On the Relativistic Transformation of Force

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The paper analyzes the relativistic law of transformation of force and some accompanied physical difficulties. We focus our attention on the complex systems, consisting of a number of sub-systems i with the velocities $\vec{u}_i$ in a laboratory frame. We establish an analogy between the total force in such system and e.m.f. in a closed deforming circuit with respect to the force transformation law. It has been concluded that for these systems the relativistic law of transformation of force contradicts the causality principle.

Keywords: force transformation law, electromotive force, causality principle

1. Introduction

At present time there are a lot of books and papers, claiming that special relativity theory (SRT) is, perhaps, physically false. Simultaneously it is stated that SRT is a perfect theory in its logic and mathematics, and thus, if SRT is incorrect physically, it could be disproved only experimentally. In contrast to this widespread opinion, the present author assumed [1] that an intrinsic problem of compatibility of SRT with the causality principle might exist. In particular, Ref. [1] points out that causality principle was tacitly applied in the proof of the theorem about invariance of space-time interval, and it was made without sufficient justification. Nevertheless, the revealed causal paradox [2] later found its resolution in [3]. We recall that the causality principle (CP) means two fundamental requirements:

1. A cause-consequence order of events is absolute.
2. The events, which can cause essential inferences (for example, collision of particles), are absolute.

One should mention that there are various definitions of the causality principle (see, *e.g.*, [4, 5]). We prefer the statements 1 and 2 as before [1, 2], because just the absolute events lie on the basis of all measurements in space-time, giving a physical interpretation to the Lorentz transformations.

Now we emphasize that CP is not restricted to its direct consequences 1, 2. There exist another its consequences, which can be referred as “indirect.” As example we mention the requirement, used in ref. [6] for analysis of the Faraday induction law: an electromotive force in a circuit has the same sign for all inertial observers, and the e.m.f. should be vanished simultaneously for all of them. The same requirement is obviously true for a torque, acting on a mechanical system. One can see that all such “indirect” consequences of CP are not intrinsically related to the measuring procedures, established in relativity. However, they obviously should be fulfilled in a correct physical theory of space-time. One sees that all of them are reduced to the original problem of transformation of force. Therefore, an analysis of the force transformation law seems important, since it elucidates the mentioned above problem: compatibility of SRT with the indirect consequences of CP. In the next section we briefly reviewed some physical difficulties, related to the definition of force and its transformation between different observers.

2. The force transformation law: state of the art and something more

It is well known that a force is a relative quantity, and its transformation between two inertial frames K and K′ has the form [7]

$$\vec{F}' = \frac{\sqrt{1-v^2/c^2}\,\vec{F} + \dfrac{\vec{v}\cdot\left(\vec{v}\cdot\vec{F}\right)\left(1-\sqrt{1-v^2/c^2}\right)}{v^2} + \dfrac{\vec{v}\cdot\left(\vec{u}\cdot\vec{F}\right)}{c^2}}{1+\left(\vec{v}\cdot\vec{u}\right)/c^2}, \qquad (1)$$

where $\vec{v}$ is the velocity of K in K′, and $\vec{u}$ is the velocity of particle in the frame K. One should stress that Eq. (1) is obtained from transformation of space-time and energy-momentum four-vectors, and it is applicable to any kind of interaction. However, a notion of force is awkward in microphysics. Hence, the law (1) is usually analyzed in macrophysics, which operates with the so-called mechanical forces and electromagnetic forces. An electromagnetic force is determined by the Lorentz force law

$$\vec{F} = q\left(\vec{E} + \vec{u}\times\vec{B}\right), \qquad (2)$$

where $\vec{E}, \vec{B}$ are the electric and magnetic fields, respectively, and $\vec{u}$ is the velocity of charged particle in the frame of observation. It is known that the electromagnetic forces between spinless particles do not provide a stability of any isolated system. That is why the "mechanical forces" are additionally introduced, which are needed to provide a stability of systems. Rigorously speaking, their nature cannot be clearly determined; there is only the general requirement (1), establishing the law of their transformation.

First time a problem of relativistic transformation of mechanical and electromagnetic forces was discussed in connection with the Trouton & Noble experiment [8-10]. This experiment dealt with a solid capacitor plates capable of supporting a bending moment. The experiment looked for a couple on the system arising out of the Earth's motion. No significant rotation was observed. Recently the Trouton-Noble type experiment was performed by Cornille [11], where he did not shield the suspended condenser from external electric field, and certainly observed a rotation effect of condenser. The authors of ref. [12] explained such a result by an influence of the magnetic field of Earth. However, the problem at the whole seems non-resolved in full up to date.

Another ambiguity is related with a nature of mechanical forces. In particular, Endean [10] referred to a known paradox, dealing with a particle sliding on a surface with a friction (Fig. 1, reproduced from [10]). In the frame in which the surface is stationary, the particle has velocity u; the normal force between the surface and the particle is N, the friction force $F_f = kN$ (k is the friction coefficient); and the length of travel of the particle across the surface is l. Then measurements in both particle and surface frames are agreed on the magnitude of the frictional force and both agree that multiplying it by the relative velocity u gives the power dissipated in frictional heating. However, they disagree on the time interval over which this power is developed. Indeed, in the frame of the surface this time interval is l/u; in the rest frame of particle this time is reduced to $l/\gamma u$. Hence, both observers get different total energy dissipated at heat. The paradox can only be resolved by introducing a continuous material velocity field and mechanical stress distribution at the sliding contact and computing quantities in the rest frame of particle [13].

There is another paradox concerning with the problem in Fig. 1, and which was not considered to the moment. It appears for observer K′, which moves along the normal to the surface at a constant velocity *v*. Then according to transformation (1), the normal force *N* remains unchanged. The friction force along the surface in the frame K′ is reduced by factor $\gamma = 1\big/\sqrt{1-v^2/c^2}$, and the additional "friction" force along the normal to the surface appears with the value

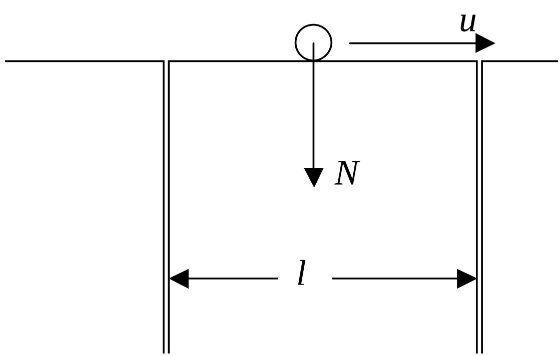

Fig. 1. Particle sliding on a surface

$$F_y' = \frac{v\left(uF_f\right)}{c^2} = \frac{uv}{c^2}kN\,. \tag{3}$$

A physical origin of the force (3) is unclear. Moreover, in the ultra-relativistic case ($v,u \approx c$), and for the friction coefficient $k > 1$, the force F_y' formally can exceed *N*. Since the friction force F_f is proportional to *N*, that it means a vanishing of both $\sqrt{1-v^2/c^2}\,F_f$ and *N* forces, and hence, no energy is dissipated at heat for the observer in

K′. Such a situation indicates that for a moving observer the friction force cannot be simply expressed as kN. One of the reasons could be a transformation of mechanical stress distribution at the sliding contact into mechanical momentum for different observers, as well as an energy-momentum transformation for a short-range field of normal forces. These effects make the problem in Fig. 1 very complicated.

Now let us consider the electromagnetic forces. There are no physical difficulties, when an external force exerts on a single point-like charged free particle. They appear when a force is computed for a system of charged particles, moving at arbitrary velocities in electromagnetic field. The analysis of force transformation law faces with a number of problems, which can be divided into formal (complicating particular calculations) and physical (creating difficulties in physical interpretation of the results obtained). Among formal problems we can mention a dependence of a position of the system's center of inertia on a velocity of external observer, as well as the impossibility to compute a total force, acting on a system, as the vector sum of forces, acting on each sub-system for an arbitrary moving observer [7].

Among physical problems we can designate the following.

I. In electromagnetic interaction the force depends on acceleration itself, which requires not two, but three initial conditions for the solution of the equation of motion of an accelerated charge [14].

II. Transformation of mechanical and electromagnetic momentum and a nature of electromagnetic force itself ([15] and references therein).

III. Transformations of electromagnetic fields and charge density [16, 17]; the involvement of force transformation law into the alternative explanation of a length contraction and time dilation in the known experiments [18].

IV. A self-force in an isolated non-radiative system of moving charges [12, 19]; the problem of "hidden" momentum [12, 20].

V. The force, acting on a complex non-radiative system of charged particles, moving at different non-zero velocities u_i in a laboratory frame.

In this paper we will not consider the problems I-IV, addressing to the mentioned references. Below we will focus our attention on the items V where, as we will show below, the force transformation law and causality requirements can contradict each other.

The law of transformation of total force, acting on a complex system, represents a multi-parametric transformation, depending on $\vec{v}$ (the velocity of system at the whole) and $\vec{u}_i$ (the velocities of sub-systems). Such a transformation is not ordinary for relativistic physics, because usually the rotation-free transformations depend on a single vectorial parameter $\vec{v}$, a relative velocity between two inertial frames involved. A multi-parametric force transformation includes a number of different transformations (1) for each sub-system *i* with the velocity parameters $\vec{v}$ and $\vec{u}_i$. In the next section we explore the properties of such transformations for the simplest complex mechanical system, which consists of two sub-systems, moving at a relative velocity $\vec{u}$. The results will be verified by the Lorentz force law. In section 4 we establish an analogy between a total force, acting on a complex mechanical system, and an e.m.f. in a deforming circuit. Finally, section 5 contains some conclusions.

3. The force acting on mechanical system, consisting of two sub-systems

Let us introduce into consideration a mechanical system, which consists of two sub-systems 1 and 2. The sub-system 1 rests in a laboratory frame K, while the sub-system 2 is moving at the velocity $\vec{u}$ at the considered time moment. An external force $\vec{F}$ acts on the sub-system 1, while the opposite in sign force $-\vec{F}$ acts on the sub-system 2. In these conditions the total force, acting on the system, is equal to zero*. Since we omit a calculation of torque exerted on the system due to these forces, then the spatial coordinates of the sub-systems 1 and 2 are not relevant. Now let us compute the total force, acting on the system, in an inertial reference frame K′, wherein the system moves at a constant velocity $\vec{v}$. To simplify consideration further, we will analyze the cases, where the vectors $\vec{v}$, $\vec{u}$ and $\vec{F}$ can lie only in two mutually orthogonal directions. One can see that four different combinations of these vectors are possible, when the signs of the velocities can be omitted (Table 1). The forces $\vec{F}_1'$, $\vec{F}_2'$ were calculated according to Eq. (1), and $\vec{F}_{total}' = \vec{F}_1' + \vec{F}_2'$. The Table shows that in two cases 2, 4 the total force acting on the system, is not equal to zero in the frame K′. This result seems to contradict the causality principle. That is why it is especially interesting to construct a physical model of such a system and specify the applied forces. The simplest way to proceed is to identify both sub-systems with the point-like opposite charged particles, whose spatial coordinates are the same at the considered time moment. (The latter requirement allows simple summing up the forces for any inertial observer, see the footnote 1). In order to consider these particles as the parts of a mechanical system, we have to make a restriction on their motion,

* In general, due to relativity of simultaneity of events we cannot simply sum up the forces applied to the system, for different inertial observers. There are special conditions, when such a summation can be carried out: either the forces are static, or they are applied to the same spatial point. We assume that at least one of these conditions is fulfilled.

being common for them. In particular, we can assume that both particles are placed inside an isolating tube with thin walls, where they are free to move only along the tube. The force, acting on the particles, is due to an external electromagnetic field. Then the formal application of the force transformation law (1) to this system can be physically verified by the Lorentz force law, applied for different inertial observers.

Table 1 - The result of calculation of $\vec{F}'_{total}$ acting on the system, consisting of two sub-systems, where $\vec{F}_{total}$ is equal to zero ($\vec{F}_1 = -\vec{F}_2$)

Case		$\vec{F}_1 = -\vec{F}_2$		F'_1		F'_2		F'_{total}	
		$\parallel \vec{v}$	$\perp \vec{v}$	$\parallel \vec{v}$	$\perp \vec{v}$	$\parallel \vec{v}$	$\perp \vec{v}$	$\parallel \vec{v}$	$\perp \vec{v}$
1	$\vec{v} \parallel \vec{u}$	F	0	F	0	$-F$	0	0	0
2		0	F	0	$\frac{F}{\gamma}$	0	$\frac{-F}{\gamma\left(1+\frac{uv}{c^2}\right)}$	0	$\frac{Fuv}{\gamma c^2\left(1+\frac{uv}{c^2}\right)}$
3	$\vec{v} \perp \vec{u}$	F	0	F	0	$-F$	0	0	0
4		0	F	0	$\frac{F}{\gamma}$	$-F\frac{uv}{c^2}$	$-\frac{F}{\gamma}$	$-F\frac{uv}{c^2}$	0

3.1. Realization of the case 2 ($\vec{v} \parallel \vec{u} \perp \vec{F}$)

Fig. 2 shows a problem, when the case 2 of Table 1 is realized. Two oppositely charged particles are placed into the neutral isolating tube. The initial coordinates of particles are equal to each other. At this time moment the particle $+q$ rests with respect to the tube, while the particle $-q$ moves at the velocity u along the tube. The particle Q, resting in the laboratory frame K, is the source of an external field

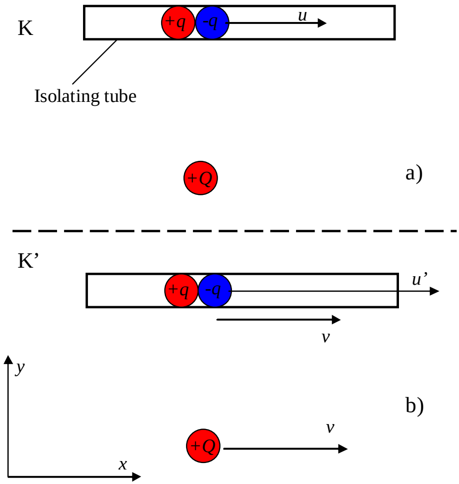

Fig. 2. Two point-like charged particles $+q$ and $-q$ are placed inside the isolating tube, where they are free to move along the axis x without friction. The spatial coordinates of both particles are equal to each other at $t = 0$. The tube and the external changed particle $+Q$ initially rest in the laboratory frame K. The x-coordinates of the particle $+Q$ and particles $+q$,$-q$ are equal to each other at the initial time moment. The particle $-q$ has the initial velocity u along the axis x. We want to compute the force, acting on the tube, in the laboratory frame K (a) and in the external inertial frame K′, wherein the frame K moves at the constant velocity v along the axis x (b).

acting on the charges in the tube. The tube also rests in the laboratory. One requires to compute the force, acting on the tube in the frame K and in the external inertial frame K′, wherein the frame K is moving at the constant velocity v along the axis x.

The problem is immediately solved in the frame K. The resting particle Q creates only the electric field. Its component along the axis y at the location of particles $+q$ and $-q$ can be denoted as E. Then the force, acting on the particles inside the tube along the axis y is qE (for positive charge) and $-qE$ (for negative charge). Hence, the total force, acting on the tube, is equal to zero, if we neglect a polarization of isolating tube.

Now compute the force, acting on the tube, in the frame K′. In this frame the external particle *Q* moves at the velocity *v* along the axis *x* at $t = 0$. Hence, its electric field along the axis *y* is

$$E'_y = \frac{E}{\sqrt{1-v^2/c^2}}, \tag{4}$$

and the magnetic field along the axis *z* is

$$B_z = \frac{vE}{c^2\sqrt{1-v^2/c^2}}. \tag{5}$$

The Lorentz force

$$\vec{F}' = q\left(\vec{E}' + \vec{u}' \times \vec{B}'\right), \tag{6}$$

acts on each particle inside the tube, where $\vec{u}'$ is the velocity of particle in the tube for an observer in K′. For positively charged particle,

$$u'_{+x} = v, \tag{7}$$

and for negatively charged particle

$$u'_{-x} = \frac{u+v}{1+uv/c^2}. \tag{8}$$

Hence, combining Eqs. (4)-(8), we obtain for both charged particles

$$F'_{+x} = 0,\ F'_{+y} = \frac{qE}{\sqrt{1-v^2/c^2}} - \frac{v^2 qE}{c^2\sqrt{1-v^2/c^2}} = qE\sqrt{1-v^2/c^2}, \tag{9}$$

and

$$F'_{-x}=0\,,\; F'_{-y}=\frac{-qE}{\sqrt{1-v^2/c^2}}+\frac{(u+v)vqE}{c^2\left(1+uv/c^2\right)\sqrt{1-v^2/c^2}}=-\frac{qE\sqrt{1-v^2/c^2}}{1+uv/c^2}\,. \quad (10)$$

From there the resulting force, acting on the tube in the frame K′ is

$$F'_{ty}=F'_{+y}+F'_{-y}=qE\sqrt{1-v^2/c^2}-\frac{qE\sqrt{1-v^2/c^2}}{\left(1+uv/c^2\right)}=\frac{qE\sqrt{1-v^2/c^2}}{\left(1+uv/c^2\right)}\frac{uv}{c^2}\,. \quad (11)$$

Note that Eqs. (9)-(11) are in a full agreement with calculated forces F'_1, F'_2 and F'_{total} for the case 2 in Table 1.

Thus, we reveal that no force acts on the tube in the laboratory frame K, while the non-vanishing force (11) acts on the tube along the axis *y* of the frame K′.

3.2. Realization of the case 4 ($\vec{v}\perp\vec{u}$, $\vec{F}\perp\vec{v}$).

This case is realized for the problem, depicted in Fig. 3. There are two differences from Fig. 2:

- now the *y*-coordinates of all charged particles are the same, and the electric force acts in the *x*-direction;
- the laboratory frame K moves along the axis *y* of the external inertial frame K′.

One again requires to find the force, acting on the tube in the frames K and K′.

Since the particle *Q* rests in K, it produces only the electric field, which is equal to E_x at the location of particles $+q$ and $-q$. Hence, the resultant force is equal to zero:

$$F_{tx}=qE_x-qE_x=0\,.$$

In the frame K′ the particle *Q* produces the electric field along the *x*-axis

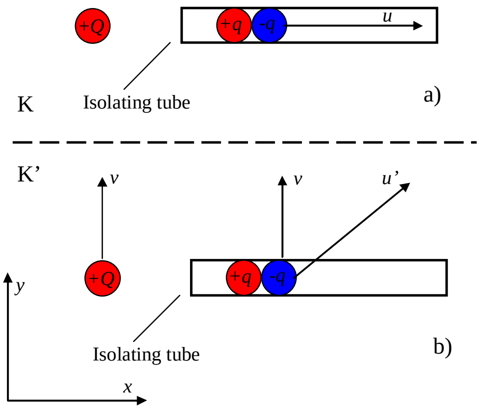

Fig. 3. Two point-like charged particles +*q* and –*q* are placed inside the isolating tube, where they are free to move along the axis *x* without friction. The spatial coordinates of both particles are equal to each other at *t* = 0. The tube and the external changed particle +*Q* initially rest in the laboratory frame K. The *y*-coordinates of particle +*Q* and particles +*q*,-*q* are equal to each other at the initial time moment. The particle –*q* has the initial velocity *u* along the axis *x*. We want to compute the force, acting on the tube, in the laboratory frame K (a) and in the external inertial frame K′, wherein the frame K moves at the constant velocity *v* along the axis *y* (b).

$$E'_x = \frac{E}{\sqrt{1-v^2/c^2}},$$

as well as the magnetic field along the *z* axis

$$B'_z = -\frac{vE}{c^2\sqrt{1-v^2/c^2}}.$$

The force acting on the particles inside the tube is determined by the Lorentz force (6). The velocity of positive charge *q* has a single component along the axis *y*

$$u'_{y+} = v, \tag{12}$$

while the negative charge $-q$ has the velocity with the components

$$u'_{x-} = u\sqrt{1-v^2/c^2}\,,\ u'_{y-} = v. \tag{13}$$

Substituting Eqs. (13) and (12) into Eq. (6), we obtain:

$$F'_{x+} = qE'_x + qu_{y+}B'_z = \frac{qE}{\sqrt{1-v^2/c^2}} - \frac{qEv^2}{c^2\sqrt{1-v^2/c^2}} = qE\sqrt{1-v^2/c^2} \quad F'_{y+} = 0,$$

and

$$F'_{x-} = -qE'_x - qu_{y-}B'_z = \frac{-qE}{\sqrt{1-v^2/c^2}} + \frac{qEv^2}{c^2\sqrt{1-v^2/c^2}} = -qE\sqrt{1-v^2/c^2}$$

$$F'_{y-} = qu_x B'_z = -\frac{qEvu}{c^2}.$$

From there we derive the following components of the total force, acting on the system:

$$F'_{xt} = F'_{x+} + F'_{x-} = 0,\ F'_{yt} = F'_{y+} + F'_{y-} = -\frac{qEvu}{c^2}. \tag{14}$$

Thus, the total force (14) is not vanishing in the frame K′, and it is collinear to the vector $\vec{v}$. It fully corresponds to the result of calculation in Table 1, case 4.

At the same time, we have to mention that for the problems in Figs. 2 and 3 we tacitly implied the equality of action and reaction for the normal forces acting on the tube due to the charged particles. Only under this condition do we derive a contradiction between the force transformation law and “indirect” requirements of CP: the tube does not experience a net force in the frame K, but is experiences the non-vanishing force in the frame K′. The equality of action and reaction for normal forces is certainly fulfilled in the laboratory frame K.

However, the same equality in the frame K′ is a matter of separate analysis. For example, it is known (see, *e.g.*, [21]) that for an electromagnetic (EM) interaction a relationship between active and reactive forces is the frame-dependent quantity. The reason is a transformation of the momentum and energy of EM field between different observers. For the problems in Fig. 2, 3 the normal forces between the charges and tube appear due to a short-range interaction $1/r^n$ $(n>2)$, which is electromagnetic in its nature. Therefore, the field of this interaction also possesses an energy and momentum, and a transformation of an energy-momentum four-vector should be also taken into account, when a net force experienced by the tube is computed. Since this problem is very complicated, we cannot conclude yet that the results of this section indicate a violation of "indirect" consequences of CP in relativity theory. A certain contradiction with CP emerges, when we leave mechanical systems and consider an electromotive force in the closed deforming circuits.

4. Transformation of an electromotive force in a deforming circuit

It is known that an electromotive force in a closed circuit Γ is defined by the equation

$$\varepsilon = \oint_{\Gamma} \vec{f} \cdot d\vec{l}\ , \tag{15}$$

where $\vec{f}$ is the force per unit charge. It is also well-known that the Faraday induction law

$$\varepsilon = -\frac{d}{dt} \int_{S} \vec{B} \cdot d\vec{S} \tag{16}$$

is valid for both fixed and deforming circuits Γ, restricting the area *S*. We will explore the case of deforming circuit $\Gamma = \Gamma(t)$, where its deformation can be described by the velocity vector field $\vec{u}\left(\vec{r},t\right)$ with $\vec{r} \in \Gamma$. Then such a circuit can be considered as a complex system, consisting of infinite set of sub-systems, representing infinitely small segments $d\vec{l}\left(\vec{r}\right)$ with the velocities $\vec{u}\left(\vec{r},t\right)$. In fact, we have a full mathematical analogy with the complex mechanical systems. Then a transformation of e.m.f. between two inertial frames K and K′, moving at a relative velocity $\vec{v}$, can be considered as a multi-parametric transformations with the parameter $\vec{v}$ and an infinite set of parameters $\vec{u}\left(\vec{r},t\right)$ as well. Analyzing this transformation, we have to stress an essential feature of Faraday's law transformation in comparison with many other relativistic problems. It is related to physical interpretation of the Lorentz transformations, suggested by Einstein in his fundamental paper [22]. Namely, if two inertial frames K and K′ are in relative motion, that each observer in his own rest frame uses his own measuring instrument to determine physical quantities in another frame. However, one can see that this is often not the case for the Faraday induction law: a measuring instrument for e.m.f. (voltmeter) usually represents an inherent part of the moving circuit, and hence, any inertial observer, regardless of his particular velocity with respect to the circuit, uses this voltmeter in his measurements. In principle, one can demand that a moving observer operates with his own voltmeter, included into a circuit by means of sliding contacts. Obviously, the problem, where all inertial observers use in their measurements a single voltmeter, integrated into a circuit, differs from the problem, where each observer uses his own voltmeter. Since the latter case has no practical significance, we analyze the transformation properties of the Faraday induction law, where all observers use the same measuring instrument. Then only in the rest frame of voltmeter K a circular integration over Γ is carried out at the same instant *t*, determining $\varepsilon(t)$. There is no physical meaning to integrate over the circuit at a fixed moment t' of some arbitrary inertial frame K′ to find $\varepsilon'(t')$ (instantaneous e.m.f.), because this value has no simple relation with an actual indication of voltmeter. In order to find such a relationship, an integration time t' should be connected with *t* by means of the Lorentz transformation

$$t' = \gamma\left(t - \vec{v}\vec{r}/c^2\right). \tag{17}$$

for $t = \text{const}$. Here $\vec{v}$ stands for the velocity of voltmeter in the frame K′, and $\vec{r}$ belongs to the closed circuit in the rest frame of voltmeter K. One sees from Eq. (17) that the time moments t' are different for different $\vec{r}$. This rule, where $t' = t'(\vec{r}, \vec{v})$, was named by Cullwick [23] as a rule for computing retarded (advanced) e.m.f. It appears due to the above-mentioned fact: a measuring instrument (voltmeter) is common for all inertial observers.

We present such a detailed explanation of a rule for computing of retarding e.m.f., because in the previous papers by the author on Faraday's law [24, 25], this rule was not explained in detail.

We notice that for an arbitrary moving deforming circuit an e.m.f. cannot be computed analytically. At the same time, in some simple cases it can be easily done. Consider, for example, a problem in Fig. 4. There is a rectangular closed circuit A-B-C-D, where the side AB slides along the sides BC and AD at the constant velocity *v* towards to the resting side CD. We assume that the constant force per unit charge *f* along the axis *y* acts in each point of the circuit. One requires finding an e.m.f. in the circuit for a laboratory observer K, and for an observer

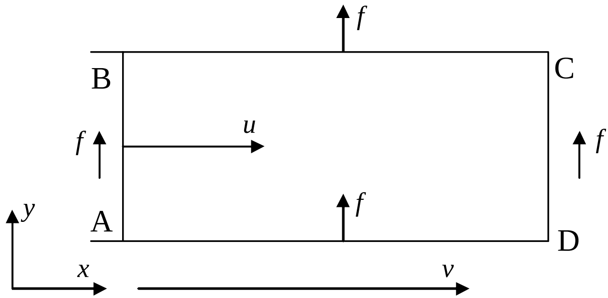

Fig. 4. A rectangular closed circuit A-B-C-D with the moving side AB can be considered as a complex system, consisting of a resting in a laboratory fragment B-C-D-A and the moving segment AB. Hence, the force transformation law is different for these sub-systems. It leads to the appearance of e.m.f. in the circuit, when it moves at the constant velocity *v* along the axis *x*. At the same time, in the laboratory frame K the constant force *f* per unit charge acts in each point on the circuit, and the e.m.f. is vanishing.

in an inertial frame K, wherein the frame K moves at the constant velocity *v* along the axis *x*.

Due to the constancy of the force *f* in the frame K, the e.m.f. in the circuit A-B-C-D is equal to zero. Under transformation of e.m.f. from K to K′ we have a simplified situation, where the retarding e.m.f. coincides with instantaneous e.m.f. in the frame K′: the constant force is orthogonal to the sides AD and BC, and hence, they do not contribute the e.m.f. Further, we can consider a circuit as a complex system, consisting of two sub-systems: resting in the laboratory fragment B-C-D-A, and moving segment A-B. Correspondingly, the force transformation law is different for these sub-systems. One can see that it corresponds to the case 2 of Table 1 ($\vec{v} // \vec{u} \perp \vec{f}$). Hence, omitting the particular calculation, we present the final result: the e.m.f. in the frame K′ is determined as

$$\varepsilon' = \frac{fLuv}{\gamma c^2 \left(1 + \frac{uv}{c^2}\right)}, \tag{18}$$

where L is the length of the segment AB. The same Eq. (18) for a similar problem was obtained in [6], proceeding from the Lorentz force law. Thus, in the frame K, $\varepsilon = 0$, while in the frame K′, ε' is determined by Eq. (18). In comparison with the problems in Figs. 2, 3, we avoid the analysis of any mechanical forces with their short-range fields, and our result certainly demonstrates a contradiction between the relativistic law of force transformation and CP. By the way, such a contradiction could be revealed earlier, since the paper by Marx [26], where he proved the Lorentz-invariance of the flux rule (16) for both fixed and deforming circuits. Indeed, we see that the *rhs* of this equation cannot be of fixed sign: in the product $\vec{B} \cdot d\vec{S} / dt$ (which is not vanishing for deforming circuits and, moreover, is dominating for slowly changed or constant $\vec{B}$) only the second multiplier has the fixed sign for all inertial observers, while the first multiplier has not. It is known that the magnetic field, constituting the components of the tensor of EM field, can change its sign for different observers. Then the product $\vec{B} \cdot d\vec{S} / dt$ is an alternating quantity. Due to the Lorentz-invariance of flux rule, an e.m.f. also represents an alternating quantity. This results means a violation of CP.

We note that a transformation of voltage U (and e.m.f. as well), as the value of fixed sign, should be guided by the law

$$U' = U \cdot F\left(v^2 / c^2\right). \tag{19}$$

That is why we referred to the result (18) as the appearance of Lorentz-invariance of Faraday's law [6]. We underline that this statement signifies the discrepancy between Eqs. (18), (19), but not a violation of the flux rule itself. Moreover, the Lorentz-invariance of flux rule is one of the factors, leading to the mentioned contradictions of Eqs. (18) and (19).

We can add that the problem in Fig. 4 cannot be simply realized in practice, because it is impossible to provide a constancy of f in real conducting circuits. Indeed, the application of constant magnetic field cannot give the equality $f = \text{const}$, when different parts of the circuit have different velocities. The application of constant electric field also does not provide a constant non-zero f, because of re-distribution of conduction electrons in a conductor, aiming to vanish an internal electric field [6]. Unfortunately, such a re-distribution of conduction electrons was not correctly analyzed in the papers [24, 25], and the experimental schemes, proposed in those papers, should be essentially complicated, in order to reveal the non-invariance of e.m.f. experimentally.

Consideration of convenient experimental schemes for test of the force transformation law by Faraday's law will be presented in a separate paper.

5. Conclusion

Thus, considering the general force transformation law (1) in relativity theory we applied it to a complex system, consisting of a number i of sub-systems, moving at the velocities $\vec{u}_i$ in a laboratory. The law of transformation of total force acting on such system represents a multi-parametric transformation, depending on $\vec{v}$ (the velocity of system at the whole) and $\vec{u}_i$. Such a transformation is not ordinary for relativistic physics, and its compatibility with the causality principle has been tested. Such a test has been carried out with the complex mechanical systems and deforming closed circuits, where an e.m.f. can be induced. We concluded that the relativistic law of transformation of force and causality principle comes into a certain contradiction, when an e.m.f. in a closed deforming circuit is calculated. In this connection the alternatives of electromagnetic force should attract more attention (in particular, Weber's force, [27] and references therein). Another alternative is to re-analyze the force transformation law on the basis of covariant ether theories [28]. It will be done in a separate paper.

Acknowledgments

The author warmly thanks Oleg V. Missevitch for helpful discussions on the subject of this paper.

References

[1] A.L. Kholmetskii, "Is the theory of relativity self-consistent?," Apeiron **8** (2001) 74-83.

[2] A.L. Kholmetskii, "Remarks on the correspondence of relativity and causality principles," Apeiron **8** (2001) 74-83.

[3] A.L. Kholmetskii, "Remarks on the causality principle," Apeiron **10** (2003) 135-153.

[4] F. Rohrlich, *Classical Charged Particles*, Addison-Wesley Publishing Co., Inc., 1990).

[5] A. Fettweis, "Can signals truly be faster than light?," Signal Processing **83** (2003) 1583-1596.

[6] A.L. Kholmetskii, "On the non-invariance of the Faraday law of induction," Apeiron **10** (2003) 32-39.

[7] C. Moller, *Theory of Relativity*, Oxford, Clarendon Press (1972).

[8] F.T. Trouton and H.R. Noble, Proc. Royal Soc. **72** (1903) 132.

[9] W. Pauli, *Theory of Relativity*, New York, Pergamon (1958).

[10] V.G. Endean, "Concerning unbalanced electromagnetic couples and forces in relativistic systems," Eur. J. Phys. **14** (1993) 108-114.

[11] P. Cornille, Galilean Electrodynamics **9** (1998) 33-39.

[12] G. Spavieri, L. Nieves, M. Rodrigues and G.T. Gillies, "Test of the Faraday law, the violation of mechanical angular momentum and the electrodynamics controversy." In: "Has the last word been said in classical electrodynamics?" (Ed. A. Chubykalo, V. Onoochin, A. Espinoza and R. Smirnov-Rueda), Pinton Press (2004). p. 255-270.

[13] R.C. Tolman, *Relativity, Thermodynamics and Cosmology*, Oxdord, Clarendon (1946).

[14] A. Harpaz and N. Soker, "Equation of motion of an electric charge," Found. Phys. **33** (2003) 1207.

[15] O.D. Jefimenko, "What is the physical nature of electric and magnetic forces?." In: "Has the last word been said in classical electrodynamics?" (Ed. A. Chubykalo, V. Onoochin, A. Espinoza and R. Smirnov-Rueda), Pinton Press (2004). p. 27-41.

[16] O.D. Jefimenko, "Correct use of Lorentz-Einstein transformation equations for electromagnetic fields," Eur. J. Phys. **18** (1997) 444-447.

[17] O.D. Jefimenko, "On the relativistic invariance of Maxwell's equations," Z. Naturforsch. **54a** (1999) 637-644.

[18] O.D. Jefimenko, "On the experimental proofs of relativistic length contraction and time dilation," Z. Naturforsch. **53a** (1998) 977-982.

[19] A.L. Kholmetskii, "Remarks on momentum and energy flux of a non-radiating electromagnetic field." Annales de la Foundation Louis de Broglie, **29** (2004) 549-582.

[20] Y. Aharonov, P. Pearle, and P. Vaidman, "Comment on "Proposed Aharonov-Casher effect: Another example of an Aharonov-Bohm effect arising from a classical lag," Phys. Rev. **A37** (1988) 4052-4054.

[21] O.D. Jefimenko, "A relativistic paradox seemingly violating conservation of momentum law in electromagnetic systems," Eur. J. Phys. **20** (1999) 39-44.

[22] A. Einstein, "On the electrodynamics of moving bodies," Ann. Phys., **17** (1905) 891-921.

[23] *E.G.* Cullwick, *Electromagnetism and Relativity*, London-New York-Toronto, Longmans, Green and Co (1957).

[24] A.L. Kholmetskii, "The Faraday induction law and field transformations in special relativity," Apeiron **10** (2003) 118-134.

[25] A.L. Kholmetskii, "The Faraday induction law in covariant ether theories," Apeiron **11** (2004) 282-302.

[26] E. Marx, "Covariant formulation of Faraday's law," J. Franklin Inst. **300** (1975) 353-364.

[27] A.K.T. Assis, "Weber's force versus Lorentz's force," Physics Essays **8** (1995) 335-341.

[28] A.L. Kholmetskii, "Covariant ether theories and special relativity," Physica Scripta **67** (2003) 381-387.