Optimization Method based on Genetic Algorithms

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The design of electromagnetic systems using methods of optimization have been carried out with deterministic methods. However, these methods are not efficient, because the object functions obtained from electromagnetic optimization problems are often highly non-linear, stiff, multi-extreme and non-differential. The lack of a single method available to deal with multidimensional problems, including those with several goals to optimize, has generated the need to use numerical processes for optimization. This paper presents a method of global optimization based on genetic algorithms. The Genetic Algorithms are a versatile tool, which can be applied as a global optimization method to problems of electromagnetic engineering, because they are easy to implement to non-differentiable functions and discrete search spaces. It is also shown how, in some cases, genetic algorithms have been applied with success in electromagnetic problems, such as antenna design, far-field prediction, absorber coatings design, etc.

Keywords: Electromagnetic Optimization, Genetic Algorithm.

Introduction

For three decades, many mathematical programming methods have been developed to solve optimization problems. However, until now, there has not been a single totally efficient and robust method to cover all optimization problems that arise in the different engineering fields. Most engineering application design problems involve the choice of design variable values that better describe the behavior of a system. At the same time, those results should cover the requirements and specifications imposed by the norms for that system. This last condition leads to predicting what the entrance parameter values should be whose design results comply with the norms and also present good performance, which describes the inverse problem.

Generally, in design problems the variables are discreet from the mathematical point of view. However, most mathematical optimization applications are focused and developed for continuous variables. Presently, there are many research articles about optimization methods; the typical ones are based on calculus, numerical methods, and random methods. The calculus based methods have been intensely studied and are subdivided in two main classes: 1) the direct search methods find a local maximum moving on a function over the relative local gradient directions and 2) the indirect methods usually find the local ends solving a set of non-linear equations, resultant of equaling the gradient from the object function to zero, i.e., by means of multidimensional generalization of the notion of the function's extreme points from elementary calculus give a smooth function without restrictions to find a possible maximum which is to be restricted to those points whose slope is zero in all directions. Both methods have been improved and extended, however they lack robustness for two main reasons: 1) they have a local focus, since they seek the maximum in the analyzed point neighborhoods; 2) they depend on the existence of their derivative, which many spaces of practical parameters respect little the notion of having derivatives and smoothness. The real world has many discontinuities and noisy spaces, which is why it is not surprising that the methods depending upon the restrictive requirements of continuity and existence of a derivative are unsuitable for all, but a very limited problem domain. A number of schemes have been applied in many forms and sizes. The idea is quite direct inside a finite search space or a discrete infinite search space, where the algorithms can locate the object function values in each space point one at a time. The simplicity of this kind of algorithm is very attractive when the numbers of possibilities are very small. Nevertheless, these outlines are often inefficient, since they do not complete the requirements of robustness in big or highly dimensional spaces, making it quite a hard task to find the optimal values. Given the shortcomings of the calculus based techniques and the numerical ones the random methods have increased their popularity.

The methods of random search are known as evolutionary algorithms. The evolutionary techniques are parallel and globally robust optimization methods. They are based on the principles of natural selection of Darwin [5] and the genetic theory of the natural selection of R.A. Fisher [7]. The application of evolutionary techniques as abstractions of the natural evolution has been broadly proven [3]. In general, all recursive approaches based on population, which use selection and random variation to generate new solutions, can be seen as evolutionary techniques. Indeed, the study of non-linear problems using mathematical programming methods that can handle global optimization problems effectively is of considerable interest. Genetic Algorithms is one such method which has been a subject of discussion by [21], [22], [23] and [24]

The genetic algorithm is an example of a search procedure that uses random selection for optimization of a function by means of the parameters space coding. The genetic algorithms were developed by Holland [10] and the most popular references are perhaps Goldberg [8] and a more recent one by Bäck [1]. The genetic algorithms have been proven successful for robust searches in complex spaces. Some papers and dissertations, like [3], state the validity of the technique in applications of optimization and robust search, crediting the genetic algorithms as efficient and effective in the approach for the search. For these reasons Genetic Algorithms are broadly used in daily activities, as much in scientific applications as in business and engineering circles. It is necessary to emphasize that genetic algorithms are not limited to the search space (relative aspects to the continuity and derivatives existence among other properties). Besides, genetic algorithms are simple and extremely capable in their task of searching for the objective improvement.

The Genetic Algorithms

The genetic algorithms (G.A.) are typically characterized by the following aspects:

- The G.A. work with the base in the code of the variables group (artificial genetic strings) and not with the variables in themselves.
- The G.A. work with a set of potential solutions (population) instead of trying to improve a single solution.
- The G.A. do not use information obtained directly from the object function, of its derivatives, or of any other auxiliary knowledge of the same one.
- The G.A. apply probabilistic transition rules, not deterministic rules.

The genetic algorithm process is quite simple; it only involves a copy string, partial string exchanges or a string mutation, all these in random form.

The fundamental theorem of genetic algorithms

A genetic algorithm is constructed by stochastic operators, and its robust search ability is based on the theorem depicted in [8], which states, "short schemata of low order with aptitude above average, exponentially increase its number by generations ", this is:

$$\mathrm{m}\left(H,t+1\right)\geq \mathrm{m}\left(H,t\right)\frac{\mathrm{f}\left(H\right)}{f_{avg}}\left[1-p_c\frac{\delta\left(H\right)}{l-1}-\mathrm{O}\left(H\right)p_m\right] \qquad (1)$$

where $\boldsymbol{m}(H,t+1)$ and $\boldsymbol{m}(H,t)$ are the schemata number H in the generation $t+1$ and t respectively, $\boldsymbol{f}(H)$ is the average aptitude value of the strings that is included on the schemata H, f_{avg} is the total population's average aptitude value, l is the total string length, $\delta(H)$ is the schemata length from H, $\boldsymbol{O}(H)$ is the schemata order from H, p_c is the crossover probability and p_m is the mutation probability.

Genetic Algorithm Operators

As shown above, a basic genetic algorithm that can produce acceptable results in many practical problems is composed of three operators:

- Reproduction
- Crossover
- Mutation

The reproduction process goal is to allow the genetic information, stored in the good fitness artificial strings, survive the next generation. The typical case is where the population's string has assigned a value according to its aptitude in the object function. This value has the probability of being chosen as the parent in the reproduction process of a new generation.

The crossover is a process by which a string is divided into segments, which are exchanged with the segments corresponding to another string. With these process two new strings different to those that produced them are generated. It is necessary to clarify that the choice of strings crossed inside those that were chosen previously in the reproduction process is random. From the point of view of problem optimization, it is equal to the exploitation of an area of the parameters space. The following outline shows the crossover process:

Before crossover After crossover

crossover point ↓

$$\left.\begin{array}{ll}\text{string } A_1 & 101001\,01 \\ \text{string } A_2 & 111100\,00\end{array}\right\} \xrightarrow{\text{crossover}} \left\{\begin{array}{ll}101001\,00 & \text{string } A_1' \\ 111100\,01 & \text{string } A_2'\end{array}\right.$$

the strings A_1' and A_2' are part of the new generation.

As with biological systems the mutation is manifested with a small change in the genetic string of the individuals. In the case of artificial genetic strings, the mutation is equal to a change in the elementary portion (allele) of the individuals' code. The mutation takes place with characteristics different to those that the individuals had at the beginning, characteristics that didn't possibly exist in the population. From the point of view of problem optimization, it is equal to a change of the search area in the parameters space. The above mentioned is illustrated with the following outline:

Before mutation After mutation

↓ mutation point

$$\text{string } A_1 \quad 10100101\} \xrightarrow{\text{mutation}} \{11100101 \quad \text{string } A_1'$$

the string A_1' belongs to the new generation.

The genetic algorithms seek their goal recurrently (by generation), evaluating each individual's aptitude in the object function which is in fact the optimization approach.

The Object Function

Frequently design problems have to comply with norms or practical constraints that either optimize cost or design performance. In general, they should cover goals for good global performance. These goals do not always match, i.e., while one goal requires the maximum of a parameter, another goal requires the same parameter to be as small as possible. Optimization goals can be expressed in a more dependent mathematical relationship form of a parameter group or design variables of which these parameters in turn can be constraints to interval values. The mathematical expression that represents the optimization goal is commonly known as the "object function".

The code and decode

As indicated before, the essential characteristic of genetic algorithms is the coding of the variables that describe the problem. The common coding method is to transform the variables to specific length binary strings. For a problem depending on more than one variable the coding involves linking with each variable code. The code length depends on the rank of the variables and the precision required by the problem.

If a design variable requires a precision A_c then the number of binary digits in the binary string can be estimated with the following equation:

$$2^m \geq \frac{X_U - X_L}{A_C + 1} \tag{2}$$

where X_U and X_L are the upper and lower bounds of the continuous variable X. It is advisable to adapt the precision to the problem, because the search process can be faulty when more precision by a longer string is required.

The decoding is basically carried out for the evaluation of the population's individual in the object function and it is applied to the population's members.

Selection Strategies

At first the genetic algorithms generate random strings for the solution population. The following generation is developed by applying the genetic operators: reproduction, crossover and mutation. The new generation is evolved based on each individual's probabilities assigned by its object function fitness; i.e., for poor object function fitness values there are few probabilities for surviving the next generation. In this way, the generations are engendered with the strings or individuals that improve the function objective fitness value. Those that do not cover these conditions disappear completely.

The reproduction is in essence a selection process. The good known selection outlines are: the proportional schema, or group one. The process of proportional selection assigns a reproduction range according to the fitness value to each individual. In the group selection process, the population is divided into groups according to their fitness value; where each group member will have the same reproduction value.

For instance, the proportional selection could be expressed mathematically in the following way:

$$P_i = \frac{f_i}{\sum f_j} \tag{3}$$

where P_i is the selection probability, f_i is the aptitude of the *i-th* individual or string and Σf_i is the sum of the population's fitness. Another form is to use the reciprocal of the object function to obtain the gross fitness f, i.e.:

$$f = \frac{1}{FO} \tag{4}$$

where FO is the object function value for the *i-th* string.

On the other hand, for the purpose of giving the most opportunity to the genetic algorithm of exploring the whole search space, the creation of the first generation should be as diverse as possible and should stay this way at least during the first generation. In a case where a string or individual has a high fitness value inside the initial generation, the individual could dominate the population. Scaling the fitness value is a form of avoiding dominance, individuals with more fitness are scaled down and those with smaller fitness are scaled up, this way the selection process can be more random.

The fitness linear scaling requires a lineal relationship between the scaled fitness f_i' and the gross fitness f, i.e.:

$$f_i' = af + b \tag{5}$$

the coefficients a and b can be chosen in several ways, however in all cases the scaled average fitness f'_{avg} is required to be similar to the average gross fitness f_{avg} because the recurrent use of this selection process will assure average contributions by the population's members with at least one offspring for the next generation.

Genetic algorithm basic parameters

The convergence of the genetic algorithms to an acceptable solution depends on its basic parameter values (reproduction, crossover, mutation, selection and population) which to find a relationship among them to maintain search robustness has been the subject of diverse studies [4], [6] and [11]. These studies have focused on the relationship between the mutation values and convergence; to the relationship between the population's size and the crossover probability values, respectively; and to the relationship among good population's size, crossover probability and selection. These studies have also focused on specific simplified problems, therefore not making it possible to use the results in practical problems. For the above-mentioned reasons it is necessary to carry out convergence tests with varying values, taking into account that the population's size, the mutation probability and the crossover probability are related for the determination of the best control parameters values. An appropriate approach [9] to begin a search is to consider population size between 30 and 50 individuals, a crossover probability of about 0.6 and a smaller mutation probability of about 0.01.

Applications

The optimizations in electromagnetic problems often involve many parameters in which the parameters may be discrete. For instance, a low side-lobes optimization of elements non-equidistantly spaced on a long array antenna, when the excitation and phase have quantized values. Although the number of possibilities in the search space is finite an exhaustive search is not practical [12] and [13]. The radiation pattern generated by an array antenna [12], is given by:

$$AF(\phi) = 2\sin\phi \sum_{n=1}^{N_{el}} \cos\left[k\left(\sum_{m=1}^{n} d_m - d_l/2 \right)\cos\phi \right] \qquad (6)$$

where $d_l/2$ is the distance from the element l to the physical center of the array, d_m is the space between the element m-1 and element m. The distance of the element m to the center of the array is given by: $\left(\sum_{m=1}^{n-1} d_m - d_l/2\right)$ which assures element n is nearest to the array center than element $n+1$, and also that the minimum distance bigger than zero is considered. It is clear that the problem gets complicated when the number of array elements is increased. In this case the most appropriate optimization method is the Genetic Algorithms.

Another case is the prediction of far field from near field measurements [14]. The mathematical pattern used in the prediction of far field involves great parameter quantity, such as complex excitation, position and orientation of the physical set of the elemental dipoles that generate the same pattern to the one obtained with measurements. In this optimization problem the parameters quantity grows in proportion with the number of elements considered (8 parameters by element). For instance, if a set of four elemental dipoles is used to predict the far field of some electronic device, the search space will have 28 parameters and each one of these in an interval. For this particular case the object function proposed is:

$$\mathrm{F}(s) = \sum_{m=1}^{M} \mathrm{g}_m \left(v_m - \mathrm{f}_m \left(r_m, s \right) \right) = 0 \qquad (7)$$

where v_m is the measured real value, $f_m(r_m,S)$ is any amplitude or phase (calculated with the field expressions for elementary dipoles [2] of any electric or magnetic field component vector radiated by the group of equivalent dipoles, both values in the point r_m); g_m is a weight function which depends on the information kind (excitation and/or phase); S is a vector formed by the excitation, position and orientation dipole parameters. A way of finding S is by minimizing $|F|$. Since $|F|$ is highly non-linear and it has too many local minima, it is only probable to find an global optimal with non-conventional optimization methods, such as the genetic algorithms.

In [15], the optimization problem between the reflectivity and the thickness of wide-band microwave absorbent coatings is presented. The reflection coefficient of the absorbent material is given by:

$$R_i(f) = \frac{\tilde{R}_i(f) + R_{i-1}(f)e^{-2jk_{i-1}(f)t_{i-1}}}{1 + \tilde{R}_i(f)R_{i-1}(f)e^{-2jk_{i-1}(f)t_{i-1}}} \tag{8}$$

$$\tilde{R}_i(f) = \frac{\mu_{i-1}(f)k_i(f) - \mu_i(f)k_{i-1}(f)}{\mu_{i-1}(f)k_i(f) + \mu_i(f)k_{i-1}(f)} \tag{9}$$

for $i > 0$, $k_i(f) = 2\pi f\sqrt{\mu_i(f)\varepsilon_i(f)}$, $\tilde{R}_0 = -1$, and $R(f) = R_{N_L}(f)$, where: N_L is the layers number of thickness t_i, $\varepsilon_i(f)$ and $\mu_i(f)$ are the permittivity and permeability of each layer, supported in a perfect electric conductive material. The process can be repeated on the group of representative frequencies inside the band B to find the frequency of the absorbent media. The total absorbent media thickness is given by: $t = \sum_{i=1}^{N_L} t_i$. In order to minimize the maximum reflection on the band:

$$R = 20\log_{10}\left\{\max\left[\mathrm{R}(f), | f \in B\right]\right\} \tag{10}$$

and the total thickness. It is clear that the goals are opposed while the maximum reflection minimization is achieved with a bigger thickness of the absorbent media; while also seeking to minimize that thickness. The technique used in this case found the trade off between the thickness of the absorbent media and the minimum reflections of the same material.

In [16] the problem of extracting the intrinsic dielectric frequency properties dependent on the media is presented. It is important to know the real and complex magnetic permeability, the real and complex electric permittivity, and the electric conductivity in circuits design when the operation frequency is in GHz. Under these conditions the dispersion losses are quite significant and their estimate is not a simple task. This document proposes a systematic method, based on genetic algorithms, to recover the material dielectric properties from the measurements of S parameters.

In [17] the design problem of electrically small auto-resonant antennas is presented. The parameter that best describes a small resonant antenna is the quality factor Q, which is defined as the relationship of the resonance frequency divided by the frequencies difference to which the radiated power falls to ½ of the power in resonance, i.e., for a smaller Q bigger antenna band width. The main problem in small antenna design is that its radiation resistance falls approaching zero according to decreases in the antenna size and its reactance approaches $\pm\infty$, depending on whether the antenna outside of resonance behaves as an inductance (loop) or as a capacitance (electric dipole). In this problem a genetic algorithm was used to find the wire configuration with both characteristics (capacitive reactance and inductive reactance) which are annulled in resonance.

In [18], the Debye & Lorentzian dispersive media parameters that characterize a material are recovered starting with measurements. The parameters recovery requires a non-linear equation set solution, which becomes quite a hard task. The method proposed; at first, using the the equation (of the telegrapher) of a transmission line to build the parameters distributed matrix with measurements of a badge parallel covered with scattering material, one which in turn constitutes an electromagnetic means of traverse propagation; secondly, using genetic algorithms to find the means scattering by minimization means of the difference between the carried out measurements and the calculated parameters.

Finally, in [19] the design problem of the geometric form absorbent coatings under such requirements as low reflection, small and lightweight volume is considered. In this case the genetic algorithms are applied to optimize the coating form and the full wave technique for form performance prediction.

Conclusions

A quick revision to current literature will show that genetic algorithms have grown in popularity to solve optimization problems in diverse scientific research subjects. The electromagnetic area is not the exception; a clear reference about it may be [20]. In this paper the few selected examples report great optimization work simplification with quite acceptable results. However, in each case the genetic algorithm should be adapted to the treated problem. In certain cases it is necessary to combine this technique with others (like in [15]) and to check them with other methods of the same class (simulated annealing). Although genetic algorithms do not demand a previous or additional knowledge (derivatives) of the function being optimized, it is necessary that one has the sense that a global optimal exists. Another aspect necessary to take into account is the growing parameters space, i.e., the characteristics of the problem plus those of the genetic algorithm control, and for these, there is no method which provides its values in an exact way, it will always be necessary to carry out tests to determine which are the best values. The only inconvenience of this technique maybe the computation time required to find the solution to a problem depending on its complexity. In general, the genetic algorithms are an excellent option for the global robust search of an optimal value from non-linear and high dimensionality functions.

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