resist acceleration (herein “particle” will mean any object with positive rest mass). A force has to be applied to accelerate a particle. This is the familiar inertial force, which pushes a person against the seat in an accelerating car or in the outward direction in rotating motion (the centrifugal force). In his famous spinning bucket experiment Isaac Newton observed that the surface of the water in a spinning bucket becomes concave and concluded that the bucket somehow “senses” that it is spinning. But, spinning relative to what? It is not the Earth because the planets are subjected to the same force in their motion around the Sun and it is not the Sun since stars in a galaxy are subjected to the same force. Newton concluded that a frame of absolute universal rest must exist and this became the subject of a celebrated debate between Clark, who spoke for Newton’s position, and Leibniz who contended that all motion is relative (Alexander, 1956).

From the time of Newton until Einstein’s special relativity theory appeared in 1905, people were convinced that there was a cosmological reference frame defined by the “ether”, which was believed to be some undefined kind of “plenum” in absolute rest carrying light and the electromagnetic field. This position was most prominently argued by Hendrik Lorentz and Henri Poincaré.

Although Einstein did away with the ether, he remained convinced that the ether did not exist only for a relatively short time - the eleven years between 1905 and 1916. After introducing General Relativity (GR) in 1916, he gradually changed his position. Here is a quote from his University of Leiden address in 1920 showing his ambivalence regarding the ether:

“But on the other hand there is a weighty argument to be adduced in favour of the ether hypothesis. To deny the ether is ultimately to assign that empty space has no physical qualities whatever. The fundamental facts of mechanics do not harmonise with this view. For the mechanical behaviour of a corporeal system hovering freely in empty space depends not only on relative positions (distances) and relative velocities, but also on its state of rotation, which physically may be taken as a characteristic not appertaining to the system in itself.”

By the end of his life Einstein was convinced that spacetime was a new form of ether that somehow served as a reference for inertia (Kostro, 2000). However, he still believed that all inertial frames were equivalent, and this might have prevented him from discovering the origin of inertia. Today the question of inertia remains unresolved.

This article will attempt to shed some light on these issues by presenting a new point of view. Inertia indicates the existence of an inertial reference frame (IRF). Presented herein are the implications of assuming that there is such an absolute reference, which might be generated by cosmic drag as modeled by the Scale Expanding Cosmos (SEC) theory.

2. The Scale Expanding Cosmos theory

In a series of papers the author introduced the SEC theory [Masreliez, Ap&SS, 1999], [Masreliez, Scale Expanding Cosmos Theory, I, II

and III, 2004], [Masreliez, Scale Expanding Cosmos Theory, IV, 2005] and [Masreliez, Ap&SS, 2005]. This essay will dwell only on a few pertinent aspects of the SEC theory that highlight a particular consequence, which implies the existence of a cosmological reference frame. The ideas of an absolute rest frame and the ether as a manifestation of spacetime geometry are revisited. The author will show how this frame of rest is consistent with special relativity and explain how the speed of light can be the same in all inertial frames. The consequence of not assuming that inertial frames are conceptually and philosophically equivalent is a possible explanation to what might cause inertia and why gravitational and inertial mass are equivalent.

Section 2.1 introduces the concepts of scale-equivalence and a cosmological reference, both of which will be crucial to this discussion.

2.1 The SEC theory implies the existence of a cosmological reference frame

The fundamental idea behind the SEC theory is the cosmological expansion of both space and time. When space expands, the pace of proper time slows down and all four metrical coefficients in the line element of general relativity change by the same scale-factor. This is four-dimensional metrical scale expansion. Since the GR equations are identical with different constant scale-factors, the Universe is '*scale equivalent*'; no particular scale takes preference. By the SEC theory the universe expands by changing its scale while preserving all physics.

The SEC theory agrees with observations and resolves many issues. It also predicts a new phenomenon, cosmic (velocity) drag, which gradually diminishes relative velocities and angular momenta

of freely moving particles. It would have the effect of generating a cosmological reference frame as the coordinate frame toward which all free motion converges. This would also explain the low relative velocities of galaxies and resolve problems with their formation. Mach's principle holds, but what determines the rest frame is not distant matter in the universe, but the cosmological scale expansion, which implies low relative velocities.

Furthermore, cosmic drag should influence the motions of planets in the solar system. Recently, several independent investigators have reported discrepancies between the optical observations and the planetary ephemerides. The SEC theory predicts these planetary drifts [Masreliez, *The Scale Expanding Cosmos Theory*, II, 2004], and the theory also explains the Pioneer anomaly [Masreliez, Ap&SS, 2005].

In what follows the *Inertial Reference Frame* (IRF) is defined as being a locally Minkowskian coordinate frame on a spacetime geodesic. *Forced acceleration* is acceleration away from this inertial frame by exerting an external force. Thus, forced acceleration always is in relation to a local Minkowskian inertial frame, the IRF.

Although cosmic drag generates a reference frame this acceleration is very small and will be ignored in this paper.

2.2 Implications of scale equivalence

According to the SEC theory the Universe expands by changing its metrical scale. However, relative to an observer in the expanding universe everything remains the same; time progresses with the scale expansion without cosmological aging. On the average the universe has always been the same as it is today. This eliminates the enigmatic big bang creation event and resolves many other issues.

The main point is that systems moving at different velocities relative to the cosmological reference may have different metrical

scales, yet they might be physically equivalent in that all laws of physics are equally valid within them. This might seem strange, but it is understood from general relativity that a gravitational field is related to the metrical coefficients (“metrics”) of the line element of GR and it is not unreasonable that motion in relation to the cosmological reference frame could influence these metrics. Thus, coordinate systems in motion relative to the IRF might have metrical scales that depend on their individual absolute velocities. Because of scale equivalence these inertial coordinate frames remain physically equivalent like they are in special relativity, but relative to the cosmological rest frame, moving frames could have different scales. This might be likened to a palace with many identical rooms. An inhabitant would experience the interior of each room the same way, yet they are obviously not equivalent.

2.3 The Special Relativity aspect

There can be no doubt that spacetime is closely Minkowskian in all inertial systems moving with constant relative velocities. This is consistent with special relativity, since the Lorentz transformation replicates Minkowskian coordinate representations, and it has been confirmed by many experiments. However, the Universe is ‘scale-equivalent’ in that similar line elements of different constant scales are physically equivalent since Einstein’s GR equations are identical. It is therefore possible that the absolute velocity of a coordinate frame might influence its metrical scale while preserving its Minkowskian character. This would make it possible to distinguish between coordinate frames moving at different absolute velocities relative to a cosmological reference frame by their different scale factors.

Consider the possibility that a coordinate system moving with *constant* velocity v relative to the IRF has the scaled Minkowskian line element:

$$ds^2 = \phi^2(v) \cdot (dt^2 - dx^2 - dy^2 - dz^2) \quad (2.1)$$

(Through this paper except in section 5 the velocity of light is $c = 1$ by convention.)

The Lorentz transformation between the cosmological rest frame for which $v = 0$ and $\phi(0) = 1$ and a moving frame may be modified accordingly. Assuming motion in the x -direction, we have the *scaled Lorentz transformation*:

$$\begin{aligned} t' &= \phi(v) \beta (t - vx) \\ x' &= \phi(v) \beta (x - vt) \\ y' &= \phi(v) y \\ z' &= \phi(v) z \\ \beta &= \frac{1}{\sqrt{1 - v^2}} \end{aligned} \quad (2.2)$$

This is the standard Lorentz transformation scaled by the factor $\phi(v)$, which depends on the (absolute) velocity relative to the cosmological rest frame. The inverse of this coordinate transformation back to the rest frame is:

$$\begin{aligned} t &= \phi^{-1}(v) \beta (t' + vx') \\ x &= \phi^{-1}(v) \beta (x' + vt') \\ y &= \phi^{-1}(v) y' \\ z &= \phi^{-1}(v) z' \end{aligned} \quad (2.3)$$

In his 1905 paper on special relativity Einstein actually considered this transformation, but concluded based on his postulate that all inertial frames are physically equivalent, that the scale factor $\phi(v)$ and its inverse must equal one.

However, in the presence of a cosmological reference frame the situation is different and it is possible that the scale factor does not equal one but depends on the absolute velocity. Transitioning from one inertial frame to another might alter the scale factor and therefore also the metrics in the line element (2.1) while retaining its scale-equivalent Minkowskian character.

This suggests that acceleration might be associated with a changing scale metric, which might explain the equivalence between gravitational and inertial mass.

3. The inertial field

General relativity is based on the assumption that inertial and gravitational mass is one and the same. Since the inertial force is generated by acceleration, the gravitational force should also be generated by acceleration. In GR the gravitational acceleration is due to gradients in the metrics of spacetime; it is generated by spacetime curvature. But, if the gravitational acceleration is generated by spacetime curvature, forced acceleration might also be associated with spacetime curvature. Thus, both forced and gravitational acceleration might be associated with gradients in the metrics of spacetime.

Assume for a moment that inertia is a gravitational-type phenomenon caused by gradients in the metrical coefficients of spacetime. Thus, the accelerating force is associated with an 'inertial' field similar to the way a gravitational force is associated with the gravitational field. Since there is no external force when a particle is

at rest or moves on a geodesic, this new inertial field will only appear when a particle accelerates relative to the IRF.

In a sense the inertial field is ‘dynamic’ in that it only arises during forced acceleration. In this respect it acts differently than the gravitational field, where the gravitational force appears for a particle at rest, but disappears in free fall. Spacetime is locally curved in a gravitational field, but relative to a freely falling particle spacetime it is (locally) Minkowskian. On the other hand, with the inertial field spacetime is curved for an accelerating particle, but is Minkowskian for a particle at rest or moving with constant velocity. Thus the inertial field may have properties that in a sense are opposite to those of the gravitational field. But conceptually there is no difference, because by GR all coordinate systems are equally valid, and a coordinate system at rest in a gravitational field is theoretically on equal footing with an accelerating coordinate system, an observation that guided Einstein in his search for GR.

A freely falling particle in a gravitational field follows a trajectory given by the geodesic equation. Similarly, if the inertial field corresponds to forced acceleration, all accelerating trajectories should be geodesics of the inertial field. If this were the case the inertial force could be viewed as a gravitational-type force corresponding to an inertial field.

However, this will only be possible if the postulated inertial field has the following property:

The geodesic acceleration given by the inertial field metrics is always identical to the forced acceleration that generates the inertial field.

Regardless of the acceleration, the inertial field will always be such that the acceleration coincides with the inertial geodesic. Thus the postulate that gravitational and inertial forces are equivalent is

generalized by also postulating that there is a corresponding inertial metrical field, which would explain the phenomenon of inertia.

4. The origin of inertia?

If the inertial Minkowskian line element were to change during forced acceleration it could have an effect similar to the changing metrics in a gravitational field, which change with position (there is a metrical gradient). Consider the situation during acceleration. The velocity changes with time, but we could also say that the velocity changes with spatial position. If the scale of spacetime were to depend on the absolute velocity, all four spacetime metrics would change equally with position. But, metrics that change with position might generate a gravitational field according to GR. Acceleration might then also be associated with a special type of gravitational field, the inertial field. Thus, an inertial field might be generated by acceleration while a gravitational field is generated by mass or energy. Note that such an inertial field only may exist in relation to some cosmological reference frame if it is to generate well-defined inertial effects.

To investigate this a bit further, suppose (like Einstein did) that Minkowskian (flat) spacetime is scaled by a scale factor $\phi^2(v)$ like in relation (2.1). However, assume that this factor depends on the absolute rather than relative velocity and consider the case where the velocity depends on position; the velocity v is no longer constant. There is a possibility that acceleration might generate a gradient in the scale metric. If there were a one to one correspondence between the geodesic acceleration for this inertial field and the forced acceleration that creates the field it would explain inertia.

Here something remarkable happens, which may provide a clue to the origin of inertia. *There exists a certain metrical scale factor, which depends on the velocity, for which the geodesic equation*

becomes an identity. This scale factor is derived in Appendix I. This result means that regardless of acceleration the inertial metric generated by this acceleration is always such that the accelerating trajectory is given by its geodesic. To further clarify this point: Each accelerating trajectory defines a certain velocity profile as a function of position. If the spacetime metrics depend on this velocity they also depend on the position. Therefore, there is a positional gradient in the metrics and a corresponding gravitational-type field - the inertial field. The geodesic acceleration for this field always exactly matches the acceleration that creates the field as postulated in section 3 and could therefore explain inertia.

In an ordinary gravitational field the gravitational acceleration always causes a particle to follow the geodesic trajectory given by the metric. Since forced acceleration always agrees with the inertial geodesic, forced acceleration may be viewed as a special case of gravitational acceleration, which means that inertial mass must be the same as gravitational mass.

This would explain the equivalence between inertial mass and gravitational mass.

To explain the inertial force felt during acceleration, consider an observer inside an accelerating vehicle. Because of the acceleration the observer, who moves together with the vehicle, will experience an inertial field and will consequently be subjected to a gravitational-type inertial force. This force is just sufficient to make the observer's acceleration match that of the box.

This explains the equivalence between inertial and gravitational force.

However, there is one fundamental difference between the inertial and gravitational field - the inertial field is generated by acceleration

relative to the IRF while the gravitational field is generated by mass (or energy, since they are equivalent). The gravitational field is static if the mass that creates it is constant and remains at rest while the inertial field is dynamic and depends on the acceleration.

4.1 The inertial scale factor

The particular Minkowskian line element scale factor for which the geodesic becomes an identity is given by $\phi^2(v) = (1 - v^2)$, where v is the velocity relative to the IRF. This scale factor is derived in Appendix I and is called the “*inertial scale metric*.” The corresponding line element is the Minkowskian line element scaled by this metric and is called the “*inertial line element*.” From the discussion above, the scaled line element is equivalent to the ordinary Minkowskian line element if the velocity v is constant. Thus, if the velocity is constant inertial coordinate systems are equivalent just as in special relativity. All physics derived from special relativity still applies. However, the inertial line element models acceleration and explains inertia. This does not contradict special relativity, which does not model transition between inertial frames.

The inertial metric implies two basic relationships from special relativity - the energy relation and the momentum relation. They both directly result from the inertial metric arriving at the same conclusion as Einstein did in 1905, but via a different route and with the difference that the velocity is not relative but absolute. Finding a metric that will model inertia has resulted in a line element that automatically gives the energy and momentum relations of special relativity. This result is also discussed in Appendix I.

However, in disagreement with special relativity the inertial line element implies that motion relative the IRF is of primary importance while relative motion is of less interest.

5. A possible ontological explanation of inertia

A few additional results from the SEC theory are required to explain why the metrics of spacetime might change with the velocity. According to the SEC theory the cosmological scale expands in increments and this incremental expansion is what makes time progress. In the paper [Masreliez, Scale Expanding Cosmos Theory, IV, 2005] the author shows that the quantum world may be explained if the metrics of spacetime oscillate at very high frequencies and he assumes that this oscillation might be due to the discrete cosmological scale expansion. In particular it appears that particles might be formed by standing waves in the metrics of spacetime at the so called Compton frequency.

5.1 A crude particle model

When modeling oscillating metrics in the Minkowskian line element in GR, the Ricci scalar of Einstein's GR equations contains sinusoidal terms. If the Ricci scalar equals zero, the result is a wave equation with solutions that are metrical waves moving at the velocity of light. Two such radial waves of equal amplitude going in opposite directions form a standing wave in the metrics. If a particle at rest is formed by such three-dimensional spherical standing waves it must be via some kind of resonating process taking place at the speed of light.

A rudimentary, provisional and heuristic, model of a moving particle can be constructed based on this conjecture, which will illuminate a fundamental aspect of motion to be elaborated.

Assume a moving square box with sides L . A particle is formed by "bouncing" waves inside the box. At rest, the frequency of these waves is given by the time it takes to reach from one side of the box to the other and back, which equals $2L/c$. (Throughout this section of

the paper the speed of light is set equal to c for clarity, while the convention $c = 1$ is used in the other sections).

However if the particle moves at the velocity v , light transmitted inside the box in a direction perpendicular to the motion (I will assume that box is aligned with the direction of motion) will move in a zigzag pattern relative to the rest frame (but back and forth in the moving frame), see figure 1a. Assume that the particle in the moving frame *oscillates at the same frequency* as the stationary particle - this assumption of synchronous oscillation of a moving particle leads to the inertial line element.

If the oscillation is synchronous the period in the zigzag pattern must be the same as for the stationary particle; it must equal $2L/c$. This occurs if the scale in the direction perpendicular to the motion becomes smaller by the inertial factor:

$$\begin{aligned} L(v) &= L \cdot \sqrt{1 - (v/c)^2} = L(0) \cdot \phi(v) \\ \phi(v) &= \sqrt{1 - (v/c)^2} \end{aligned} \tag{5.1}$$

This would make the diagonal path of the light across the box (as seen from the rest frame) equal to L and the period of the oscillation in the moving frame would then be the same as in the stationary frame. Thus, dimensions in the direction perpendicular to the motion are diminished by the scale-factor ϕ . This is illustrated in figure 1a.

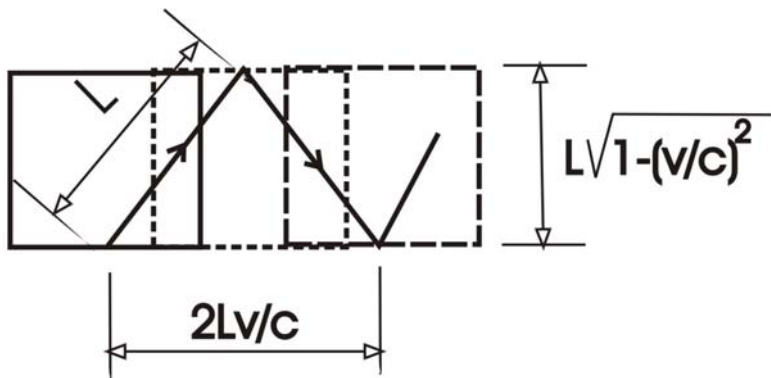


Figure 1a: Perpendicular light ray

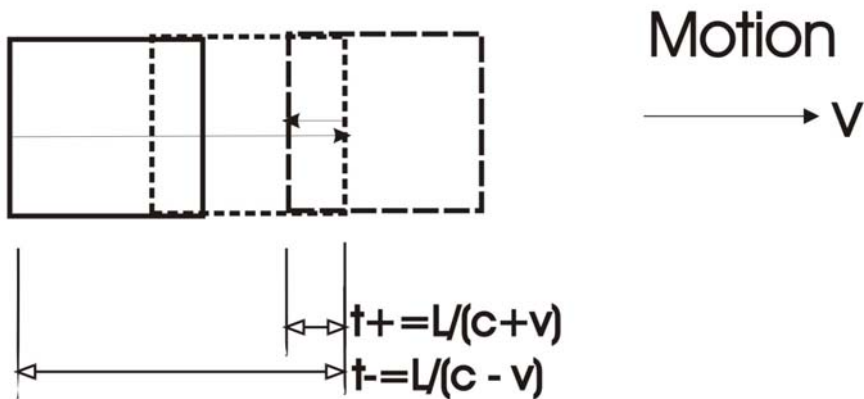


Figure 1b: Parallel light ray

In the direction parallel to the motion the light time to reach the forward wall of the box is $L/(c - v)$ since the box moves away with

the speed v . The return time to rear wall is shorter, $L/(c + v)$. The time it takes to move back and forth then is (see figure 1b):

$$\left(\frac{L}{c-v} + \frac{L}{c+v} \right) = \frac{2L}{c(1-(v/c)^2)} \quad (5.2)$$

This means that in a direction parallel to the motion the period is longer than in the rest frame by the factor $1/(1 - (v/c)^2)$. This can be compensated for by changing the scale of the box in the direction of motion by the factor $1 - (v/c)^2$, which is the square of the scale change in the perpendicular direction. If these scale changes are incorporated, the moving particle box oscillates at the same frequency as the stationary box as seen from the rest frame.

The reader familiar with special relativity might already have noticed that the spatial metrical coefficients of the moving box now are a factor ϕ smaller than the metrics of special relativity. In special relativity dimensions perpendicular to the motion remain the same, but dimensions in the direction of the motion diminish due to length contraction by the factor ϕ .

As a result of this additional factor the corresponding line element is no longer Minkowskian, but is scaled by the factor $\phi^2 = 1 - (v/c)^2$.

But, this is the inertial line element for which the geodesic becomes an identity.

Therefore, during acceleration moving particles may adjust their scale in order preserve their oscillation frequencies.

6. The co-moving observer's perspective

How does this look from the perspective of a co-moving observer? If the velocity is constant the line element of this observer has changed by the square of the inertial scale factor ϕ . Such an observer will not notice this scale change because all her references in space and time

change. Relative to such an observer spacetime will remain Minkowskian like in the rest frame and the transformation between the rest frame and the moving frame could from this point of view be modeled by the standard Lorentz transformation. This explains why Einstein arrived at a different result in special relativity; he assumed that all moving frames are equivalent.

Since the observer in the moving frame does not notice the scale change, her particle-box will still have the relative width and length L and the time for a light pulse to traverse the box will still be L/c *provided the right time coordinate transformation is chosen*. There is a particular choice of coordinates for which the speed of light becomes the same in all directions in the moving frame. But, how can this be possible? It is accomplished by using a trick employed by Lorentz, Poincaré and Einstein; simply redefine the time coordinate in the moving frame so that it compensates for the different light velocity in the direction parallel to the motion. By defining time differently in the moving frame than in the rest frame, so that it includes the spatial dimension as well as the velocity, it can compensate for the fact that the speed of light is not equal in all directions. With the ‘right’ choice of coordinates the particle will satisfy the same wave equation in the moving frame as in the rest frame and the speed of light will appear to be constant in all directions.

The transformation that accomplishes this is the Lorentz transformation (and the scaled Lorentz transformation). With this transformation the moving spacetime appears to be identical to that of the rest frame, preserving the Minkowskian metrics. A wave function modelling the particle’s oscillation in the rest frame would also hold in the moving frame. It might seem like ‘cheating’ by choosing the coordinates so that the velocity of light becomes constant in the moving frame when in reality this only is true in the rest frame. But,

this is permitted by GR where all coordinate representations connected by continuous variable transformations are physically equivalent. The coordinates are simply found that make the moving frame look the same as in the stationary frame. This concept is further discussed in Appendix II.

The Minkowskian coordinate frame might be required in order to preserve the structure of particles during motion and this might explain the Lorentz transformation! It could be said that motion ‘distorts’ material objects as seen from the rest frame in order to preserve them in the moving frame. Thus, Einstein’s assumption that the speed of light is the same in frames moving with constant relative velocities does not rule out the existence of a cosmological reference frame.

But, a cosmological reference frame is a standard against which different coordinate choices may be compared. And, the rest frame may be considered to represent the nominal state of affairs.

7. An apparent temporal discrepancy

The alert reader might have noticed something strange in the development above. According to the inertial line element all moving particles oscillate at the same absolute frequency. If atomic clocks were to oscillate similarly, a moving clock would count the same number of cycles as a stationary clock in a given absolute time interval. A stationary observer would conclude that the clock in the moving frame runs at the same pace as in the absolute frame. On the other hand, there is experimental evidence that the pace of atomic clocks change with motion, for example the Hafele-Keating experiment (Hafele and Keating, 1972). How can we reconcile these diverse interpretations?

Since the change of pace of proper time has been experimentally confirmed, it can be assumed that clocks moving in relation to the IRF run slower than in the IRF. However, if this is true it means that the pace of proper time in an inertially moving frame is different from the pace in the reference frame, yet both these frames are in special relativity modeled by the same Minkowskian line element. This disagrees with GR where the reference increment ds is fixed and reflects the pace of proper time.

Identical line elements connected by a continuous variable transformation like the Lorentz transformation cannot model different paces of proper time in GR, since this would imply that the pace of proper time is undefined.

Therefore, if there is an IRF the Lorentz transformation of special relativity somehow implements a change in the pace of proper time depending on the absolute velocity in violation with GR. To explain this puzzle, let's review the SEC scale expansion process.

According to the SEC theory the Universe expands by adjusting the scale in an incremental process that changes the pace of (proper) time. Although the pace of time changes relative to a fictitious observer who does not expand together with the Universe, relative to a co-expanding observer (like all of us) time appears to progress at the same pace and the spacetime geometry remains the same, since the scales of space and time both expand.

The cosmological expansion may be modelled as a series of brief time intervals during which the scale expands continuously and the pace of time is constant. These short intervals are checked by incremental scale adjustments. The continuously expanding intervals permit us to use GR, which also bridges the discrete steps since GR does not "see" these discrete scale adjustments, which adjust the pace

of time (Einstein's GR equations are not affected). The SEC expansion cycle is illustrated in figure 2.

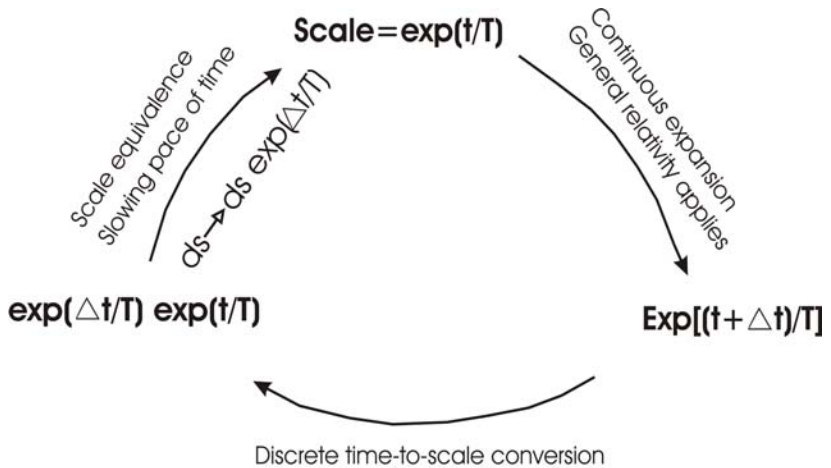


Figure 2: Proposed SEC expansion cycle

There is new physics in this cosmological expansion model—the incrementally slowing progression of proper time. *But, this is the only way by which the pace of proper time can change without changing the line element.*

Thus, the temporal expansion process may be modelled by an incrementally and discretely changing four-dimensional scale, which fortunately does not alter Einstein's GR equations. This means that GR can still be used; it is valid during the continuous part of the cycle, but that there are additional hidden aspects of fundamental importance that lead to improved understanding. GR is generalized to include discrete changes of the scale with adjustment of proper time (the element ds), abandoning the assumption that spacetime is a

continuous manifold. This could be a stumbling block for the SEC theory, since it implies a leap of faith into new territory. However, in the previous series of papers the author has shown that this leap is well worth taking because it would resolve numerous issues in physics and cosmology.

Acceleration in space may be modelled similarly to the SEC expansion cycle. During short consecutive intervals the scale of spacetime changes continuously as modelled by the inertial line element. GR applies during these intervals and models inertia via the changing metrical scale. These short intervals of continuous scale adjustment are checked by discrete scale increments that “reset” the scale and restore the Minkowskian line element. Acceleration might then be thought of as a cycle comprised of two segments: a continuous phase during which the scale changes, followed by a second discrete step where the scale adjusts. The first phase is modeled by the inertial metric, and together the two segments are in accordance with a special relativity boost, i.e. the two steps combined implement the Lorentz transformation. However, the discrete step, to which GR is “blind”, adjusts the pace of proper time, just as in the cosmological expansion process. This acceleration cycle is shown in figure 3.

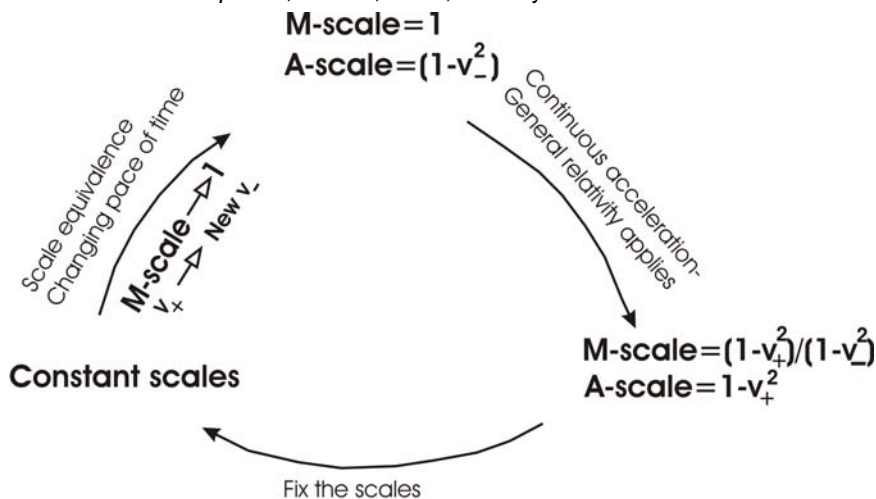


Figure 3: Proposed acceleration cycle
M-scale = Minkowskian scale
A-scale = Absolute scale

Note that the pace of (proper) time is constant during the continuous part of the cycle, *which permits us to model inertia by GR*. The discrete scale adjustment at the end of the cycle changes the pace of time and adjusts the Minkowskian line element to the updated velocity. Although the Minkowskian line element is preserved by this scale adjustment, the proper time increment of GR actually changes due to the incremental scale expansion:

$$ds(v) = \sqrt{1 - v^2} \cdot ds(0) \quad (7.1)$$

Here $ds(v)$ is the proper time increment at the absolute velocity v and $ds(0)$ the corresponding increment in the rest frame. Energy and momentum in the moving frame are in proportion to the ratio between

these proper time increments showing how they depend on the pace of proper time and the absolute velocity:

$$\frac{ds(0)}{ds(v)} = \frac{1}{\sqrt{1-v^2}} = \frac{1}{\phi} = \beta \quad (7.2)$$

Relative to the moving observer the scale also changes during the continuous portion of the cycle:

$$ds_+ = \sqrt{\frac{1-v_+^2}{1-v_-^2}} \cdot ds_- \approx \left[1 - (v_+^2 - v_-^2)/2 \right] \cdot ds_- \quad (7.3)$$

The Minkowskian line element is restored by a Lorentz boost and the pace of time is adjusted at the end of each cycle. This adjustment of the pace of time corresponds to a positive or a negative kinetic energy increment.

Thus, the line element remains Minkowskian but with a different pace of proper time depending on the velocity. This relationship remains hidden in special relativity, which deals with continuous Lorentz transformations that conceal these scale adjustments. Although special relativity recognizes time dilation, it is not reflected in the line element, which always remains Minkowskian. The Lorentz transformation is continuous and is therefore allowed by GR, but gives the impression that the increments ds in various moving frames are identical. With the new interpretation *the increments ds for Minkowskian line elements at different velocities are not the same, but depend on the absolute velocities.* This hidden dependency brings us beyond GR and might have prevented us from finding the origin of inertia.

The discrete scale adjustments have the effect of changing the pace of (proper) time just like in the SEC cycle. The absolute scale depends on the absolute velocity and keeps track of the pace of time

in the corresponding Minkowskian frame, which is the frame experienced by a moving observer.

8. Similarities between gravitational and inertial acceleration

The classical Newtonian gravitational acceleration is given by:

$$a_g = -\frac{mG}{r^2} \quad (8.1)$$

with the corresponding gravitational potential:

$$P_g = \int_r^\infty a_g du = -\int_r^\infty \frac{mG}{u^2} du = -\frac{mG}{r} \quad (8.2)$$

For inertial motion the geodesic acceleration is given by the identity (AI.13) in Appendix I:

$$\bar{a} = \nabla \left(\frac{v^2}{2} \right)$$

The corresponding potential is given by:

$$P_i = \int_0^L \bar{a} \cdot d\bar{l} = \int_0^v \nabla \left(\frac{v^2}{2} \right) d\bar{l} = \frac{V^2}{2} \quad (8.3)$$

Thus, kinetic energy might be viewed as ‘inertial potential’ and forced acceleration as being associated with the spatial gradient of this inertial potential. The accelerating force may be interpreted as being a gravitational-type force due to changing metrics. Conversely, force causes acceleration, which ‘curves’ spacetime, changing line element.

Also, gravitational time dilation is due to the temporal metric in Schwarzschild’s exterior solution [Schwarzschild, 1916]:

$$d\tau = dt\sqrt{1 - r_0 / r} \quad (8.4)$$

Similarly, the time dilation for a moving frame can be viewed as being due to the inertial metric:

$$d\tau = dt\sqrt{1 - v^2} \quad (8.5)$$

9. Further speculation on the nature of motion

The SEC expansion may be viewed as consecutive intervals with continuously changing scale, each interval terminated by discrete scale adjustments, which change the pace of proper time. This way of visualizing the expansion is motivated by the need to use known physics in modeling it. It might not be the best possible description, but if this cycle actually models the dynamical nature of spacetime, moving particles must comply with this scheme.

During acceleration the scale may change continuously in order to maintain the physical properties of particles. If this is done without changing the pace of proper time (*i.e.*, the increment ds) it may be modelled by GR and would then explain the origin of inertia. This acceleration phase mirrors the continuous portion of the SEC cycle. The discrete scale adjustment that follows models a discretely changing pace of time. This dynamic, partly continuous, partly discrete, nature of motion has not been recognized in the past because Einstein's GR equations do not change for discrete scale changes.

Although not covered by GR, discrete scale adjustments might change the pace of time both during the cosmological expansion and during acceleration in space. During acceleration the energy may change incrementally with the changing pace of time according to (7.2) and (7.3). This discrete scale adjustment process would explain why energy is quantized in quantum theory.

The difficulty in modeling this quasi-continuous process might be due to inappropriate mathematics. Using GR to model inertia by the inertial line element (which is compatible with GR) is inappropriate in a universe that is not a continuous manifold and therefore not compatible with GR. To make up for this shortcoming the Minkowskian line element for the moving frame repeatedly is restored via a discrete process. This latter step brings us beyond GR by the changing pace of proper time (the increment ds changes).

It is interesting (but perhaps not surprising) that the inertial acceleration cycle of figure 3 so closely mirrors the cosmological scale expansion of the SEC theory. Both are modeled relative to co-moving observers in a process whereby the Minkowskian line element repeatedly is being restored by scale adjustments. But, there is an important difference; during forced acceleration an observer in the cosmological reference frame plays the same role as the fictitious observer (for which the scale and the pace of time remain constant) does in the SEC theory's temporal acceleration scenario. Thus the "fictitious observer" of the SEC theory is no longer fictitious in spatial acceleration, but corresponds to an observer in the cosmological rest frame.

It is also interesting that accelerating motion may be modeled by a dynamically scaled Minkowskian line element. This makes one wonder if motion in general might consist of consecutive intervals with scaled Minkowskian line elements during which acceleration is 'registered' by the scale metric. Note that as demonstrated in Appendix I the dynamically scaled inertial metric will model acceleration in any direction. After each of these brief continuous segments the Minkowskian line element is updated by applying a special relativity 'boost' that also adjusts the pace of proper time via the element ds . According to this scenario all motion is mediated by an oscillatory process between space and time.

Incremental change in the pace of time and kinetic energy would also explain quantization of the cosmological redshift from distant sources (William Tiftt, 1978 a,b), which could be due to discrete changes in a photon's pace of proper time.

10. Summary

This paper suggests that inertia might be modeled as an acceleration-induced gravitational-type spacetime effect, which would explain the equivalence between inertial and gravitational mass and the inertial force. However, this explanation crucially depends on the existence of a cosmological reference frame. If there is such an inertial reference, absolute acceleration might induce a gradient in the metrics of spacetime, generating a gravitational-type inertial field. The geodesic relation for this field expresses an identity; the acceleration that generates the inertial field always equals the geodesic acceleration induced by the field. By this scenario acceleration is modeled as a series of short intervals with continuously changing scale metrics checked by discrete scale adjustments that restore the Minkowskian line element and change the pace of proper time. Combined these two intervals implement the Lorentz transformation and therefore agree with special relativity, but reveal a hidden feature of the Universe, which could explain to the origin of inertia. For example, it is shown in Appendix I how absolute circular motion might generate inertial acceleration.

It appears that Nature could resort to the same mechanism during forced acceleration as with the cosmological scale expansion of the SEC theory. In both cases the pace of proper time is adjusted by discrete scale increments. These discrete scale changes may take place without influencing Einstein's GR equations, which would explain why they previously have not been discovered. They have

remained hidden and with them the link between quantum theory and general relativity and the mystery of the progression of time. The possibility of resolving these longstanding puzzles as well as the enigmatic question of inertia should help alleviate any initial discomfort one might experience with the SEC model.

Acknowledgements

I acknowledge Dr. Alexander Kholmetskii for helpful comments and my daughter, Malin Young, for her editorial input.

Appendix I: The inertial scale metric

Consider the scaled Minkowskian line element:

$$ds^2 = \phi^2(v) \cdot (dt^2 - dx^2 - dy^2 - dz^2) \quad (\text{AI.1})$$

Here the velocity v is assumed to be a function of the spatial coordinates, $v = v(x,y,z)$. If acceleration is due to changing metrics it should satisfy the geodesic equation of GR:

$$\frac{d^2 x^\mu}{ds^2} + \Gamma_{\nu\lambda}^\mu \frac{dx^\nu}{ds} \frac{dx^\lambda}{ds} = 0 \quad (\text{AI.2})$$

For the x-coordinate this is with $x^0 = t$, $x^1 = x$, $x^2 = y$ and $x^3 = z$:

$$\begin{aligned} \frac{d^2 x}{ds^2} = & -\Gamma_{00}^1 \left(\frac{dt}{ds}\right)^2 - \Gamma_{11}^1 \left(\frac{dx}{ds}\right)^2 - \Gamma_{22}^1 \left(\frac{dy}{ds}\right)^2 - \\ & -\Gamma_{33}^1 \left(\frac{dz}{ds}\right)^2 - 2\Gamma_{12}^1 \left(\frac{dx}{ds}\right)\left(\frac{dy}{ds}\right) - 2\Gamma_{13}^1 \left(\frac{dx}{ds}\right)\left(\frac{dz}{ds}\right) \end{aligned} \quad (\text{AI.3})$$

We have:

$$\frac{d^2x}{ds^2} = \frac{d}{ds} \left(\frac{dx}{dt} \frac{dt}{ds} \right) = \frac{d}{dt} \left(\frac{dx}{dt} \frac{dt}{ds} \right) \left(\frac{dt}{ds} \right) =$$

$$\left(\frac{d^2x}{dt^2} \right) \left(\frac{dt}{ds} \right)^2 + \left(\frac{dx}{dt} \right) \left[\frac{d}{dt} \left(\frac{dt}{ds} \right) \right] \left(\frac{dt}{ds} \right)$$
(AI.4)

From the line element (AI.1):

$$\frac{dt}{ds} = \frac{1}{\phi \sqrt{1-v^2}}; \text{ where } v \triangleq \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \text{ and } \dot{x} \triangleq \frac{dx}{dt}$$
(AI.5)

The bracket factor in the last term of (AI.4) therefore is:

$$\frac{d}{dt} \left(\frac{dt}{ds} \right) = - \frac{\dot{\phi}}{\phi^2 \sqrt{1-v^2}} + \frac{v \cdot \dot{v}}{\phi (1-v^2)^{3/2}}$$
(AI.6)

We may get rid of the dependence on s by dividing all terms in the geodesic by $(dt/ds)^2$. Rewriting the last term of (AI.4) by multiplying and dividing by dt/ds using (AI.5):

$$\dot{x} \left[\frac{d}{dt} \left(\frac{dt}{ds} \right) \right] \left(\frac{dt}{ds} \right) = \left[- \frac{\dot{x} \cdot \dot{\phi}}{\phi} + \frac{\dot{x} \cdot v \dot{v}}{(1-v^2)} \right] \left(\frac{dt}{ds} \right)^2$$
(AI.7)

The geodesic relation may now be written:

$$\left[\ddot{x} - \frac{\dot{x} \cdot \dot{\phi}}{\phi} + \frac{\dot{x} \cdot v \dot{v}}{(1-v^2)} \right] \left(\frac{dt}{ds} \right)^2 = \text{Right hand side of (AI.3)}$$
(AI.8)

The right hand side may also be written:

$$\left[-\Gamma_{00}^1 + \Gamma_{11}^1 \dot{x}^2 - \Gamma_{22}^1 \dot{y}^2 - \Gamma_{33}^1 \dot{z}^2 \right] \left(\frac{dt}{ds} \right)^2$$

$$\left[-2\dot{x} \{ \Gamma_{11}^1 \dot{x} + \Gamma_{12}^1 \dot{y} + \Gamma_{13}^1 \dot{z} \} \right] \left(\frac{dt}{ds} \right)^2$$
(AI.9)

The Christoffel symbols are:

$$\Gamma_{00}^1 = \Gamma_{11}^1 = -\Gamma_{22}^1 = -\Gamma_{33}^1 = \frac{1}{\phi} \frac{\partial \phi}{\partial x}$$

$$\Gamma_{12}^1 = \frac{1}{\phi} \frac{\partial \phi}{\partial y}; \quad \Gamma_{13}^1 = \frac{1}{\phi} \frac{\partial \phi}{\partial z}$$
(AI.10)

Substituting this into the bracket of (AI.9):

$$-\frac{1}{\phi} \left[\frac{\partial \phi}{\partial x} - \frac{\partial \phi}{\partial x} (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + 2\dot{x} \left(\frac{\partial \phi}{\partial x} \dot{x} + \frac{\partial \phi}{\partial y} \dot{y} + \frac{\partial \phi}{\partial z} \dot{z} \right) \right] =$$

$$-\frac{1}{\phi} \left[\frac{\partial \phi}{\partial x} (1 - v^2) + 2\dot{x} \frac{d\phi}{dt} \right]$$
(AI.11)

(AI.8) becomes:

$$\ddot{x} + \frac{\dot{x} \cdot v\dot{v}}{(1 - v^2)} = -\frac{1}{\phi} \left[\frac{\partial \phi}{\partial x} (1 - v^2) + \dot{x} \dot{\phi} \right]$$
(AI.12a)

Similarly for the other two components:

$$\ddot{y} + \frac{\dot{y} \cdot v\dot{v}}{(1 - v^2)} = -\frac{1}{\phi} \left[\frac{\partial \phi}{\partial y} (1 - v^2) + \dot{y} \dot{\phi} \right]$$
(AI.12b)

$$\ddot{z} + \frac{\dot{z} \cdot v\dot{v}}{(1 - v^2)} = -\frac{1}{\phi} \left[\frac{\partial \phi}{\partial z} (1 - v^2) + \dot{z} \dot{\phi} \right]$$
(AI.12c)

Multiplying AI.12a by \dot{x} , AI.12b by \dot{y} and AI.12c by \dot{z} and adding:

$$\dot{x}\ddot{x} + \dot{y}\ddot{y} + \dot{z}\ddot{z} + \frac{v^3\dot{v}}{(1 - v^2)} = -\frac{1}{\phi} \left[\dot{\phi} (1 - v^2) + v^2 \dot{\phi} \right]$$

$$v\dot{v} + \frac{v^3\dot{v}}{(1-v^2)} = -\frac{\dot{\phi}}{\phi}$$

$$\frac{v\dot{v}}{(1-v^2)} = -\frac{\dot{\phi}}{\phi} \quad (\text{AI.13})$$

This relation is *identically* satisfied with the *inertial scale metric*:

$$\phi = \text{constant} \cdot \sqrt{1-v^2} \quad (\text{AI.14})$$

In this relation the velocity is a function of the spatial coordinates. Substituting the inertial metric into AI.12:

$$\ddot{x} = v \frac{\partial v}{\partial x} \quad (\text{AI.15a})$$

$$\ddot{y} = v \frac{\partial v}{\partial y} \quad (\text{AI.15b})$$

$$\ddot{z} = v \frac{\partial v}{\partial z} \quad (\text{AI.15c})$$

Therefore the geodesic acceleration for the inertial line element satisfies:

$$\bar{a} = \text{grad} \left(\frac{v^2}{2} \right) = \nabla \left(\frac{v^2}{2} \right) \quad (\text{AI.16})$$

This relation always holds for forced acceleration in any coordinate representation since the gradient vector is covariant. The geodesic acceleration equals the gradient of the *inertial field potential* $V^2/2$.

The *inertial line* element is:

$$ds^2 = \text{constant} \cdot (1-v^2) \cdot (dt^2 - dx^2 - dy^2 - dz^2) \quad (\text{AI.17})$$

For the absolute line element in figure 3 this constant equals one and for the moving Minkowskian line element the constant is:

$$\text{M-scale constant} = \frac{1}{1 - v_-^2} \quad (\text{AI.18})$$

During the acceleration phase the dynamical scale is:

$$\text{M-scale} = \frac{1 - v^2}{1 - v_-^2} \quad v_- \leq v \leq v_+ \quad (\text{AI.19})$$

Note that the inertial line element varies with the velocity; each acceleration profile generates a corresponding metrical gradient.

Forced acceleration would then, like gravitational acceleration, always be accompanied by changing metrics of spacetime. This might explain the equivalence between inertial and gravitational mass and the phenomenon of inertia.

As an example consider rotational motion with fixed radius of the Minkowskian frame modeled by:

$$x = r \cos(\omega t); \quad y = r \sin(\omega t)$$

$$\dot{x} = -r\omega \sin(\omega t) = -\omega y; \quad \dot{y} = r\omega \cos(\omega t) = \omega x$$

$$v^2 = \dot{x}^2 + \dot{y}^2 = \omega^2 (y^2 + x^2)$$

From (AI.16):

$$a_x = \frac{\omega^2 \partial (y^2 + x^2)}{2 \partial x} = \omega^2 x; \quad a_y = \omega^2 y$$

$$|a| = \sqrt{a_x^2 + a_y^2} = \omega^2 r = \text{centrifugal acceleration}$$

Spherical or cylindrical coordinates immediately yield:

$$a_r = \frac{1}{2} \frac{\partial v^2}{\partial r} = \frac{1}{2} \frac{\partial (r\omega)^2}{\partial r} = r\omega^2$$

Thus, with the inertial scale metric $(1-v^2)$ the centrifugal acceleration equals the geodesic acceleration. This illustrates how circular motion of the inertially scaled Minkowskian frame generates an inertial force.

The energy corresponding to the inertial line element (AI.17) is given by:

$$E = m_0 \frac{dt}{ds} = \frac{m_0}{\sqrt{1-v^2}} \quad (\text{AI.20})$$

And the momentum is:

$$|p| = m_0 \left| \frac{dx}{ds} \right| = m_0 \left| \frac{dx}{dt} \frac{dt}{ds} \right| = m_0 \frac{|v|}{\sqrt{1-v^2}} \quad (\text{AI.21})$$

These relations are familiar from special relativity, but the interpretation here is very different. In special relativity these relations may be found using the Lorenz transformation plus additional considerations based on for example Maxwell's equations, as was done by Einstein in his 1905 paper. Here these relations follow directly from the inertial line element and hold for motion relative to the IRF.

Surprisingly, with the inertial metric the geodesic relation (AI.13) becomes the *identity*, which holds for all forced accelerating trajectories.

Appendix II: How to make the speed of light constant in a moving frame

We saw that a light beam moving from the rear to the front of the particle-box in section 5 takes the time $t(0)$ as measured in the rest frame:

$$t(0) = \frac{L}{c - v} \quad (\text{AII.1})$$

Let's re-define the time coordinate in the moving frame as follows:

$$t(v) = t(0) - xv / c^2 \quad (\text{AII.2})$$

Thus, the moving time coordinate now also depends on the position x as well as on the velocity. A light beam will move in the positive x -direction with speed c and since $x = ct(0)$ we have:

$$t(v) = t(0) - [c \cdot t(0)]v / c^2 = t(0) \cdot (1 - v / c) \quad (\text{AII.3})$$

Combining (AII.1) and (AII.3):

$$t(v) = L / c \quad (\text{AII.4})$$

This is the same time as in the stationary box. Similarly, light moving in the opposite direction satisfies $x = -ct(0)$ and the time to move from the front to the rear of the box also becomes L/c . Therefore, it appears that the speed of light is constant in all directions in the moving box. The transformation (AII.2) involving the spatial distance is permitted by GR, since all coordinate representations that may be formed by continuous variable transformation are physically equivalent. It takes into account an error of synchronization of two distant clocks, which emerge due to possible light velocity anisotropy in an inertial frame, moving at the constant absolute velocity v .

One can show [Kholmetskii, 2003] that for Galilean transformations this synchronization error due to the difference in the

speed of light in different directions accounts for the term vx/c^2 in (AII.2). Thus, (AII.2) is a consequence of the Galilean transformation when adopting Einstein's temporal synchronization method and therefore not all that surprising. Therefore, the inertial scale factor is consistent with the Galilean transformation, but implies additional scale compression in the direction perpendicular to the motion.

References

- Alexander H. G., *The Clark-Leibniz correspondence* (Manchester University Press, 1956)
- Einstein A., On the Electrodynamics of Moving Bodies. *Annalen der Physik* **17** (1905): 891 – 921.
- Einstein A., *University of Leiden address*, (1920)
- Hafele J.C. and Keating R. E., *Science* **177**, 166 (1972)
- Kholmetskii A. L., *Physica Scripta* **67**, 381 – 387, 2003
- Kostro L, “Einstein and the Ether”, *Apeiron Montreal*, (2000)
- Masreliez C.J., “The Scale Expanding Cosmos Theory”, *Ap&SS*, **266**, 399-447, (1999)
- Masreliez C. J., Scale Expanding Cosmos Theory I – An Introduction, *Apeiron April*, (2004)
- Masreliez C. J., Scale Expanding Cosmos Theory II– Cosmic Drag , *Apeiron Oct.*, (2004)
- Masreliez C. J., Scale Expanding Cosmos Theory III – Gravitation, *Apeiron Oct.*, (2004).
- Masreliez C. J., Scale Expanding Cosmos Theory IV – A possible link between General Relativity and Quantum Mechanics, *Apeiron Jan.* , (2005)
- Masreliez C. J., *Ap&SS*, v. **299**, no. 1, pp. 83-108 (2005)