The Role of Retarded Momentum and Spin in Explaining the Meissner Effect and Other Electrodynamic Phenomena

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The paper focuses on the problem of developing an electrodynamic model of the supercurrent-growing transition stage in the Meissner effect. The London theory giving the quantum-mechanical explanation for the steady superconducting state says nothing about the forces that evoke the electric current in the transition stage. The classical electrodynamics is also inapplicable in the transition stage through the zero forces predicted by Faradey's law. This gap caused the author to look deeply inside the fundamentals of the classical electrodynamics. The analysis is focused on the basic problem of calculating the force on a stationary point charge from a uniformly moving point charge and reveals two arguable assumptions in the classic theory. The first is the extension of Coulomb's law for calculating the force exerted by a stationary charge on a uniformly moving charge. This assumption is in contradiction with the retarded action principle and overlooks the existence of the longitudinal force that might be responsible, in particular, for the Meissner effect and for maintaining the stability of Rutherford's atom. The second assumption about the point-like charges makes the classic formula for Lorentz force physically unsound at normal temperatures when we deal with fermions rather than point charges with zero spin. Therefore a more complex model of the electron should be used, something like a spinning charged ring. One can safely regard free electrons as point charges only for superconductors due to the formation of Cooper pairs. The above considerations have formed the basis of the computing model describing the transition stage in the Meissner effect and explaining the absence of electric current in the system comprising "normal conductor – permanent magnet".

Keywords: Meissner effect, Cooper pairs, retarded momentum, longitudinal electrodynamic forces, spinning ring model of the electron

1. Introduction

In this paper, we start by investigating the problem of describing of the transition stage in the Meissner-Ochsenfeld experiment. It seems to be likely that in the *macroscopic* system like "conductor–permanent magnet" the transition between the normal state and the superconducting state when the conductor is cooled below the transition temperature should take some time before being detected by the *macroscopic* observer because any instantaneous jump would contradict the causality principle in the macroworld. Hence, the problem of describing the dynamics of the transition state arises. As for the steady superconducting state, there is the quantum-mechanical explanation by the London theory [1]. However, as for the supercurrent-growing transition state no explanation exists currently.

We will define the beginning and the end of the transition stage with two time labels: the moment of creation of Cooper pairs initially stationary, and the moment when they attain the (nearly) constant velocity in the superconducting state. Since the transition state should last a non-zero time interval, the accelerated motion of electrons must be described in terms of electrodynamics, i.e., by differential equations connecting distance, velocity, and forces on a Cooper pair. The London theory explaining how "a transport of electricity can only be effected in the *presence of a magnetic field*" [1], says nothing about the forces (and their origin, magnitude and direction) that evoke the electric current repulsing the external magnetic flux in the stationary system "superconductor—permanent magnet". The London theory didn't try to discuss these hypothetical equations.

On the other hand, classical electrodynamics fails to provide such equations as well because it gives a zero accelerating force according to Faraday's law of induction. Therefore, the next problem arises to find out the reasons of such inability of classical electrodynamics for the particular case and to construct more adequate theoretical model of an electrodynamic interaction problem.

2. Retarded action of a moving point charge on a charge at rest

We will start the investigation with the treatment of the fundamental problem in any electrodynamics – calculating the force exerted on the stationary test point charge by the uniformly moving source point charge. But it would be logical to consider first the simpler electrostatic problem of interaction of two point charges having zero relative velocity. This problem can be examined in two inertial reference frames, namely, $S_{(0)}$ in which the charges are stationary (Figure 1-a), and $S_{(1)}$ in which they are moving with equal velocities transverse to the line connecting the charges (Figure 1-b). We will focus on the x component of the total force on the test charge q_0.

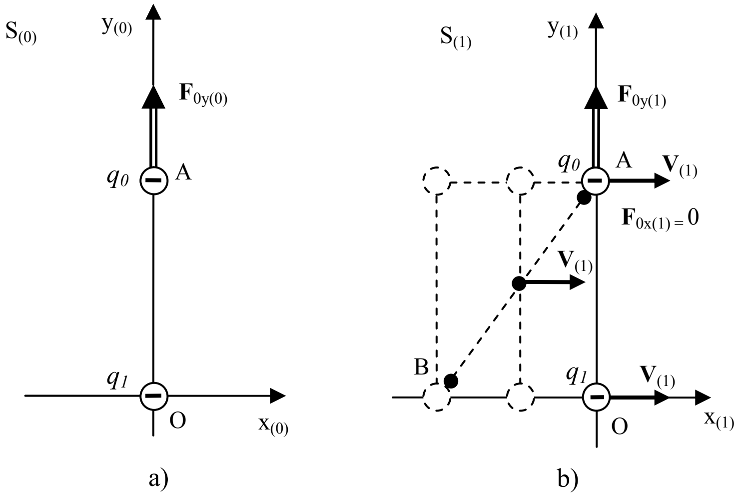

Fig. 1. Disclosing the "retarded" nature of the electrostatic action when passing an observer from the reference frame $S_{(0)}$ where charges are stationary (a) to the frame $S_{(1)}$ where charges move uniformly (b).

We will examine this problem from the viewpoint of two competitive and mutually exclusive fundamental physical interaction models, namely, the "instantaneous action-at-a-distance" model, and the generally acknowledged "retarded action" model where the speed of interaction is assumed to be finite. In the frame $S_{(0)}$ the force $F_{0(0)}$ on the test charge q_0 is determined by Coulomb's law and is directed along the y axis, so its x component $F_{0x(0)}$ is equal to zero. This picture is equally true both for the "action-at-a-distance" model and for the "retarded action" one, so we cannot distinguish them in this case. The picture shown in the frame $S_{(1)}$, Figure 1-b, also corresponds to the "instantaneous action-at-a-distance" model because the force $F_{0(1)}$ is directed along the y axis, that is, along the line connecting the two charges at the *same* time $t_{(1)}$. However, this picture corresponds to the "retarded action" model as well – as we can see from the successive "snapshots" showing the process of the momentum propagation started from the retarded position of the source charge q_0 at the retarded time

$$t_{(1)ret} = t_{(1)} - \Delta t_{(1)}, \tag{1}$$

where

$$\Delta t_{(1)} = \frac{r_{01(1)}}{c\sqrt{1-\beta_{(1)}^2}}, \quad \beta_{(1)} = \frac{V_{(1)}}{c}, \quad r_{01(1)} = OB.$$

This process is similar to the aberration of light in the transverse light clock. Obviously, in the frame $S_{(1)}$ the test charge q_0 must not experience any acceleration confined to the x axis because it is not accelerated along the x direction in the frame $S_{(0)}$. This requirement is met in our analysis—no momentum is transferred in the x direction because of equal velocities of the test charge and the momentum in the x direction. However, the momentum is transferred *along the y axis* due to non-zero relative velocity of the test charge and the momentum carrier in the y direction.

It should be noted that our analysis doesn't depend on any assumptions about the physical nature of the momentum carrier – field, corpuscle, virtual photons, or some else. Therefore, within the framework of the electrostatic problem in point, Coulomb's law is in agreement with the "retarded action" model despite its "action-at-a-distance"-like mathematical formulation.

Another situation takes place when we consider the following case depicted in Figure 2-a: the test charge q_0 is moving but the source charge q_1 is at rest. In the classical electrodynamics, there is an assertion that "*Coulomb's law correctly gives the force on the test charge, for any velocity of the test charge (however high), provided the source charge is at rest*", [2], p. 232. The assertion is in accordance with experimental measurements of the particle's deflection. As it follows from this assertion expressed mathematically in the formula for the Lorentz force, the *x* component $F_{0x(0)}$ of the total force on the test charge must be equal to zero.

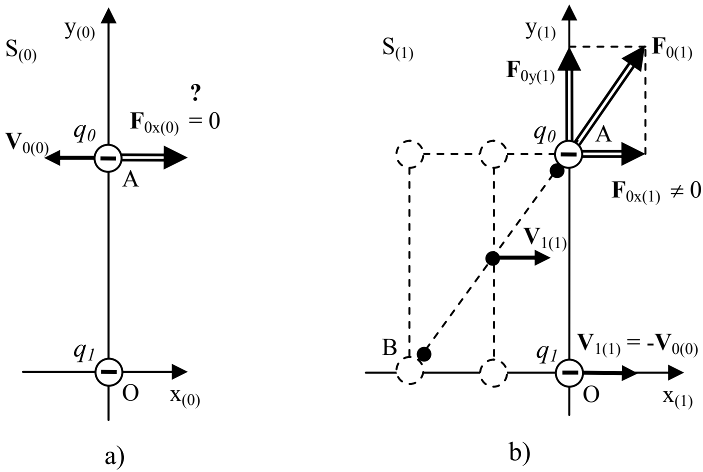

Fig. 2. Transferring the longitudinal component of momentum to the test charge due to the non-zero transverse relative velocity between the test charge and the momentum carrier.

Needless to say that such an extension of Coulomb's law can give the force value with the acceptable level of error for a lot of particular problems involving assemblies of charged particles, but it means, strictly speaking, that the extended Coulomb's law may be considered as a merely perfect *ad hoc* model and no more. A well-known historical precedent in question is a competition between Ptolemaic and Copernican systems. In order to be a candidate for a true *physical* model of the problem in point, the extended application of Coulomb's law should satisfy at least the principle of the "retarded action". But it doesn't, as we can see from the picture shown in the frame $S_{(1)}$, Figure 2-b. Indeed, the momentum must be transferred on *both* the *x* axis and the *y* axis because the test charge q_0 is at rest in the frame $S_{(1)}$, so the *x* component $F_{0x(1)}$ will not be equal to zero. Therefore, returning to the frame $S_{(0)}$, we have to admit that the test charge q_0 experiences the acceleration along the *x* axis as well, i.e., the force $F_{0x(0)}$ must not be equal to zero.

3. Formula for the force on a point charge and the possible role of its longitudinal component in maintaining the stability of an atom

The formulas for computing the components of the total force $F_{0(1)}$ on the test charge q_0 in the frame $S_{(1)}$ may be derived using the picture shown in Figure 3, where A and C represent positions of the test and source charges in the same instance *t*, as well as B is the retarded position of the source charge at the time $t_{ret} = t - \Delta t$

$$F_{0x(1)} = -F_{0(1)} \cos\theta , \tag{2}$$

$$F_{0y(1)} = F_{0(1)} \sin\theta , \tag{3}$$

where

$$\theta = \arccos\left(\frac{x - V_{1(1)}\Delta t}{R_{10(1)ret}}\right) = \arccos\left(\frac{x}{c\cdot\Delta t} - \beta_{1(1)}\right) \tag{4}$$

$$R_{10(1)ret} = c\cdot\Delta t \tag{5}$$

$$\beta_{1(1)} = \frac{V_{1(1)}}{c} \tag{6}$$

$$\Delta t = -0.5p + \sqrt{0.25p^2 - q} \tag{7}$$

$$q = -\frac{R_{10(1)}^2}{c^2\left(1-\beta_{1(1)}^2\right)} \tag{9}$$

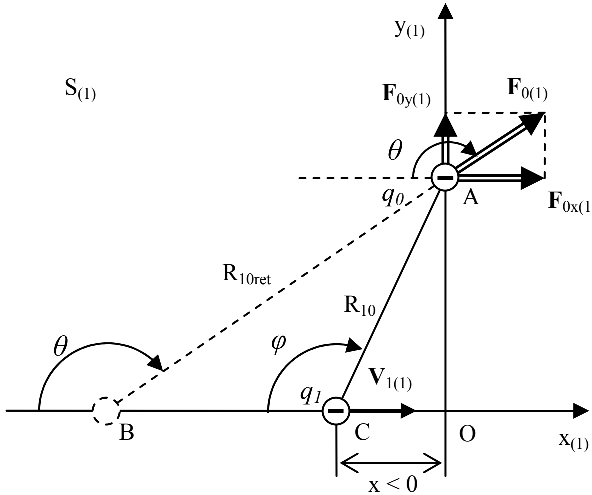

Fig. 3. Retarded action of the source charge on the stationary test charge in case of arbitrarily directed vector of velocity of the source charge.

Concerning the total force $F_{0(1)}$ exerted by the moving charge on the stationary charge, it should be noted that the only model for its computation is known for the present – the inverse-square Coulomb's law:

$$F_{0(1)} = k\frac{q_0 \cdot q_1}{R_{10(1)ret}^2} \tag{10}$$

The force $F_{0(1)}$ is directed radially from the *retarded* position of the source charge, since otherwise the retarded action principle would be violated as shown in the previous section. As a result, there exists the additional non-zero longitudinal component of the electrodynamic force on the stationary test charge proportional to the velocity of the source charge.

Having changed the sign of the source charge to positive and having moved the observer to the reference frame $S_{(0)}$ where the source charge is stationary, we come up closely to the interaction scheme that takes place in Rutherford's model of the atom of hydrogen – the nucleus is the positive source charge and the orbital electron is the test charge, see Figure 4.

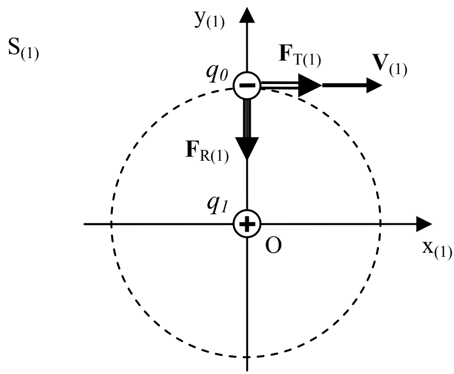

Fig. 4. Longitudinal force on the orbital electron in the model of Rutherford's atom.

In principle, the longitudinal force on the moving point charge coincident in direction with the velocity vector of the orbital electron may compensate the energy loss due to the accelerating motion around the nucleus as we can see by comparing the corresponding powers. The total power radiated by an accelerated charge in the relativistic case is given by the formula obtained by Liénard [3], p. 667:

$$P_{rad} = \frac{2}{3} k \frac{e^2 c}{R^2} \beta^4 \gamma^4 \tag{11}$$

where $\gamma = (1-\beta^2)^{-1/2}$, e is the absolute value of the charge of the electron.

The power provided by the longitudinal force F_T for the given case of transverse motion ($x = 0$) is expressed by the following formula

$$P_{income} = F_T \cdot V = k\frac{e^2 c}{R^2}\frac{\beta^2}{\gamma^2} \tag{12}$$

The orbital velocity of the electron $V = c \times \beta_S$ corresponding to the energy balance can be obtained by equating Eqs.(11) and (12):

$$2\beta_s^2 = 3\cdot\left(1-\beta_s^2\right)^3 \tag{13}$$

The solution is $\beta \approx 0.6098$, that is, the motion of orbital electron should be treated as relativistic.

Needless to say, the above considerations are related only to the classical planetary Rutherford's atom model and cannot be applied to real atoms directly. The main reason is that both the protons and orbital electrons are not point charges because they possess the magnetic moment, and in the interaction scheme it should be taken into account the mutual orientation of spins of charged particles.

3. Computing the longitudinal force on a stationary point charge due to a line of uniformly moving charges

Taking as a basis the formulas for two-particle problem derived in the previous section, we can develop the more complex model for computing the total force on a stationary test point charge exerted by the collective of source point charges moving along the infinite line, as depicted in Figure 5.

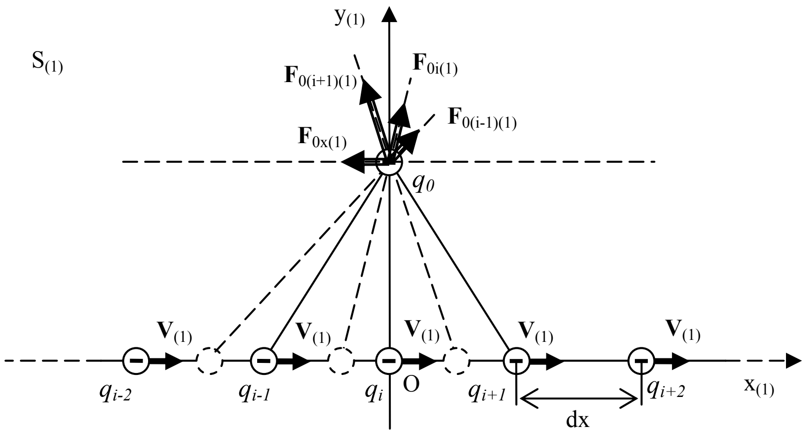

Fig. 5. Longitudinal force $F_{0x(1)}$ on the stationary point test charge q_0 as the result of retarded action of point charges uniformly moving along the *x* axis.

The model allows calculating the total force on the stationary test charge being on the *y* axis at the point *(0, d)* when the moving source charges are distributed symmetrically relative to the frame origin *O*. The model (in CGS units) is implemented in C++ and encapsulated in the class *ChargedLine*. Its definition is presented in Appendix.

The simulation shows that there is the non-zero *x* component of the total force directed opposite to the velocity vector of the source charges. For example, in case of $N = 20001$ charges distributed along the *x* axis with intervals $dx = 0.01$ cm, moving with the relative velocity $\beta = 10^{-8}$, and passing the test charge at impact parameter d = 10 cm, the end of the output is as follows:

```
. . .
i = 19999  x=  99.980  teta = 0.57
               Fx =  0.000198459203  Fy = 199.9899997
i = 20000  x=  99.990  teta = 0.57
               Fx =  0.000098454202  Fy = 199.9900007
i = 20001  x= 100.000  teta = 0.57
```

$$Fx = -0.000001530802 \quad Fy = 199.9900017$$

As we can see, the total longitudinal force $F_{0x(1)}$ accelerates the test charge q_0 in the direction *opposite* to the velocity vector of the source charges, and this force is smaller than the total force $F_{0(1)}$ approximately by the factor β.

4. Dynamic model of the Meissner effect

Strictly speaking, the model described in the previous section cannot be applied directly for calculating the interactions taking place at normal temperatures, because in this case the magnetic moment of the charged particles should be taken into account. Indeed, let us consider the model of retarded action of a permanent magnet on the free electron in a conductor, depicted in Figure 6.

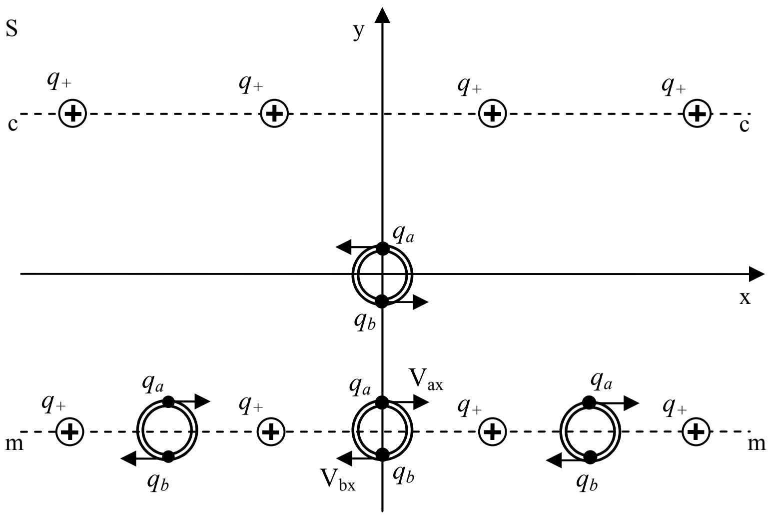

Fig. 6. Interaction model "permanent magnet – conduction electron – conductor lattice" at normal temperatures.

Since the magnetic field of ferromagnetic is almost totally defined by the magnetic moments of electrons, the magnetic field of a permanent magnet is simulated by the bound electrons having definite spin orientation and, therefore, represented in the *xy*-plane as spinning charged rings as described in [4-6]. These electrons are evenly distributed along the line *m*-*m* together with the positive ions represented as point charges due to undefined orientations of their spins. For the same reason, the test free electron placed in the origin *0* of the reference frame *S* is also represented as a spinning charged ring. The spinning charged ring representation of an electron can be simplified in computing model by splitting the ring in two halves: upper and lower equal sub-charges q_a and q_b correspondingly, which move in opposite directions along *x* axis with velocities V_{ax} and $V_{bx} = -V_{ax}$. The total force on the test electron in the reference frame *S* along the *x* axis will be amounted to the following sum:

$$F_{0x} = \sum_{i=-\infty}^{\infty} F_{a0aix}\left(q_{a0}, V_{a0x}, q_{ai}, V_{aix}, x_{ai}, d\right) +$$
$$\sum_{i=-\infty}^{\infty} F_{a0bix}\left(q_{a0}, V_{a0x}, q_{bi}, V_{bix}, x_{bi}, d + 2r_e\right) +$$
$$\sum_{i=-\infty}^{\infty} F_{b0aix}\left(q_{b0}, V_{b0x}, q_{ai}, V_{aix}, x_{ai}, d - 2r_e\right) +$$
$$\sum_{i=-\infty}^{\infty} F_{b0bix}\left(q_{b0}, V_{b0x}, q_{bi}, V_{bix}, x_{bi}, d\right) +$$
$$\sum_{j=-\infty}^{\infty} F_{a0+jx}\left(q_{a0}, V_{a0x}, q_{+j}, V_{+jx}, x_{+j}, d + r_e\right) +$$
$$\sum_{j=-\infty}^{\infty} F_{b0+jx}\left(q_{b0}, V_{b0x}, q_{+j}, V_{+jx}, x_{+j}, d - r_e\right) +$$
$$\sum_{j=-\infty}^{\infty} F_{a0+jx}\left(q_{a0}, V_{a0x}, q_{+j}, V_{+jx}, x_{+j}, d - r_e\right) +$$
$$\sum_{j=-\infty}^{\infty} F_{b0+jx}\left(q_{b0}, V_{b0x}, q_{+j}, V_{+jx}, x_{+j}, d + r_e\right), \tag{14}$$

where $x_{ai} = x_{bi} = x_i$,, r_e-radius of the electron's ring.

The first four terms in Eq. (14) are partial forces on the test electron due to the bound electrons in the permanent magnet. The 5th and 6th terms represent the action of the stationary ion lattice of the permanent magnet on the test electron, where ions velocity $V_{+jx} = 0$ and charge $q_{+j} = +e$. The 7th and 8th terms give the force on the test electron due to the stationary ion lattice of the conductor. Taking into account that $V_{b0x} = -V_{a0x} = V_{aix} = -V_{bix} = V$ we can find for the symmetrical charge distributions that in the frame S the total force on the test electron $F_{0x} = 0$. This result corresponds to the experimental evidence that there is no electric current in the normal conductor placed stationary near the permanent magnet.

However, another situation arises when the temperature falls below the transition temperature for the given conductor. In this case, according to the Cooper model and the BCS theory (see, for example, [7], pp. 737-747), free electrons can form Cooper pairs, so the resulting quasi-particle with a zero spin can be treated as a point charge from the viewpoint of the fundamental electrodynamic interaction. The corresponding model is shown in the Figure 7.

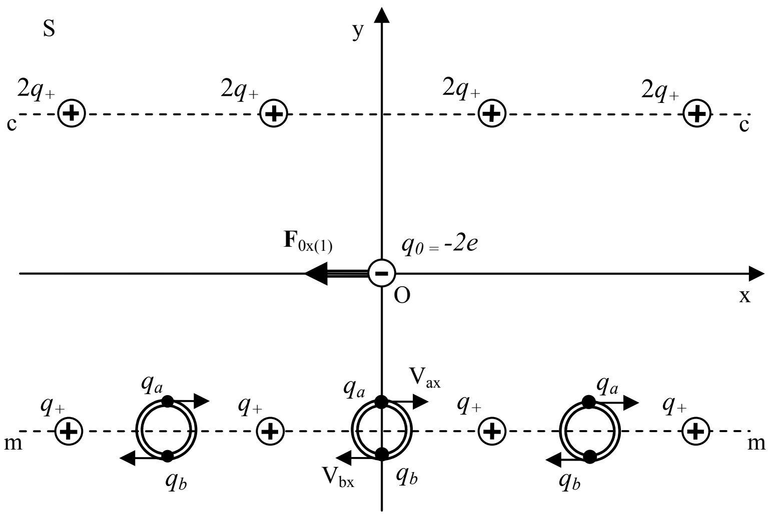

Fig. 7. Interaction model "permanent magnet - Cooper pair in a superconductor"

The total force on the test point charge $q_0 = -2e$ is defined by

$$
\begin{aligned}
F_{0x} = & \sum_{i=-\infty}^{\infty} F_{0aix}\left(q_0, V_{0x}, q_{ai}, V_{aix}, x_{ai}, d - r_e\right) + \\
& \sum_{i=-\infty}^{\infty} F_{0bix}\left(q_0, V_{0x}, q_{bi}, V_{bix}, x_{bi}, d + r_e\right) + \\
& \sum_{j=-\infty}^{\infty} F_{0+jx}\left(q_0, V_{0x}, q_{+j}, V_{+jx}, x_{+j}, d\right) + \\
& \sum_{j=-\infty}^{\infty} F_{0+jx}\left(q_0, V_{0x}, 2q_{+j}, V_{+jx}, x_{+j}, d\right) ,
\end{aligned}
\tag{15}
$$

where $q_{+j} = e$. The last sum represents the force exerted by the ion lattice of superconductor.

Now, in the initial state when the test charge is stationary, $V_{0x}(t_0) = 0$, the total force F_{0x} is not equal to zero and accelerates the test charge in the negative direction of the x axis. The motion of the test charge is described by the following system of differential equations:

$$
\begin{aligned}
\frac{dx_0}{dt} &= V_{0x} \\
\frac{dV_{0x}}{dt} &= \frac{1}{2m_e} F_{0x}
\end{aligned}
\tag{16}
$$

with initial conditions: $x_0(t_0) = 0$, $V_{0x}(t_0) = 0$. This initial-value problem has been solved numerically using the 4$^{\text{th}}$-order Runge-Kutta method with the time step $h_t = 10^{-20}$ sec. Due to the very small estimated value of the electron radius

$$
r_e = \frac{\hbar}{mc} = 3.86 \cdot 10^{-11}\ cm
$$

that entails evident computational difficulties, the permanent magnet as a source of external magnetic field has been replaced by the equivalent current-bearing wire with the velocity of conduction electrons $V = c \times 10^{-10}$. The solution $V_{0x}(t)$ describing the transition stage of the Meissner effect is graphically represented in Figure 8.

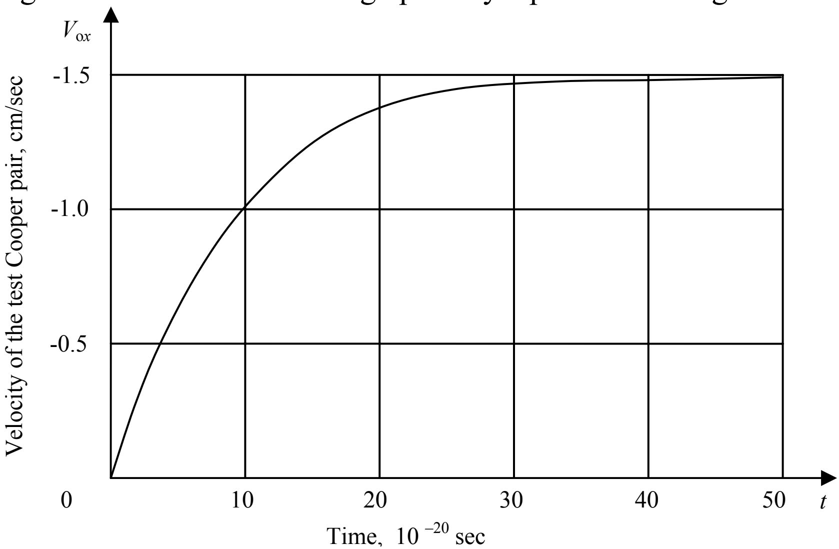

Fig. 8. Increasing the velocity of Cooper pairs in the transition stage of the Meissner effect

It is evident that the asymptote of the graph fully corresponds to the velocity derived from the London equation. That is, the London equation can be treated as a solution of the system (16) when $t \to \infty$.

5. Conclusion

As we can see from the above considerations, a much more sophisticated analysis as compared with classical electrodynamics is required when studying some electrodynamic phenomena. In the paper, the author has tried to follow the principle of retarded action as thoroughly and consistently as possible. As has been shown concerning the fundamental model of acting the moving charge on the stationary charge, the classical electrodynamics applies the principle of retarded action incorrectly, ignoring the *longitudinal* momentum transfer and, therefore, has got the result concurring with one derived on the "instantaneous action-at-a-distance" model. Needless to say that even the tally in great many engineering applications cannot compensate this logical fault. As a result, the classic formula for Lorentz force contradicts the principle of retarded action and, generally speaking, has no physical sense. However, it gives very good approximate results in diverse applications, and can be used as an engineering *ad hoc* formula. Nevertheless, its usage as a tool for fundamental research in physics should be restricted. Obviously, such a situation cannot be acceptable as for the theory pretending to be a background of theoretical physics.

Only the consistent follow-up of the principle of retarded action, as well as taking into account the existence of particle's spin and its orientation can approach us to explaining such phenomena as the transient stage in the Meissner effect, stability of atom, and some manifestations of longitudinal electrodynamic forces described, e.g., in [8]. While completing the article, the author has learnt about some new evidence showing the role of electron spin orientation in electrodynamic interaction - so-called "quantum spin dependent tunnelling effect" controlled by a magnetic field [9].

Acknowledgements

The author wishes to thank Dr. Stanley Jeffers for his help in making the article more readable and clear.

References

[1] F. London, "A new conception of superconductivity," *Nature* (November 6, 793-796, November 13, 834-836, 1937).

[2] A. P. French, *Special relativity*, Chapman & Hall. (1968, reprint 1997) 286 pp.

[3] J. D. Jackson, *Classical electrodynamics*, John Wiley & Sons, 3rd edn., (1999) 808 pp.

[4] A. L. Parson, "A magneton theory of the structure of the atom," *Smithsonian Miscellaneous Collection* **65**, No.11, Publication No. 2371, (29 November, 1915) 1-80.

[5] H. S. Allen, "The case for a ring electron," *Proc. Phys. Soc. London* **31** (1919) 49-68.

[6] D. L. Bergman, and J. P. Wesley, "Spinning charged ring model of electron yielding anomalous magnetic moment," *Galilean Electrodynamics* **1** (No. 5, 1990) 63-67.

[7] N. W. Ashcroft, and N. D. Mermin, *Solid state physics*, Saunders College, Philadelphia. (1976) 848 pp.

[8] P. Graneau, and N. Graneau, *Newtonian electrodynamics*, World Scientific Publishing Co. (1996) 288 pp.

[9] ""Spintronics" hold potential for future electronic devices," *APS News* **7** (American Physical Society, No. 6, June 1998) 4.

Appendix

The C++ code below presents the declaration and definition of the class *ChargedLine* encapsulating the computing model of electrodynamic interaction of motionless and moving point charges:

```
class ChargedLine
{
private:
  const double k, c, PI;
  double d, x0, x, dx, q0, q1, beta;
  double R, Rret, teta, dt, p, q,
         dF, dFx, dFy, Fx, Fy;
  int i;
public:
```

```
ChargedLine():k(1), c(2.99793e10),
      PI(3.141592654), d(1), x0(-100),
      dx(0.01), q0(1), q1(1), beta(1e-8),
      Fx(0), Fy(0){}
void ForceXY()
{
  R = sqrt(x*x + d*d);
  p = 2*beta*x/(c*(1-beta*beta));
  q = -R*R/(c*c*(1-beta*beta));
  dt = -0.5*p + sqrt(0.25*p*p - q);
  Rret = c*dt;
  teta = acos(x/(c*dt) - beta);
  dF = k*q0*q1/(Rret*Rret);
  dFx = - dF*cos(teta);
  dFy = dF*sin(teta);
}
void TotalForce()
{
  x = x0;
  i = 0;
  while(i <= -2*x0/dx)
  {
    ForceXY();
    Fx += dFx;
    Fy += dFy;
    ++i;
    ShowFXY();
    x += dx;
  }
}
void ShowFXY()
{
  cout.setf(ios::fixed);
  cout << "i = "  << setw(5) << i
       << " x= " << setw(7) <<
       setprecision(2) << x << "  "
       << "teta = "<< setw(6)<<
       setprecision(2) << teta*180/PI << "  "
```

```
            << "Fx = " << setw(15) <<
            setprecision(12) << Fx << "  "
            << "Fy = " << setw(12) <<
            setprecision(7) << Fy << endl;
    }
};
```